Total marks (120)

Attempt Questions 1-8

All questions are of equal value

Answer each question starting a FRESH SHEET with your name and the question number at the top. Extra writing booklets are available.

Question 1 *(15 marks)* Use a SEPARATE writing booklet **Marks**

(a) Find $\int x\cos(x^2)dx$ 1

(b) Using the substitution $x = 2\sin\theta$ evaluate $\int_0^2 \sqrt{4-x^2}\,dx$ 4

(c) Using the method of partial fractions find $\int \frac{-4dx}{x^2+2x-3}$ 4

(d) Find $\int \frac{x^2+2x-3}{x+1}dx$ 4

(e) Using integration by parts evaluate $\int_1^e \ln x.dx$ 2

Marks

Question 2 *(15 marks)* Use a SEPARATE writing booklet

(a) If A = 3+4i and B = 2-i

Express the following in the form $x + iy$ where x and y are real numbers :

(i) AB **1**

(ii) $\sqrt{A}$ **2**

(iii) $\dfrac{A}{B}$ **2**

(b) If $z = \sqrt{3} + i$ **2**

(i) Find the exact values of mod(z) and arg(z)

(ii) By using your answers to (i) and De Moivre's theorem write z^5 in the form $a + ib$ **2**

(c) On an Argand diagram shade the region containing all the points representing the complex numbers z such that: **2**

$$1 \le |z-1| \le 2 \qquad \text{and} \qquad \frac{\pi}{4} < \arg(z-1) < \frac{\pi}{2}$$

(d) Explain algebraically or geometrically why the locus described by **2**

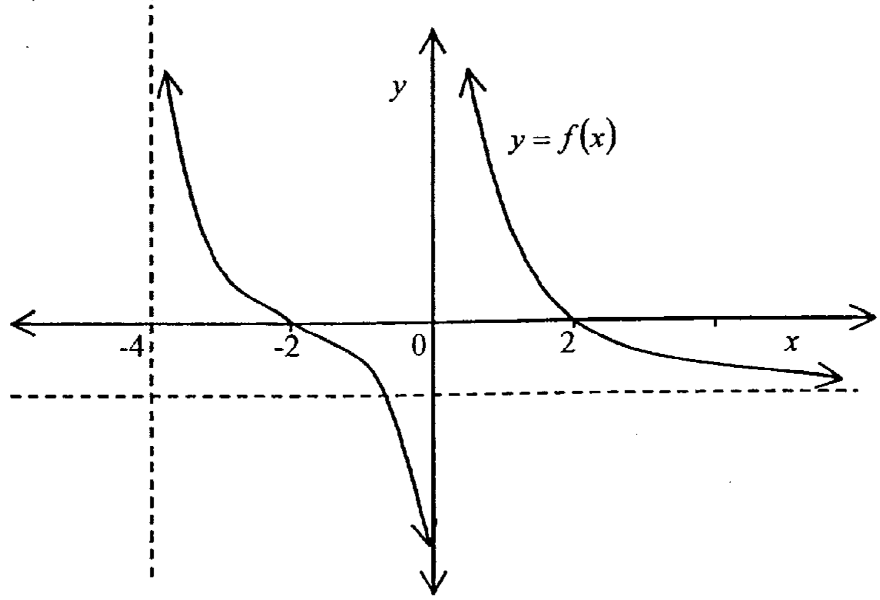
$\arg\left(\dfrac{z}{z-4}\right) = \dfrac{\pi}{2}$ is a circle.

(e) Given that z and w represent two complex numbers, explain why **2**

$$|z| + |w| \ge |z - w|$$

Question 3 *(15 marks)* Use a SEPARATE writing booklet **Marks**

(a)



The sketch above shows the graph of the function $y = f(x)$. There is a horizontal asymptote at $y = -1$ and vertical asymptotes at $x = 0$ and $x = -4$. Draw separate sketches of the following functions

(i) $y = |f(x)|$ **2**

(ii) $y = \dfrac{1}{f(x)}$ **2**

(iii) $y = \int f(x)dx$ **2**

(b) An ellipse has equation $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

(i) Show that this is the equation of the locus of a point $P(x,y)$ moving such that the sum of its distances from A (4, 0) and B (- 4, 0) is 10 units. **4**

(ii) Calculate the eccentricity of this ellipse. **2**

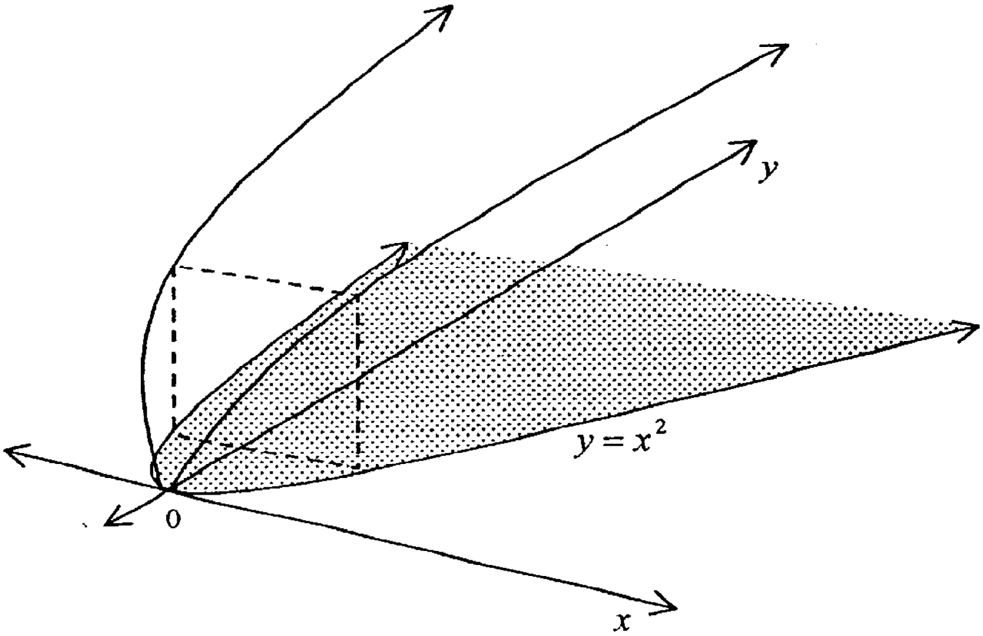
(iii) State the equations of the directrices of this ellipse. **1**

(iv) Find the equation of the tangent to the curve at a point Q (a, b) which lies on the ellipse. **2**

Marks

Question 4 *(15 marks)* Use a SEPARATE writing booklet

(a) A solid shape is formed as shown in the sketch below . It has its base on the XY plane in the shape of the parabola $y = x^2$ The vertical cross-section is a square as shown (base in XY plane)



By using the method of slicing calculate the volume of the solid between the values of y = 0 and y = 3. **4**

(b) The equation $x^3 - 6x^2 + 7x - 3 = 0$ has roots $\alpha, \beta,$ and γ

(i) Write an equation which has roots $\alpha^2, \beta^2,$ and γ^2. **2**

(ii) Write an equation which has roots $\frac{1}{\alpha}, \frac{1}{\beta},$ and $\frac{1}{\gamma}$. **2**

(iii) It is known that the solution to given a problem is the average of the roots of the equation $x^3 - 6x^2 + 7x - 3 = 0$ **1**
Without finding the roots determine the solution to the problem.

(c) (i) Find the domain of $f(x) = \sin^{-1}(2x - 1)$ **1**

(ii) Sketch the graph of $y = \sin^{-1}(2x - 1)$ **1**

(iii) Solve $\sin^{-1}(2x - 1) = \cos^{-1} x$. **4**

Question 5 *(15 marks)* Use a SEPARATE writing booklet. **Mar**

(a) Solve the equation $4x^3 - 8x^2 + 5x - 1 = 0$ given that it has a double root. **3**

(b) Factorize $x^4 - 16$ fully over the complex field. **2**

(c) A particle of mass m is suspended by a light inextensible string and describes a horizontal circle at a constant speed. The centre of the circle is at a distance of h units below the point of suspension. **3**

Show that the angular velocity depends on the value of h only.

(d) Determine the angle of banking of a roadway to allow a car to round a curve of radius 100 m at a speed of 100 km/h with no side thrust on the wheels. (use g = 10 ms^{-2}) **3**

(e) Given the equation $x^2 + xy + y^2 = 1$

(i) Make y the subject **2**

(ii) Hence or otherwise find $\dfrac{dy}{dx}$ **2**

Question 6 *(15 marks)* Use a SEPARATE writing booklet. Marks

(a) A particle of mass m is released and allowed to fall vertically in a medium in which the resistance is mk times the square of the speed (v) of the particle.

(i) Write an equation for the motion of the particle. 1

(ii) Show that the terminal velocity (V_T) is given by $V_T = \sqrt{\dfrac{g}{k}}$ 2

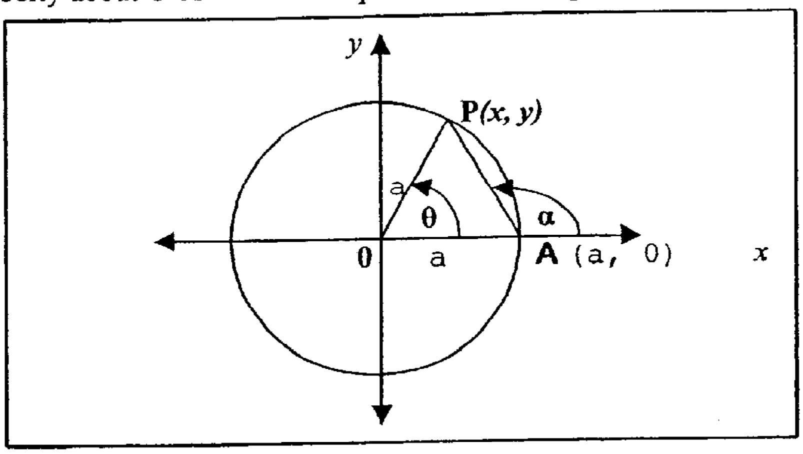
(iii) Show that the distance fallen (x) in terms of the velocity of the particle (v) is 4

$$x = \frac{1}{2k}\ln\left(\frac{g}{g - kv^2}\right)$$

(iv) Hence find an expression for the distance that the particle must fall for the velocity to reach $\frac{1}{2}V_T$. 2

(v) Explain briefly why, if the particle is projected upwards at a speed of $\frac{1}{2}V_T$, the distance it travels before coming to rest will be less than the distance x from (iii). 2

(b) A point P (x, y) is moving on the circumference of the circle $x^2 + y^2 = a^2$ with an angular velocity about O of π radians per second. At a particular instant it's position is as shown.



(i) Show that an expression for the angular velocity of P about A is given by 3

$$\frac{d\alpha}{dt} = \frac{\frac{d\theta}{dt}\cos(\alpha - \theta)}{\cos(\alpha - \theta) - \cos\alpha}$$

(ii) Find the value of $\dfrac{d\alpha}{dt}$ when $\theta = \dfrac{\pi}{2}$ 1

Marks

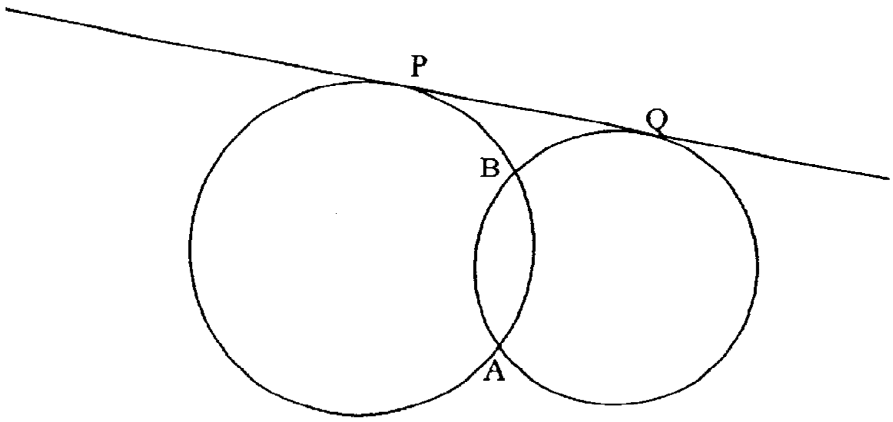
Question 7 *(15 marks)* Use a SEPARATE writing booklet.

(a) A triangle ABC is right angled at A and has sides of lengths a ,b and c units (the side a is opposite to $\angle$ A etc). A circle of radius r units is drawn such that the sides of the triangle are tangents to the inscribed circle.

(i) Sketch the above information. **1**

(ii) Prove that $r = \frac{1}{2}(c + b - a)$. **3**

(b) Two circles intersect at A and B and a common tangent touches them at P and Q as shown.



(i) A chord PR is drawn parallel to QA. RA produced meets the other circle at S. Copy the above diagram and complete it. **1**

(ii) Prove that PRSQ is a cyclic quadrilateral. **4**

(iii) Prove that PA is parallel to QS. **2**

(c) Given that a and b are two unequal positive numbers, show that the average of the squares of a and b is greater than the square of the average of a and b. **4**

Question 8 *(15 marks)* Use a SEPARATE writing booklet. Ma

(a) Given the complex number $Z = r(\cos\theta + i\sin\theta)$, use the method of mathematical induction to prove De Moivre`s Theorem. 5

[ie $z^n = r^n(\cos n\theta + i\sin n\theta)$] For any real number *n*.

(b) By noting that $z^n + z^{-n} = 2\cos n\theta$ and that z is the complex number $\cos\theta + i\sin\theta$, show that 4

$$\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$$

(c) A box contains 6 cards, two of which are identical. From this box 3 cards are drawn without replacement.

(i) How many different selections could be made. 2

(ii) What is the probability that a selection will include the two identical cards. 1

(iii) If this process of selecting three cards was repeated, with all cards being replaced after each selection, how many repetitions would be necessary to make the probability of drawing a combination containing the two identical cards at least once, exceed 99%. 3