2 UNIT MATHEMATICS FORM VI

Time allowed: 3 hours

Exam date: 7th August, 1995

Instructions:

There will be five minutes reading time.
All questions may be attempted.
All questions are of equal value.
Part marks are shown in boxes in the left margin.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.

Collection:

Each question will be collected separately.
Start each question in a new answer booklet.
If you use a second booklet for a question, place it inside the first. Don't staple.
Write your candidate number on each answer booklet.

QUESTION ONE (Start a new answer booklet)

Marks

[1] (a) Evaluate $\dfrac{\sqrt{17{\cdot}62 + 3{\cdot}4}}{9{\cdot}8}$ correct to two decimal places.

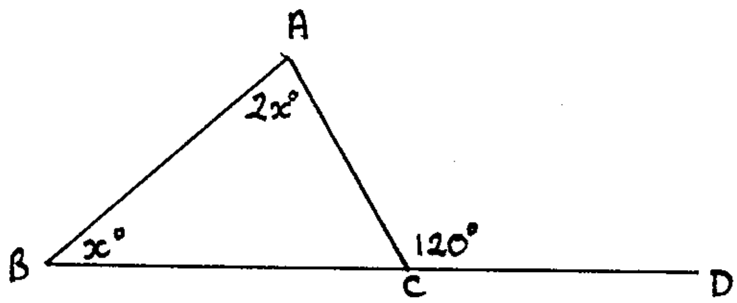
[1] (b) Solve $3(x-1) > 2x$.

[2] (c) Solve $|x-4| = 3$.

[2] (d) Express $\dfrac{2}{\sqrt{7}-1}$ with a rational denominator in simplest form.

[1] (e) Find the area of a rhombus with diagonals of length 10 cm and 15 cm.

[2] (f)



Find the value of x with reasons.

[3] (g) (i) Solve $x^2 + 2x - 8 = 0$.

(ii) Solve $x^2 + 2x - 8 > 0$.

QUESTION TWO (Start a new answer booklet)

Marks

[1] (a) On a number plane, mark the points $L(-2,-1)$, $M(0,3)$ and $N(4,0)$.

[1] (b) Find the gradient of MN.

[1] (c) Show that the equation of MN is $3x + 4y - 12 = 0$.

[1] (d) Show algebraically that the midpoint of LN is $(1, -\frac{1}{2})$. Call this point D.

[2] (e) Find the point K such that D is the midpoint of MK.

[2] (f) What type of quadrilateral is $KLMN$? Give a reason for your answer.

[2] (g) Find the perpendicular distance from L to MN.

[2] (h) Find the area of $KLMN$.

(Exam continues next page ...)

QUESTION THREE (Start a new answer booklet)

Marks

[5] (a) Differentiate:

(i) e^{4x},

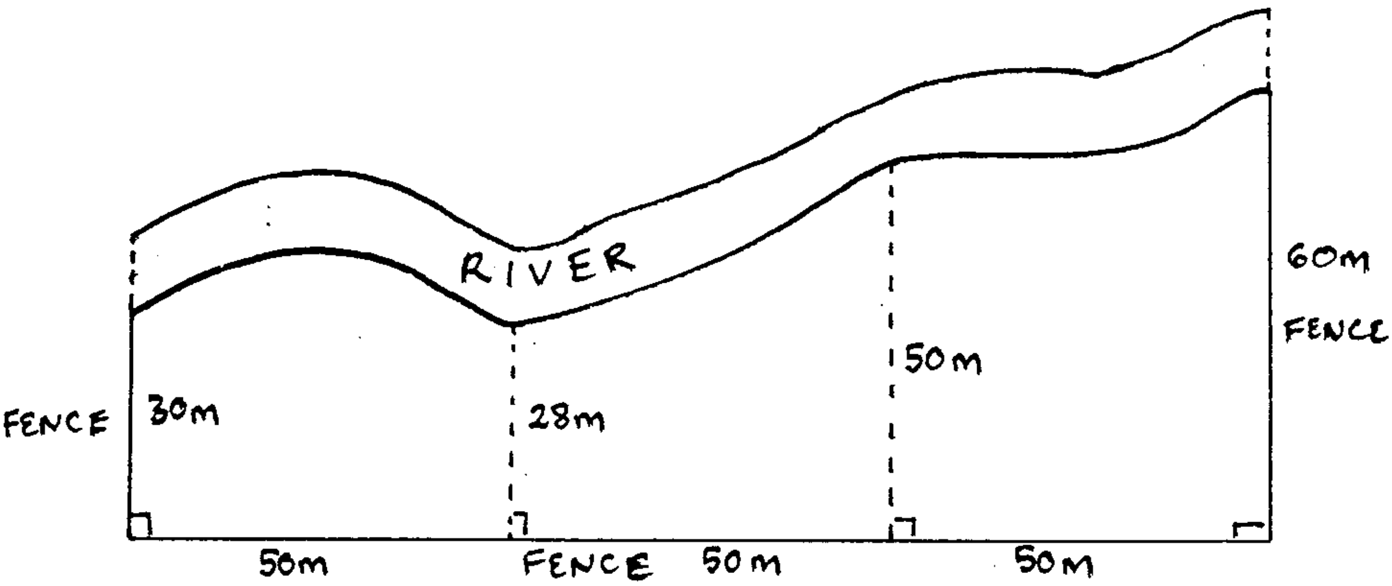
(ii) $\dfrac{1}{\sqrt{x}}$,

(iii) $x\ln(x+1)$.

[1] (b) Find $\int \sec^2 3x\,dx$.

[3] (c) Evaluate $\int_0^1 (3-2x)^4\,dx$.

[3] (d)

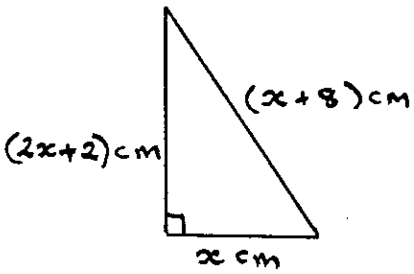


The diagram above is of a field bounded by a river and three fences. A farmer wishes to calculate the area of the field and has obtained the measurements above. Estimate the area of the field using the trapezoidal rule with four function values.

QUESTION FOUR (Start a new answer booklet)

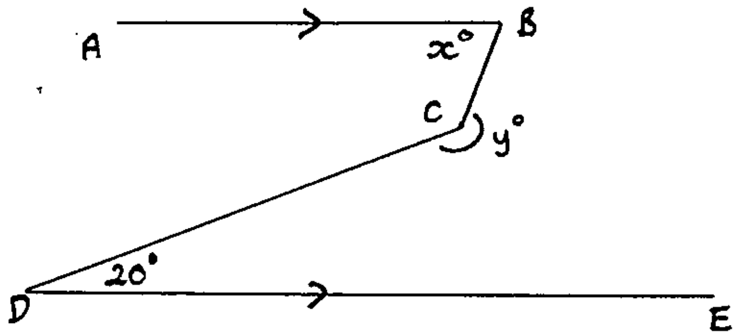
Marks

3 (a)



Find the value of x in the above diagram.

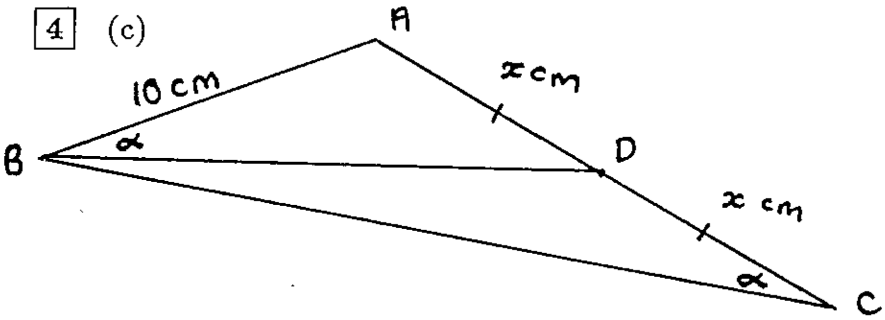
4 (b)



(i) Copy the above diagram into your answer booklet.

(ii) Prove that $y - x = 160$.

4 (c)



In the diagram, $AB = 10$cm, $AD = DC = x$cm and $\angle ABD = \angle BCD$.

(i) Prove $\triangle ABD \;|||\; \triangle ACB$.

(ii) Hence find the exact value of x.

1 (d) Write down an inequality which represents the locus of all points which lie less than 3 units from the origin.

(Exam continues next page ...)

QUESTION FIVE (Start a new answer booklet)

Marks

[4] (a) If α and β are the roots of the quadratic equation $3x^2 - 4x - 1 = 0$, find the value of:

(i) $\alpha + \beta$,

(ii) $\alpha\beta$,

(iii) $\alpha^2\beta^3 + \alpha^3\beta^2$,

(iv) $\alpha^2 + \beta^2$.

[3] (b) A factory produces 1000 football jerseys in its first week of operation and each week its production level is 10% greater than that of the previous week.

(i) How many jerseys will it produce in its 2nd and 3rd weeks of production?

(ii) Find the number of jerseys produced in its 60th week of production? Answer correct to the nearest hundred.

[3] (c) Find in exact form the sum of the first 20 terms of the following arithmetic sequence:

$$\ln 8, \ln 16, \ln 32, \ldots.$$

[2] (d) Write down the domain of $y = \sqrt{1 - x}$.

QUESTION SIX (Start a new answer booklet)

Marks

[5] (a) Consider the parabola $(y - 4)^2 = 8(x + 2)$.

(i) Write down the co-ordinates of the vertex.

(ii) Find the focus and the directrix.

(iii) Find the y-intercepts.

(iv) Sketch the curve showing clearly all the above information.

[2] (b) Water is flowing into a bath and the depth of the water D cm, at time t min, is given by:

$$D = 20 + \frac{t}{2} + \frac{t^3}{6}.$$

Find the rate at which the depth is increasing after 4 minutes.

[5] (c) The number, N, of bacteria in a colony after t minutes is given by $N = 10\,000e^{0\cdot 06t}$.

(i) Find the number of bacteria after 5 minutes, correct to the nearest hundred.

(ii) Find the time, in minutes, required for the initial population to double, correct to one decimal place.

(Exam continues overleaf ...)

QUESTION SEVEN (Start a new answer booklet)

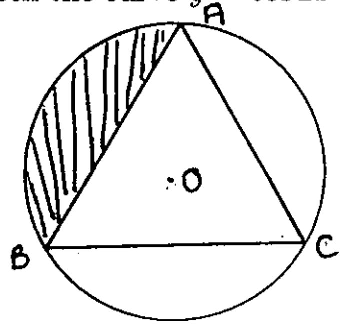
Marks

3 (a) Solve $\sin^2 x - \frac{1}{2} = 0$ for $0 \leq x \leq 2\pi$.

4 (b) (i) Sketch $y = \cos 2x$ for $0 \leq x \leq 2\pi$.

(ii) Find the area between the curve $y = \cos 2x$ and the x-axis from $x = 0$ to $x = \pi$.

3 (c)

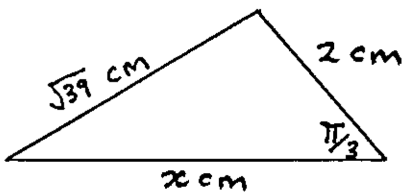


An equilateral triangle is inscribed in a circle of radius 1 unit.

(i) Explain why $\angle AOB = \dfrac{2\pi}{3}$.

(ii) Hence show that the shaded area is equal to $\dfrac{4\pi - 3\sqrt{3}}{12}$ u^2.

2 (d)



(i) Using the cosine rule, show that $x^2 - 2x - 35 = 0$.

(ii) Hence find the perimeter of the triangle.

QUESTION EIGHT (Start a new answer booklet)

Marks

8 (a) Consider the curve $y = x^3 - x^2 - 5x + 1$.

(i) Find any turning points and determine their nature.

(ii) Find any points of inflexion.

(iii) Sketch the curve for $-2 \leq x \leq 2$. You need not find the x-intercepts.

(iv) For what values of x is the curve decreasing but concave up?

4 (b) (i) Show that $\sin^2 x \cos x = \cos x - \cos^3 x$.

(ii) Hence show that $\dfrac{d}{dx}(\sin x - \frac{1}{3}\sin^3 x) = \cos^3 x$.

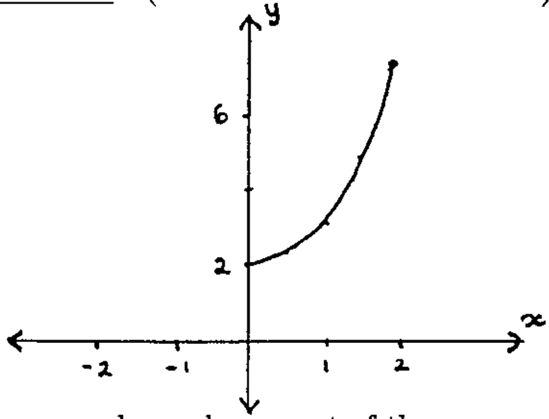
(iii) Hence find $\displaystyle\int 3\cos^3 x \, dx$.

(Exam continues next page ...)

QUESTION NINE (Start a new answer booklet)

Marks

4 (a)



The diagram above shows part of the curve $y = e^x + e^{-x}$.

(i) Show that $y = e^x + e^{-x}$ is an even function.

(ii) Copy the diagram onto your answer booklet and complete the curve for $-2 \le x \le 0$.

(iii) The region bounded by the curve $y = e^x + e^{-x}$, the x-axis and the lines $x = -2$ and $x = 2$, is rotated about the x-axis. Find the volume of the resulting solid of revolution, correct to one decimal place.

5 (b) A couple plan to buy a home and they wish to save a deposit of \$40 000 over five years. They agree to invest a fixed amount of money at the beginning of each month during this time. Interest is at 12% per annum compounded monthly.

(i) Let \$$P$ be the monthly investment. Show that the total investment \$$A$ after five years is given by

$$A = P(1{\cdot}01 + 1{\cdot}01^2 + \cdots + 1{\cdot}01^{60}).$$

(ii) Find the amount \$$P$ needed to be deposited each month to reach their goal. Answer correct to the nearest dollar.

3 (c) For what values of k does the quadratic equation

$$kx^2 - 4kx - k + 5 = 0,$$

have real, distinct roots?

(Exam continues overleaf ...)

QUESTION TEN (Start a new answer booklet)

Marks

2 (a) Given that $p+q=1$, prove that $(p^2-q^2)^2+pq=p^3+q^3$.

5 (b) An open cylindrical can is to have a surface area of $20\pi\ \text{cm}^2$. (The can has no lid.)

(i) Let r centimetres be the radius of the can and h centimetres be its height. Show that $h=\dfrac{20-r^2}{2r}$.

(ii) Hence, show that the total volume of the can is given by $V=10\pi r-\frac{1}{2}\pi r^3$.

(iii) Show that the maximum volume is obtained when the height of the can equals its radius.

5 (c) A particle is moving in a straight line with acceleration at time t seconds given by

$$\ddot{x}=\frac{-1}{(1+t)^2}.$$

Initially, the particle is at the origin and is moving with a velocity of -1 m/s.

(i) Find in exact form the displacement of the particle after 3 seconds.

(ii) Another particle starts with a displacement of 10 metres and moves with a constant velocity of -2 m/s. When do the particles collide?

FMW

The following list of standard integrals may be used:

$$\int x^n\,dx = \frac{1}{n+1}\,x^{n+1},\quad n \neq -1;\quad x \neq 0, \text{ if } n < 0,$$

$$\int \frac{1}{x}\,dx = \ln|x|,\quad x \neq 0,$$

$$\int e^{ax}\,dx = \frac{1}{a}\,e^{ax},\quad a \neq 0,$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0,$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0,$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0,$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0,$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0,$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\quad -a < x < a,$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left|x + \sqrt{x^2 - a^2}\right|,\quad |x| > |a|,$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right).$$

3) (a) $LHS = (p^2 - q^2)^2 + pq$

$= ((p-q)(p+q))^2 + pq$

$= p^2 - 2pq + q^2 + pq$

$= p^2 - pq + q^2$

$RHS = p^3 + q^3$

$= (p+q)(p^2 - pq + q^2)$

$= p^2 - pq + q^3$

$= LHS$

(b) (i) $\pi r^2 + 2\pi r h = 20\pi$

$2\pi r h = 20\pi - \pi r^2$

$h = \dfrac{20\pi - \pi r^2}{2\pi r}$

$= \dfrac{20 - r^2}{2r}$ as required

(ii) $V = \pi r^2 h$

$= \pi r^2 \left(\dfrac{20 - r^2}{2r}\right)$

$= \pi r \left(10 - \dfrac{r^2}{2}\right)$

$= 10\pi r - \frac{1}{2}\pi r^3$ as required

(iii) $V' = 10\pi - \frac{3}{2}\pi r^2$

$V'' = -3\pi r$

if $V' = 0$, $10\pi - \frac{3}{2}\pi r^2 = 0$

$r^2 = \dfrac{20}{3}$

$r = \pm\sqrt{\dfrac{20}{3}}$ or $\pm\frac{2}{3}\sqrt{15}$

at $r = \sqrt{\frac{20}{3}}$, $V'' = -3\pi\sqrt{\frac{20}{3}} < 0$, ∴ max volume here.

at $r = \sqrt{\frac{20}{3}}$, $h = \dfrac{20 - \frac{20}{3}}{2 \times \sqrt{\frac{20}{3}}}$

$= \dfrac{60 - 20}{3} \times \dfrac{\sqrt{3}}{2\sqrt{20}}$

$= \dfrac{40}{\sqrt{3}} \times \dfrac{1}{2\sqrt{20}}$

$= \sqrt{\frac{20}{3}} = r$ as required

(c)(i) $\ddot{x} = \dfrac{-1}{(1+t)^2}$

$= -1(1+t)^{-2}$

$\dot{x} = (1+t)^{-1} + c$

$= \dfrac{1}{1+t} + c$

at $t = 0$, $\dot{x} = -1$, ∴ $\dfrac{1}{1+0} + c = -1$ $\quad 1 + c = -1$, $c = -2$

∴ $\dot{x} = \dfrac{1}{1+t} - 2$

$x = \ln(1+t) - 2t + K$

at $t = 0$, $x = 0$, ∴ $K = 0$

hence $x = \ln(1+t) - 2t$

at $t = 3$, $x = \ln(1+3) - 2(3)$

$= (\ln 4 - 6)$ m (or $2\ln 2 - 6$)

(ii) for the 2nd particle,

$\dot{x} = -2$

$x = -2t + c$

when $t = 0$, $x = 10$, ∴ $c = 10$

hence $x = -2t + 10$

if the particles collide,

$\ln(1+t) - 2t = -2t + 10$

$\ln(1+t) = 10$

$1 + t = e^{10}$

$t = (e^{10} - 1)$ min

③ (a) (i) $y = e^{4x}$

$y' = 4e^{4x}$

(ii) $y = \frac{1}{\sqrt{x}}$

$= x^{-1/2}$

$\left.\begin{aligned} y' &= -\frac{1}{2}x^{-3/2} \\ &= \frac{-1}{2\sqrt{x^3}} \end{aligned}\right\}$ (either)

(iii) $y = x \ln(x+1)$

$y' = x \times \frac{1}{x+1} + 1 \times \ln(x+1)$

$= \frac{x}{x+1} + \ln(x+1)$

(b) $\int \sec^2 3x \, dx = \frac{1}{3}\tan 3x + C$ (ignore absence of C)

(c) $\int_0^1 (3-2x)^4 dx = \left[\frac{(3-2x)^5}{-10}\right]_0^1$ (-1 each error)

$= -\frac{1}{10} + \frac{243}{10}$

$= 24\frac{1}{5}$ (or 24.2)

(d) $A \doteqdot \frac{50}{2}\left[30 + 2(28) + 2(50) + 60\right]$ (-1 each error)

$= 6150 \text{ m}^2$

④ (a) $x^2 + (2x+2)^2 = (x+8)^2$ (pythag. thm) (ignore absence of reason)

$x^2 + 4x^2 + 8x + 4 = x^2 + 16x + 64$

$4x^2 - 8x - 60 = 0$

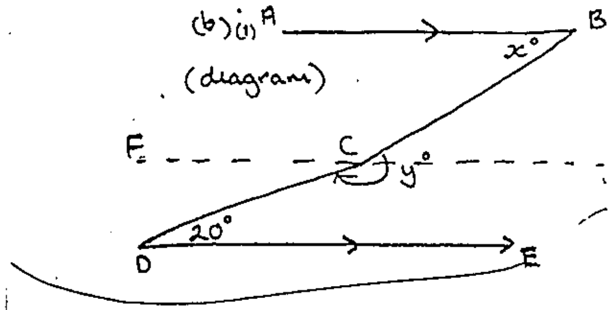
$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5$ or -3

x is positive $\therefore$ choose $x = 5$

(b) (i)



(diagram)

(ii) construct FC ∥ AB (ignore absence of FC ∥ DE (both ∥ AB))

$\therefore \angle FCD = 20°$ (alt. ∠'s FC ∥ [illegible]

$\angle FCB + x° = 180°$ (coint. ∠'s, FC ∥ AB)

$\angle FCB = 180° - x°$

$\therefore y + 180 - x + 20 = 360$ (∠ sum revolut[illegible]

$y - x = 160$

(as required)

(c) (i) in Δ's ABD + ACB

$\left.\begin{aligned} &\angle A \text{ is common} \\ &\angle ABD = \angle ACB \text{ (given)} \end{aligned}\right\}$ both

$\therefore \Delta ABD \;|||\; \Delta ACB$ (A.A.)

(ii) $\therefore \frac{2x}{10} = \frac{10}{x}$ (matching sides of ||| Δ's) (ignore reason but grizzle)

$2x^2 = 100$

$x^2 = 50$

$\left.\begin{aligned} x &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}\right\}$ either

(d) $x^2 + y^2 < 9$

10 Questions, 12 marks each

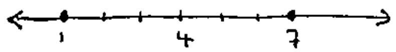
total 120

① (a) 0.47 (2 d.p)

(b) $3x - 3 > 2x$

$x > 3$

(c) distance from x to 4 = 3 $\quad x = 1$ or $x = 7$



(d) $\frac{2}{\sqrt{7}-1} \times \frac{\sqrt{7}+1}{\sqrt{7}+1} = \frac{2(\sqrt{7}+1)}{6}$

$= \frac{\sqrt{7}+1}{3}$

(e) $A = \frac{1}{2} \times 10 \times 15$

$= 75 \text{ cm}^2$ (ignore units but grizzle)

(f) $3x = 120$ (ext. ∠ of Δ thm) (must have reason)

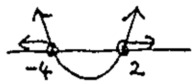
$x = 40$

(g)(i) $x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

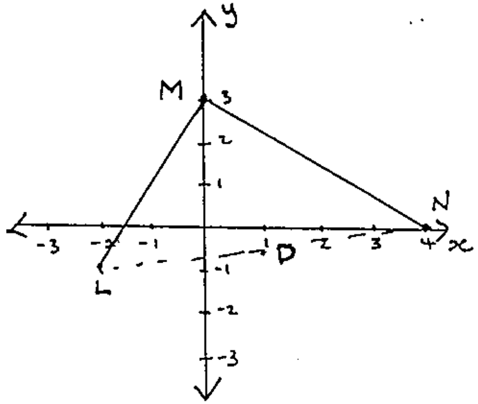
$x = -4$ or $x = 2$ (must have both)

(ii)



$x < -4$ or $x > 2$

② (a)



for M, N, L placed correctly

(b) $m_{MN} = -\frac{3}{4}$

(c) $y = -\frac{3}{4}x + 3$ (or $y - 3 = -\frac{3}{4}(x - 0)$)

$4y = -3x + 12$

$3x + 4y - 12 = 0$ as required.

(d) $D = \left(\frac{-2+4}{2}, \frac{-1+0}{2}\right)$

$= (1, -\frac{1}{2})$ as required.

(e) $\frac{x+0}{2} = 1 \qquad \frac{y+3}{2} = -\frac{1}{2}$

$x = 2 \qquad y + 3 = -1$

$y = -4$ } working both x & y

∴ K is $(2, -4)$ (must have as a point)

(f) parallelogram (diagonals bisect each other)

(g) $d = \left| \frac{3(-2) + 4(-1) - 12}{\sqrt{3^2+4^2}} \right|$

$= \frac{22}{5}$ units

(h) $d_{MN} = \sqrt{(-4)^2 + (3)^2}$

$= 5$

$\therefore A = \frac{22}{5} \times 5$

$= 22 \ u^2$

⑤ (a)(i) $\alpha+\beta = \frac{4}{3}$ (ii) $\alpha\beta = -\frac{1}{3}$ (practice NSE here)

(iii) $\alpha^2\beta^3+\alpha^3\beta^2 = \alpha^2\beta^2(\beta+\alpha)$

$= (-\frac{1}{3})^2(\frac{4}{3})$

$= \frac{4}{27}$

(iv) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (\frac{4}{3})^2 + 2\times\frac{1}{3}$

$= 2\frac{4}{9}$

(b) (i) $u_2 = 1000(1.1)$

$= 1100$

$u_3 = 1100(1.1)$

$= 1210$ (both)

(ii) GP, $a = 1000$, $r = 1.1$, $n = 60$

$u_{60} = 1000(1.1)^{59}$

$= 276800$ (nearest hundred)

(must have correct approx)

(c) $\ln 8 = 3\ln 2$

$\ln 16 = 4\ln 2$

$\ln 32 = 5\ln 2$

A.P with $a = 3\ln 2$

$d = \ln 2$

$n = 20$

(some explanation required)

$S_{20} = \frac{20}{2}\left[6\ln 2 + 19\ln 2\right]$

$= 250\ln 2$

(award ✓/x for $\ln 8 + 247\ln 2$ or other combinations)

(d) $1-x \geq 0$

$x \leq 1$

(award ✓✓ for answer only)

⑥ a) (i) $(-2, 4)$

(ii) here, $4a = 8$

$a = 2$

∴ focus is at $(0,4)$

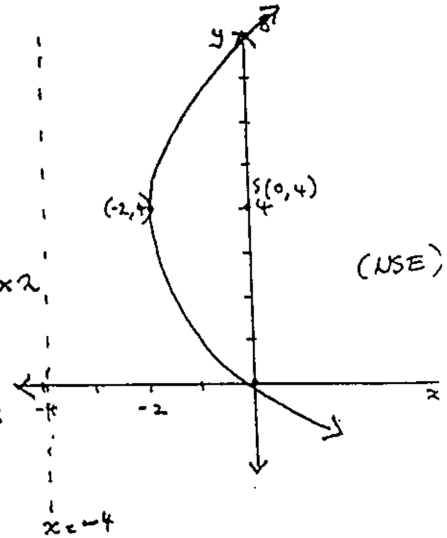
& directrix is at $x = -4$

(iii) when $x = 0$, $(y-4)^2 = 8\times 2$

$y^2 - 8y = 0$

$y(y-8) = 0$

$y = 0$ or $y = 8$



(NSE)

(b) $\frac{dD}{dt} = \frac{1}{2} + \frac{3t^2}{6}$

$= \frac{1}{2} + \frac{t^2}{2}$

at $t = 4$, $\frac{dD}{dt} = \frac{1}{2} + \frac{4^2}{2}$

$= 8\frac{1}{2}$ cm/min

(must have correct units)

(c) (i) $N = 10\,000\,e^{0.06\times 5}$

$= 13\,500$ (nearest hundred)

(ii) $20\,000 = 10\,000\,e^{0.06t}$

$2 = e^{0.06t}$ } either

$\ln 2 = \ln e^{0.06t}$

$t = \frac{\ln 2}{0.06}$ } either

$= 11.552\ldots$

$= 11.6$ min (1 d.p)

(penalize incorrect approx. once only)

(b)(i) $\sin^2 x \cos x = (1-\cos^2 x)\cos x$

$= \cos x - \cos^3 x$ as required

(ii) $\frac{d}{dx}\left(\sin x - \frac{1}{3}\sin^3 x\right) = \cos x - \frac{d}{dx}\left(\frac{1}{3}(\sin x)^3\right)$

$= \cos x - \sin^2 x \cos x$

$= \cos x - (\cos x - \cos^3 x)$

$= \cos^3 x$ as required.

(iii) $\therefore \int 3\cos^3 x\, dx = 3\sin x - \sin^3 x + C$

(ignore C but grizzle)

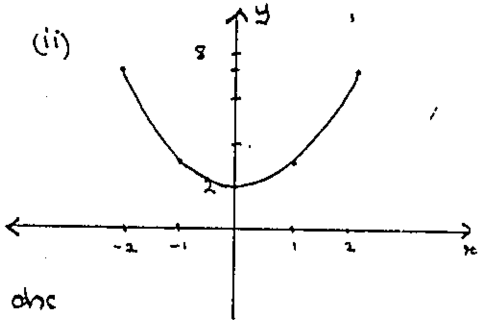
9 (a) (i) $f(-x) = e^{-x} + e^{-(-x)}$

$= e^{-x} + e^{x}$

$= f(x)$

$\therefore$ the function is even.

(ii)



(iii) $V = 2\pi \int_0^2 (e^x + e^{-x})^2\, dx$

$= 2\pi \int_0^2 (e^{2x} + 2 + e^{-2x})\, dx$

$= 2\pi \left[\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x}\right]_0^2$

$= 2\pi \left[\frac{1}{2}e^4 + 4 - \frac{1}{2}e^{-4} - \frac{1}{2} + \frac{1}{2}\right]$

$= 196.6\ u^3$ (1 d.p)

(b) (i) 12% p.a = 1% per month

first investment $\rightarrow P(1.01)^{60}$

2nd investment $\rightarrow P(1.01)^{59}$

this pattern continues

(some explanation required)

$\therefore$ total investment $= P(1.01)^{60} + P(1.01)^{59} + \ldots + P(1.01)$

$= P(1.01 + 1.01^2 + \ldots + 1.01^{60})$

as required

(ii) now, they wish to save \$40,000

$\therefore 40{,}000 = P(1.01 + 1.01^2 + \ldots + 1.01^{60})$

GP with $a = 1.01$, $n = 60$, $r = 1.01$

$S_{60} = \frac{1.01(1.01^{60}-1)}{1.01-1}$

$= \frac{1.01(1.01^{60}-1)}{0.01}$

$\therefore 40\,000 = P \times \left[\frac{1.01(1.01^{60}-1)}{0.01}\right]$

$P = \frac{0.01 \times 40\,000}{1.01(1.01^{60}-1)}$

$= \$485$ (nearest dollar)

(c) the quadratic will have real, distinct roots if $\Delta > 0$

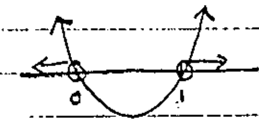
ie: $(-4k)^2 - 4k(-k+5) > 0$

$16k^2 + 4k^2 - 20k > 0$

$20k^2 - 20k > 0$

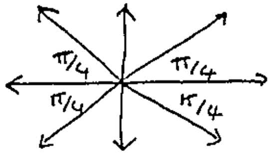
$20k(k-1) > 0$

$k < 0$ or $k > 1$



⑦ (a) $\sin^2 x = \frac{1}{2}$

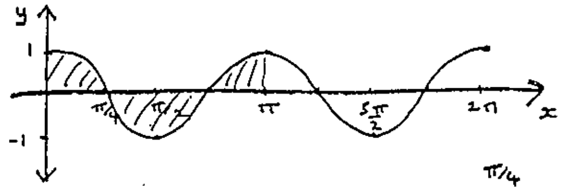
$\sin x = \pm\frac{1}{\sqrt{2}}$



(award ✓✓× for if correctly solve $\sin x = \frac{1}{\sqrt{2}}$)

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(b) (i)



(ii) Using symmetry, $A = 4\int_0^{\pi/4} \cos 2x \, dx$ (or equivalent)

$= 4\left[\frac{1}{2}\sin 2x\right]_0^{\pi/4}$

$= 2\sin\pi/2$

$= 2 \; u^2$

(c) (i) $\angle AOB = \angle BOC = \angle AOC$ ($\angle$'s opp. = sides)

$\therefore 3\angle AOB = 2\pi$ ($\angle$ sum revolution) (or equivalent)

$\angle AOB = \frac{2\pi}{3}$ as required.

(ii) $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}(1)\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$

$= \frac{1}{2}\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

$= \frac{2\pi}{6} - \frac{\sqrt{3}}{4}$

$= \frac{4\pi - 3\sqrt{3}}{12} \; u^2$ as required.

(d) (i) $39 = x^2 + 2^2 - 2\times 2\times x\times\cos\frac{\pi}{3}$

$39 = x^2 + 4 - 2x$

$x^2 - 2x - 35 = 0$ as required

(ii) $(x-7)(x+5) = 0$

$x = 7$ as $x > 0$

& $P = (\sqrt{39} + 9)$ cm

⑧ (a) $y = x^3 - x^2 - 5x + 1$

$\frac{dy}{dx} = 3x^2 - 2x - 5$, $\frac{d^2y}{dx^2} = 6x - 2$

(i) if $\frac{dy}{dx} = 0$, $3x^2 - 2x - 5 = 0$

$(3x-5)(x+1) = 0$

$x = \frac{5}{3}$ or $x = -1$

$y = -5\frac{13}{27}$ $y = 4$ (OK to read off graph)

$\frac{d^2y}{dx^2} = 8$ $\frac{d^2y}{dx^2} = -8$

$\therefore \left(\frac{5}{3}, -5\frac{13}{27}\right)$ is a min. turning pt

& $(-1, 4)$ is a max. turning pt

(ii) if $\frac{d^2y}{dx^2} = 0$ check sign change:

$6x - 2 = 0$

$6x = 2$

$x = \frac{1}{3}$

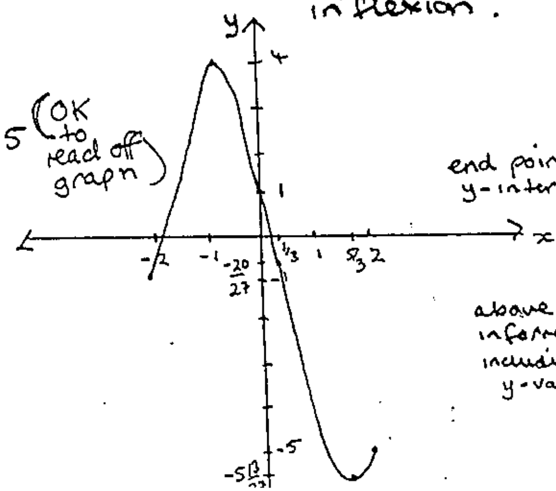
$y = -\frac{20}{27}$

x	0	1/3	1
y''	-2	0	4
	−	0	+

$\therefore \left(\frac{1}{3}, -\frac{20}{27}\right)$ is a pt of inflexion.

(iii) if $x = -2$, $y = -1$

if $x = 2$, $y = -5$ (OK to read off graph)



end points y-interce

above informa includin y-valu

(iv) $\frac{1}{3} < x < \frac{5}{3}$