Question 1 (15 marks) **Marks**

(a) Sketch the hyperbola $f(x) = \dfrac{x+1}{x-1}$ showing asymptotes and intercepts with the axes. [6]

On separate diagrams, sketch the following graphs, showing clearly any intercepts with the axes and the equations of any asymptotes.

(i) $y = f(-x)$

(ii) $y = |f(x)|$

(iii) $y = f(|x|)$

(iv) $y = e^{f(x)}$

(b) Find $\int \tan^4 x dx$ [2]

(c) (i) Expand $(\sqrt{3}+1)^2$ [7]

(ii) The polynomial equation $x^4 + 4x^3 - 2x^2 - 12x - 3 = 0$ has roots α, β, γ and δ.
Find the polynomial equation whose roots are α+1, β+1, γ+1 and δ+1.

(iii) Hence or otherwise, solve the equation
$x^4 + 4x^3 - 2x^2 - 12x - 3 = 0$

Question 2 (15 marks) Start a new page **Marks**

(a) Use the substitution $x = \sin\theta$ to find $\int \frac{1}{(1-x^2)^{\frac{3}{2}}} dx$ [3]

(b) Show that $\int_0^{\frac{\pi}{6}} x \cos x dx = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$ [3]

(c) Find $\int \frac{1}{2+\sqrt{x}} dx$ by using the substitution $u = \sqrt{x}$ [3]

(d) (i) Show that $\int_0^1 \frac{1}{(5x+3)(x+1)} dx = \frac{1}{2} \log \frac{4}{3}$ [3]

(ii) Hence find $\int_0^{\frac{\pi}{2}} \frac{1}{4\sin x - \cos x + 4} dx$ [3]

using the substitution $t = \tan\frac{x}{2}$

Question 3 (15 marks) Start a new page **Marks**

(a) (i) Write down the expansion of $(1+i\alpha)^4$ in ascending powers of α [3]

(ii) Hence find the value/s of α such that $(1+i\alpha)^4$ is real.

(b) Find all the complex numbers $z = a + ib$, where a and b are real, [3]
such that $|z|^2 - 7 = 2i(z+2)$

(c) Given that $z = \dfrac{3+i}{1-i}$, find $z + \dfrac{1}{z}$ in the form $a + ib$, [3]
where a and b are real.

(d) Solve for z over the complex field: $z^2 - 6z + 25 = 0$ [3]

(e) The locus of a complex number z is defined by the equation [3]
$\arg(z+1) = \dfrac{\pi}{4}$.

(i) Sketch the locus of z on an Argand diagram

(ii) Find the least value of $|z|$

Question 4 (15 marks) Start a new page **Marks**

(a) The equation $(\sin^2\theta)z^2 - 2(\sin\theta\cos\theta)z + 1 = 0$, where $0 < \theta < \frac{\pi}{2}$ [4]
has roots α and β.

(i) Show that the roots of the equation are $\cot\theta + i$ and $\cot\theta - i$

(ii) Rewrite the roots α and β and apply De Moivre's theorem to show that $\alpha^n + \beta^n = \dfrac{2\cos n\theta}{\sin^n\theta}$

(b) Consider the function $y = e^{-2x}\tan x$ for $0 \le x < \frac{\pi}{2}$ [5]

(i) Show that $\dfrac{dy}{dx} = e^{-2x}(1 - \tan x)^2$

(ii) Sketch the graph of the function showing critical points, the equation of the asymptote and the coordinates of the stationary point.

(c) (i) Sketch the circle $(x-1)^2 + y^2 = 4$ and shade the region where $(x-1)^2 + y^2 \le 4$ and $x \ge 0$ are both satisfied. [6]

(ii) If the shaded region forms the base of a solid with every cross section perpendicular to the x axis forming a square, one side of which lies in the base, find the volume of the solid.

Question 5 (15 marks) Start a new page **Marks**

(a) (i) If $I_n = \int_0^1 x^n e^x dx$ where *n* is a positive integer, show that [5]

$I_{n+1} = e - (n+1)I_n$.

(ii) Hence evaluate $\int_0^1 t^3 e^t dt$

(b) (i) Sketch the part of the curve $y = x^{2/3}$ for $0 \le x \le 8$ [5]

(ii) The region enclosed by the curve, the ordinate $x = 8$
and the *x*-axis is rotated about the line $x = 8$.
Using cylindrical shells find the volume generated.

(c) (i) Show that $\sin x + \sin 3x = 2\sin 2x \cos x$ [5]

(ii) Hence or otherwise, find all solutions of
$\sin x + \sin 2x + \sin 3x = 0$, for $0 \le x \le 2\pi$

Question 6 (15 marks) Start a new page **Marks**

(a) (i) Find the equation of the normal to $x^2 - xy - y^2 = 1$ at the point (2 , 1) [4]

(ii) Find the coordinates of the point where the normal meets the curve again.

(b) The level of water in a river changes with the tide. The top of an old post placed upright in the river is 0.8m below the water at high tide and 0.2m above the water at low tide. [4]

High tide occurred at 6.30am and low tide occurred at 12.35pm on the same day that the measurements were taken. Assuming that the motion of the tide is simple harmonic, when was there at least 0.5m of water above the top of the post?

Express your answer correct to the nearest minute.

(c) Show that the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by [4]

Area= πab

(d) The polynomial $P(x) = x^3 + cx + d$ has zeros α, β and 2(α - β) [3]

(i) Show that $c = -13\alpha^2$

(ii) Show that $d = 12\alpha^3$

Question 7 (15 marks) Start a new page **Marks**

(a) For the ellipse $\frac{x^2}{36}+\frac{(y-1)^2}{16}=1$, find the [5]

(i) coordinates of the foci
(ii) equations of the directrices and
(iii) draw a neat sketch of the ellipse.

(b) Let the point P(*x* , *y*) be on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ [4]

(i) State the coordinates of the foci S and S' and the equations of the directrices.
(ii) Using the ratio of the distance from P to the focus and to the directrix, find the length SP+PS'

(c) Find the equations of the tangent and normal to the curve whose parametric equations are $x = 2\cos\theta$ and $y = 3\sin\theta$ at $\theta=\frac{\pi}{3}$ [6]

Question 8 (15 marks) Start a new page **Marks**

(a) (i) Use the substitution $u = \pi - x$ to show that for any function $f(x)$, $\int_0^{\pi} x.f(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ [8]

(ii) Hence show that $\int_0^{\pi} \frac{x\sin^3 x}{1+\cos^2 x}dx = \frac{\pi}{2}(\pi - 2)$

(b) The complex numbers z_1 and z_2 have modulus 1 and arguments α_1 and α_2 respectively, where $0 < \alpha_1 < \alpha_2 < \frac{\pi}{2}$ [7]

(i) Draw a diagram showing all the given information

(ii) Show that $\arg(z_1 - z_2) = \frac{1}{2}(\alpha_1 + \alpha_2 - \pi)$

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