Question 1 Western Region 1996 4 Unit maths trial
(Start a new page) **Marks**

a. Find $\int_0^3 \frac{6dx}{x^2+6x+18}$ 3

b. Find $\int \frac{dx}{\sqrt{x+5}\sqrt{4-x}}$ for $x<4$, using the substitution $v^2=4-x$ 4

c. Use the method of partial fractions to prove 4

$$\int \frac{2dx}{x^2+4x+3} = \ln\left|\frac{x+1}{x+3}\right| + c$$

hence evaluate $\int_{-2}^{0} \frac{2dx}{x^2+4x+3}$

d. Find $\int \frac{dx}{1+\cos x-\sin x}$ 4

Question 2 (Start a new page)

a. Solve the following equation for z giving your answer in modulus-argument form. 3

$$z^2+z+1=0$$

b. If $z_1 = 2cis\frac{2\pi}{3}$ and $z_2 = cis\frac{\pi}{3}$ express your answer to the following in the form $a+bi$ 3

i. $z_1 z_2$ ii. $\frac{z_1}{z_2}$

c. The equation $z^3-3z^2+7z-5=0$ has one root equal to $(1-2i)$. 3
Factorise this equation.

d. What are the complex equations for the following loci: 3
i. The circle with centre $(-1,i)$ and radius 2.
ii. The ellipse with foci $(-1,0)$, $(2,0)$ that passes through $(2,4i)$.

e. Sketch the locus of z such that $|z-2-2i|=2$ 3
i. Find the range of $|z|$
ii. Find the range of $\arg z$

Question 3 (Start a new page) **Marks**

a. i. Given the equation of the hyperbola $xy = c^2$ 3

establish the equation of the tangent at $T\left(ct, \frac{c}{t}\right)$

ii. P, Q, R are three points on one branch of this hyperbola, with parameters p, q, r respectively. The tangents at P and Q intersect at U. If O, U, R are collinear, find the relationship between p, q and r. 6

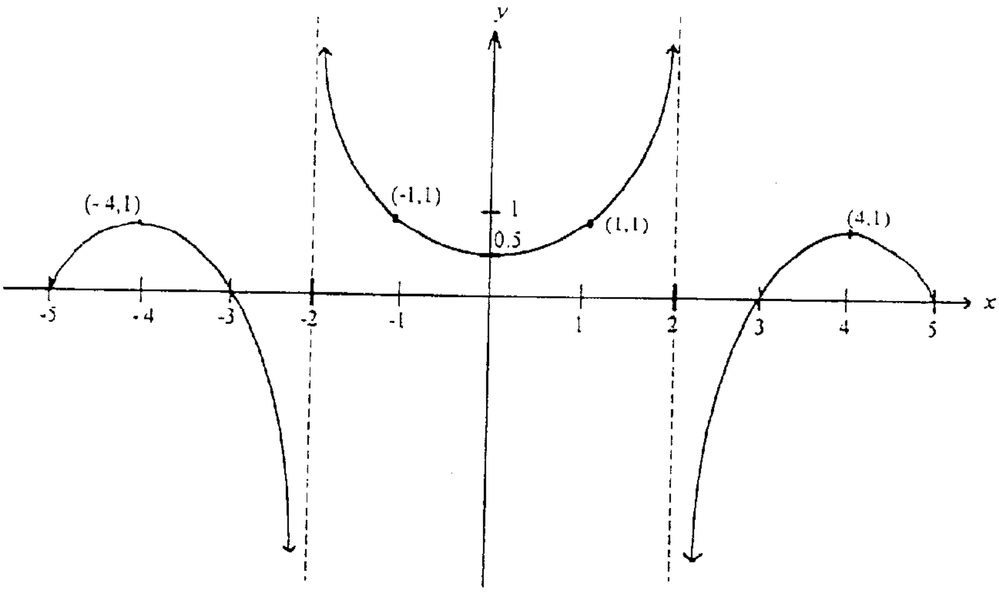
b. i. Show that $P(a\cos\theta,\ b\sin\theta)$ lies on the ellipse 2

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ii. If P and Q $(a\cos\varnothing,\ b\sin\varnothing)$ are two points on this ellipse, prove that the locus of the mid-point of PQ is a straight line, given that $\theta + \varnothing = \frac{\pi}{2}$ for all positions of P and Q. 4

Question 4 (Start a new page) Marks

a. 4



This is a graph of the curve $y = f(x)$. $-5 \le x \le 5$

Sketch separate graphs of the following and mark any important points on each sketch.

i. $y = \sqrt{f(x)}$

ii. $y = f'(x)$

b. i. Show that $\dfrac{1}{x-2} - \dfrac{4}{x+3} + 3 = \dfrac{3x^2 - 7}{x^2 + x - 6}$ 1

ii. Find the vertical and horizontal asymptotes of $f(x) = \dfrac{3x^2 - 7}{x^2 + x - 6}$ 2

iii. Find the turning points and determine their nature. 4

iv. Sketch the curve, showing all important points. You may omit the point of inflexion. 3

v. Use the sketch to solve $0 < f(x) < 3$. 1

Question 5 (Start a new page) **Marks**

a. Prove $a = \dfrac{d\left(\frac{1}{2}v^2\right)}{dx}$ 2

b. i. A small mass, attached by a string to a fixed point, describes a horizontal circle at the uniform angular speed of 1 revolution per second. Prove that the distance of the mass below the fixed point is independent of the mass and of the length of the string. 4

ii. Find the tension in the string when the mass is 2.5kg, the string is 45cm long and the angular velocity is 2 revolutions per second. 3

c. A mass moves in a straight line against frictional resistance of 0.2 of its weight and air resistance of $0.1v$ per unit mass, where v is the velocity of the mass. If the initial velocity was $40ms^{-1}$, find the distance travelled and the time taken for it to come to rest. 6
$\left[\text{Take } g = 10ms^{-2}\right]$

Question 6 (Start a new page)

a. A solid has a base in the shape of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. If every cross section perpendicular to the base is a semi-circle, with its diameter at right angle to the major axis of the ellipse, find the volume of the solid by slicing. 4

b. Use integration to show that the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is πab. 5

c. i. If $I_n = \int x^n e^{-2x} dx$ prove $I_n = \dfrac{-x^n e^{-2x}}{2} + \dfrac{n}{2} I_{n-1}$ 3

ii. Hence find $\int x^3 e^{-2x} dx$ 3

Question 7 (Start a new page) **Marks**

a. PQR is an acute angled triangle inscribed in a circle. The altitudes PM and QN intersect at A. PM is produced and cuts the circle at B.

i. Draw a neat diagram showing this information. 2

ii. Prove $AM=MB$. 4

b. A student council consists of 6 girls and 8 boys. A committee of 5 members is chosen at random. What is the probability that the girls will have a majority on the committee? 4

c. The equation $8x^4+44x^3+54x^2+25x+4=0$ has a triple root. Solve this equation. 5

Question 8 (Start a new page)

a. If a, b, c, d are unequal real numbers, prove $(a^2+b^2)(c^2+d^2)\geq(ac+bd)^2$. 3

b. Find the relationship between the co-efficients of $P(x)=x^3+ax^2+bx+c=0$ given that the roots are in Arithmetic progression. 4

c. The circle $x^2+y^2=4$ is rotated about the line $x=3$ to form a Torus. Use the method of slicing the torus perpendicular to the y axis to form an annulus to show the volume of the torus is $24\pi^2$. 5

d. Consider the function $y=x^x$ for $x>0$ 3
Show that its derivative is $(\log_e x+1)x^x$.

End of paper