

Name: _____

Teacher: _____

**STRATHFIELD GIRLS HIGH
SCHOOL**

**2003
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Write your name and teacher's name at the top of each page
- All necessary working should be shown in every question

Total marks - 84

- Attempt questions 1 - 7
- All questions are of equal value

Exam Requirements

- 1 exam paper
- 1 table of standard integrals (Detach from back of exam)
- 20 sheets of writing paper

This is a trial examination only and does not necessarily reflect the format of the paper in the HSC

Answer each question on a new page

Question 123 (12 marks)	Marks
(a) Find the exact value of $\cos(\sin^{-1} \frac{3}{4})$	2
(b) Find the coordinates of the point P(x, y) which divides the interval AB externally in the ratio 1:3 with A(1, 4) and B(5, 2).	2
(c) Solve the inequality $\frac{x}{2-x} \leq 4$	3
(d) By graphing, or otherwise, simplify $\sin^{-1} x - \cos^{-1}(-x)$	2
(e) Find $\int_0^{\frac{\pi}{16}} \cos^2(4x) dx$	3

End of Question 1

Question 2 (12 marks) START A NEW PAGE

Marks

(a) Find $\frac{d}{dx}(3 \tan^{-1}(2x))$

1

(b) Find $\int \frac{x}{\sqrt{1+x^2}} dx$ using the substitution $u = 1+x^2$

3

(c) It is known that $\log_e x + \sin x = 0$ has a root close to 0.5.

3

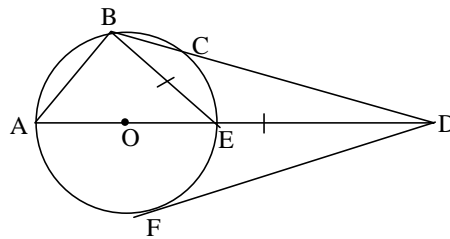
Use one application of Newton's Method to obtain a better approximation of the root. (Round your answer to 3 decimal places)

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(d) In the diagram A, B, C and E are points on the circle with centre O. Diameter AE is produced to D such that BE = DE.

5

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(i) Copy the diagram onto your answer page

(ii) Show that $\angle BEO = 2\angle CDE$

2

(iii) Show that $\angle BAO = 90^\circ - \angle BEO$

4

(iv) Tangent DF is drawn to meet the circle at F. If BE = 5 cm and the circle has a radius 3.5 cm, find the length of DF in exact form.

2

End of Question 2

Question 3 (12 marks) **START A NEW PAGE**

Marks

(a) Two of the roots of $x^3 + qx + r = 0$ are 5 and 1.

(i) Show that the other root is -6 .

3
4

(ii) What are the values of q and r ?

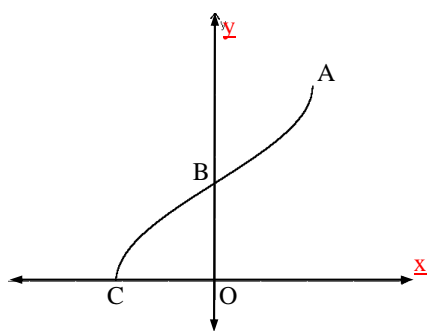
2

(b) Evaluate $\int_0^{1.5} \frac{dx}{\sqrt{9-x^2}}$

2

(c) The diagram below shows the graph of $y = \pi + 2 \sin^{-1}(2x)$

5



(i) Write down the coordinates of the endpoints A and C

2

(ii) Show that the coordinates of B are $(0, \pi)$

4

(iii) Find the equation of the tangent to the curve $y = \pi + 2 \sin^{-1}(2x)$ at the point B.

2

(d) For the equation $\tan \alpha = -\frac{1}{\sqrt{3}}$,

2

(i) Find the general solution, expressing your answer in terms of π .

4

(ii) Use your general solution to find the value of α in the domain

4

$$-\frac{3\pi}{2} < \alpha < -\pi$$

End of Question 3

Question 4 (12 marks) START A NEW PAGE

Marks

(a) A biased coin is found to have a probability of 0.7 of showing a head when tossed.
What is the probability that at least one head occurs in a sequence of 5 tosses of the coin?

2

(b) Consider the function $f(x) = \frac{\log_e x}{x}$

10

(i) State the domain of the function

4

(ii) Find the turning point and determine its nature
(given that $f''(x) = \frac{2\log_e x - 3}{x^3}$)

3

(iii) Find any points of inflexion.

2

(iv) Describe the tendency of y as $x \rightarrow \infty$

4

(v) Sketch the function showing all essential features

2

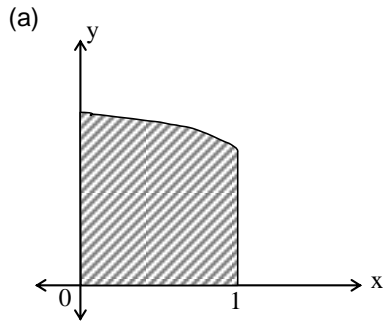
(vi) For what restricted domain would the inverse of $f(x)$ be a function?

4

End of Question 4

Question 5 (12 marks) **START A NEW PAGE**

Marks



The region bounded by the function $y = \frac{2}{\sqrt{x^2 + 3}}$, the x-axis, the y-axis and the line $x = 1$ is shaded in the diagram.

3

Find the exact volume of the solid formed when this region is rotated about the x-axis.

- (b) A particle moves in a straight line so that at time t seconds ($t \geq 0$) its acceleration, a , is given by $a = 4x$ where x is the displacement of the particle from the origin. The particle starts its journey one metre to the right of the origin with a velocity of -2 ms^{-1} . (Assume that the right of the origin is positive)

5

- (i) Show that $v = -2x$ **2**
- (ii) Express x as a function of t . **2**
- (iii) Explain whether or not the particle will ever move to the left of the origin. **4**

- (c) The points P, Q and R on the parabola $x = 2at$, $y = at^2$ have parameters p , q and r respectively. The chord QR is parallel to the tangent at P. **4**

Show that:

- (i) $q + r = 2p$ **2**
- (ii) the line joining P to the midpoint of QR is parallel to the axis of the parabola. **2**

End of Question 5

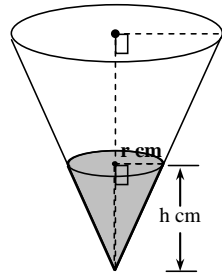
Question 6 (12 marks) START A NEW PAGE

Marks

(a) Find the derivative of $\log_5 x$

2

(b) Rain is falling steadily and is collected in an inverted cone, as shown in the diagram. The volume collected increases at a constant rate of $6\pi \text{ cm}^3/\text{hour}$. If the radius, $r \text{ cm}$, of the cone of the water is always equal to two thirds of its height, $h \text{ cm}$, find the rate, in cm/hour , at which the height is increasing when $h = 3$.



3

$$[V = \frac{1}{3} \pi r^2 h]$$

(c) Find all solutions of $3 \cos 2A - \sin A + 2 = 0$ for $0 \leq A \leq 2\pi$
Answer correct to three decimal places where necessary.

4

(d) By considering the domain and range of $y = |\log_e(3x - 1)|$, sketch the graph of the function

3

End of Question 6

Question 7 (12 marks) START A NEW PAGE

Marks

- (a) A woman borrows \$3000 on the following terms:
- Interest free period for the first 6 months
 - 18%pa interest compounded monthly (at the end of each month before the payment has been made)
 - the borrowed amount is to be paid back in equal monthly payments over 3 years. (Note: payments must be made during the interest free period)

5

(i) Develop an expression for the amount owing after 9 months

4

(ii) Find an expression for the amount owing after n months (for $n > 6$)

~~2~~

(iii) Calculate the amount of each monthly payment

~~2~~

(b)

7
~~2~~

(i) Show that $\sqrt{2} \sin\left[f(x) + \frac{\pi}{4}\right] = \sin f(x) + \cos f(x)$ where $f(x)$ is a function of x .

(ii) Given that $y = e^x \sin x$,

(1) show that $\frac{dy}{dx} = \sqrt{2}e^x \sin\left[x + \frac{\pi}{4}\right]$

2

(2) prove, by mathematical induction, that the n^{th} derivative of y with respect to x (where $n \geq 1$) is given by:

3

$$\frac{d^n y}{dx^n} = \sqrt{2}^n e^x \sin\left[x + \frac{n\pi}{4}\right]$$

$$\left[\text{Hint: } \frac{d^{k+1}y}{dx^{k+1}} = \frac{d}{dx}\left(\frac{d^k y}{dx^k}\right)\right]$$

END OF PAPER

Name: _____

GRAPH FOR QUESTION 24 (b) DETACH AND HAND IN WITH YOUR ANSWERS