1994 WESTERN REGION 4U TRIAL

Question 1

a. Use the method of integration by parts to determine

$$\int x^2 e^x dx$$

b. i. If $\dfrac{7x^2-3x+2}{(x-2)(x^2+x+2)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+x+2}$

find the values of A, B and C.

ii. Hence determine

$$\int \frac{7x^2-3x+2}{(x-2)(x^2+x+2)} \cdot dx$$

c. i. Derive a reduction formula for

$$I_n = \int \tan^n\theta . d\theta$$

ii. By using your answer from (i) or otherwise, evaluate

$$\int_0^{\frac{\pi}{4}} \tan^6\theta . d\theta$$

Question 2

a. i. If the polynomial $P(x)$ has a zero of multiplicity n at $x = a$, show that its derivative $P'(x)$ will have a zero of multiplicity $n-1$ at $x = a$.

ii. Given that $P(x) = x^4 + 2x^3 - 12x^2 + 14x - 5 = 0$ has a triple root, find all its real roots.

b. When a polynomial $P(x)$ is divided by $(x-3)$ the remainder is 5, and when it is divided by $(x-4)$ the remainder is 9.

Find the remainder when $P(x)$ is divided by $(x-4)(x-3)$

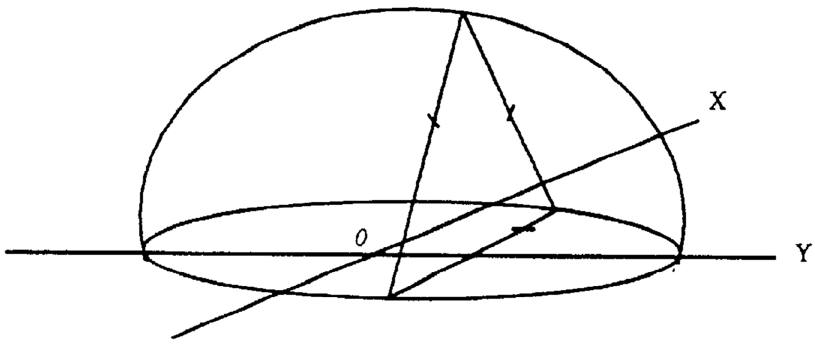
c. By applying DeMoivre's theorem, find an expansion for $\sin 4\theta$ in terms of $\sin\theta$ and $\cos\theta$.

Question 3

a. The base of a certain solid is the circle $x^2 + y^2 = 4$.

Each plane section of this solid perpendicular to the y axis is an equilateral triangle with one side in the base of the solid.

By using the technique of slicing, find the volume of the solid.



b. Reduce $P(x) = x^4 - x^2 - 12$ to its factors over the field of

i. Rational Numbers

ii. Complex Numbers

c. If $(x-i)$ and $(x+1-i)$ are two factors of a degree 4 monic polynomial, write down the other two factors.

d. Write down the complex cube roots of unity and show that they may be written as 1, ω and ω^2.

Question 4

i. Show that at the locus of the point $P(x, y)$ moving so that the sum of its distances from $A(4,0)$ and $B(-4,0)$ is always 10 units is an ellipse with equation

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

ii. Find the eccentricity of this ellipse.

iii. Write down the equations for the directrices of this ellipse.

iv. Find the equation of the tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ at the point $\left(\frac{5}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$

v. Determine the area enclosed by the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$

(HINT: Use a substitution $x = a\sin\theta$ where 'a' is some suitable number)

Question 5

a. The complex numbers Z_1, Z_2, Z_3 and Z_4 are represented by the points A, B, C and D respectively on the Argand diagram.

If $Z_1 + Z_3 - Z_2 - Z_4 = 0$ and $Z_1 - Z_4 - 2iZ_1 + 2iZ_2 = 0$, describe the quadrilateral with vertices A, B, C and D.

b. Given that P and Q represent the complex numbers $\sqrt{3} + i$ and $3 - 4i$ respectively:

i. Find mod P and Arg P.

ii. Write $\sqrt{Q}$ in the form $a + ib$

c. Sketch the locus of the point representing the complex number Z on the Argand diagram if $\operatorname{Arg}\left(\frac{Z-1}{Z+3}\right) = \frac{\pi}{2}$

State any important features and give its Cartesian equation.

Question 6

a. A body falling from rest experiences resistance directly proportional to its velocity squared (Resisting Force = mkv^2 where k is some constant).

i. Write the equation of motion for this body.

ii. Show that the distance fallen when the velocity is V is given by

$$x = \frac{1}{2k}\ln\left(\frac{g}{g - kV^2}\right)$$

iii. Explain why terminal velocity is given by $V^2 = \frac{g}{k}$

b. A car racing circuit has its corners (which are arcs of a circle) banked so that a car travelling at 180km/hour experiences no sideways force.

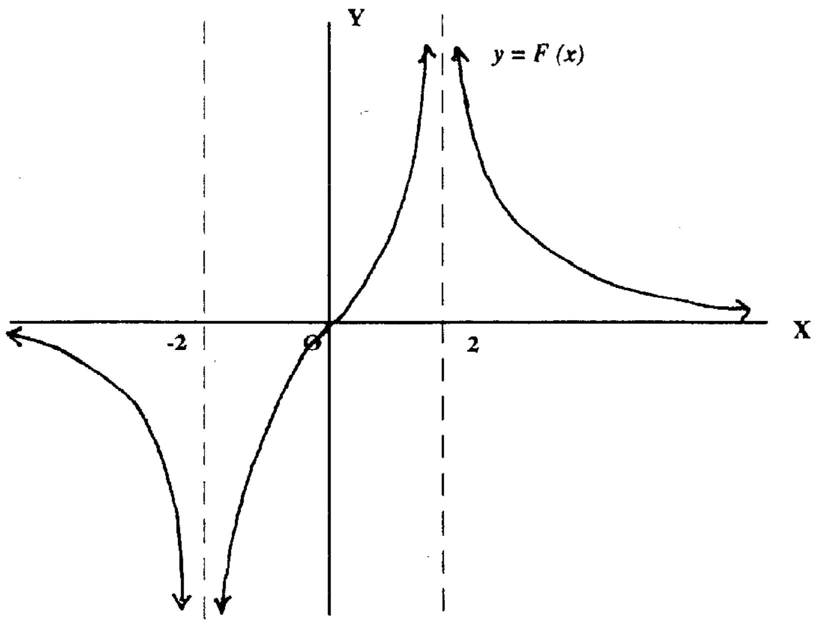
i. Draw a sketch of the forces acting on the car.

ii. If the radius of one corner is 70m, calculate the angle of the banking to the nearest minute.

c. If the probability of a rocket hitting its target is 0.6, how many rockets must be fired so that the probability of at least one hitting is 99.9%.

Question 7

a.



Above is a sketch of $y = F(x)$.

On different sets of axes, sketch possible graphs of:

i. $y = \dfrac{1}{F(x)}$ ii. $y = [F(x)]^2$

iii. $y = F^1(x)$

b. The graph $f(x) = \dfrac{ax^2 + bx + c}{x^2 + qx + r}$ has the lines $x = 1$, $x = 3$ and $y = 2$ as asymptotes. It also has a turning point at (0, 1).

Determine the values of a, b, c, q and r.

Question 8

a. Find the equations of the two bisectors of the angles formed by the intersection of the lines $3x+4y=0$ and $5x-12y+1=0$.

(HINT: All points on a bisector of an angle are equidistant from the arms of the angle).

b. If α, β and γ are the roots of $2x^3+3x^2+x-5=0$

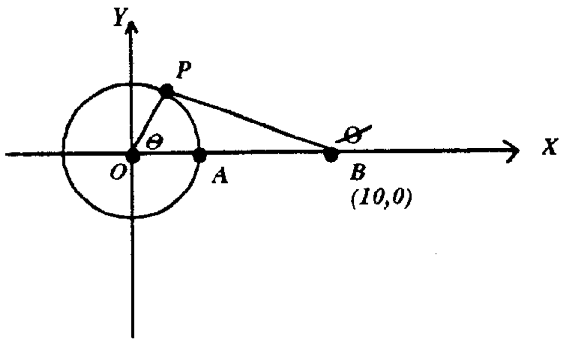
Find an equation with roots of $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$

c. A point P is moving on the circle $x^2+y^2=25$ with an angular velocity of 2π radians per second about the centre of the circle.

i. Find the angular velocity of P about the point $A(5,0)$.

ii. If B is the point $(10, 0)$, and letting $\angle POX$ be θ and $\angle PBX$ be $\varnothing$, show that the angular velocity of P about B is given by

$$\frac{d\varnothing}{dt}=\frac{2\pi\cos(\varnothing-\theta)}{\cos(\varnothing-\theta)-2\cos\varnothing}$$



HINT: Use the SINE RULE to find a relationship between θ and $\varnothing$.