



Student Number: _____

2023 TRIAL HSC EXAMINATION

Mathematics Standard 2

General Instructions

- Reading time – 10 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: **Section I – 15 marks** (pages 2–4)
100

- Attempt Questions 1–15
- Allow about 25 minutes for this section

Section II – 85 marks (pages 5–21)

- Attempt Questions 16–38
- Allow about 2 hours and 5 minutes for this section

Section I

15 marks

Attempt Questions 1–15

Allow about 25 minutes for this section

Use the multiple-choice answer sheet for Questions 1–15.

1 Which of the following is the same as $3(x - 2) - 2(x - 1)$?

- A. $x - 1$ B. $x - 3$ C. $x - 4$ D. $x - 8$
-

2 How many significant figures does the number 5.081×10^2 contain?

- A. 2 B. 3 C. 4 D. 5
-

3 Each of our four Houses can finish 1st, 2nd, 3rd or 4th in each House event. What is the classification of this type of data?

- A. Continuous quantitative B. Discrete quantitative
C. Nominal categorical D. Ordinal categorical
-

4 Zara works for 4 days in a week for the following hours and rates.

- 6 hours on Thursday at ordinary time
- 7 hours on Friday at ordinary time
- $4\frac{1}{2}$ hours on Saturday at time-and-a-half
- 3 hours on Sunday at double-time

On which day does she earn the highest pay?

- A. Thursday B. Friday C. Saturday D. Sunday
-

5 Lucia measured her height as 1.74 m, correct to the nearest centimetre. What is the percentage error in her measurement?

- A. 0.003% B. 0.006% C. 0.3% D. 0.6%
-

6 Jason lives in Perth (32°S , 115°E). He wants to watch an ice hockey game being played in Toronto (44°N , 80°W) starting at 10:00 pm on Wednesday. What is the time in Perth when the game starts?

- A. 9:00 am Wednesday B. 11:00 am Thursday
C. 7:40 pm Wednesday D. 12:20 am Thursday
-

7 Charlie is about to go on holidays for four weeks. Her weekly salary is \$760 and her holiday loading is $17\frac{1}{2}\%$ of four weeks pay. What is Charlie's total pay for the four weeks holiday?

- A. \$532 B. \$893 C. \$3173 D. \$3572
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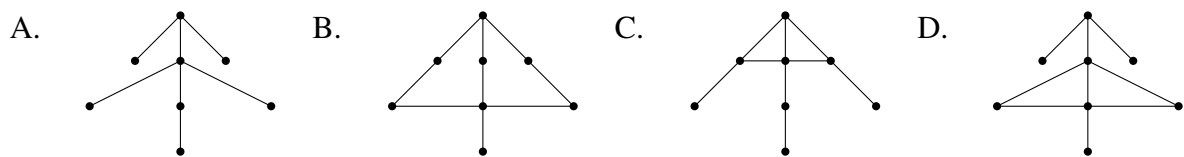
8 An investor has 500 shares with a current market value of \$6.82. The company declares a dividend yield of 3.5%. What is the dividend on this investment?

- A. \$23.87 B. \$119.35 C. \$3410 D. \$11 935
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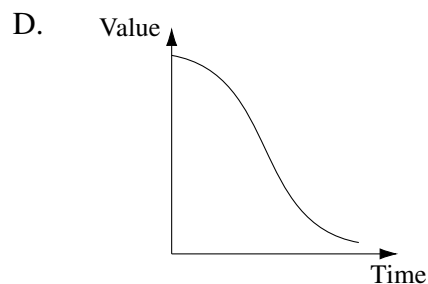
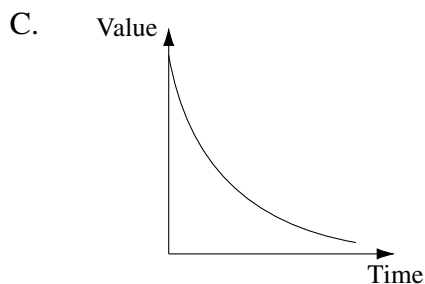
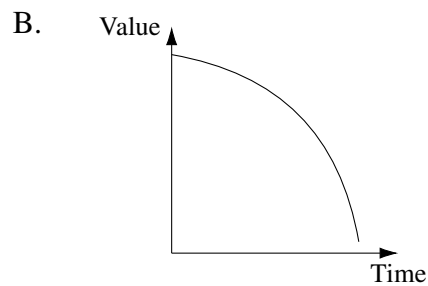
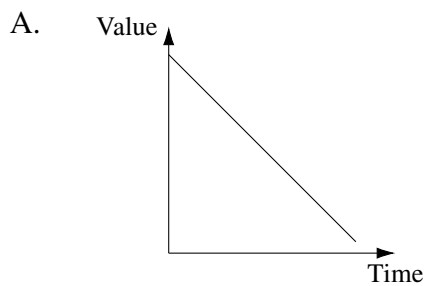
9 Which of the following gives the best description of the relationship between a person's alcohol consumption and their reaction time?

- A. Negative correlation B. Perfect correlation
C. Positive correlation D. Zero correlation
-

10 Which of the following networks is a tree?



11 The value of a car is depreciated using the straight line method. Which graph best illustrates the value of the car over time?



- 12 Olivia takes out a new credit card that has no interest free period and charges interest at a flat rate of 20.4% p.a. on all purchases. Interest is charged on amounts from (and including) the date of purchase up to (and including) the payment date.

On 22 June Olivia uses her credit card for the first time to buy a handbag for \$420. If she makes no further purchases, what amount would Olivia need to repay if she wants to pay the outstanding balance on 31 July?

- A. \$429.15 B. \$429.25 C. \$429.39 D. \$429.49

- 13 A group of 347 people was tested for flu and the results were recorded. The flu test results are not always accurate.

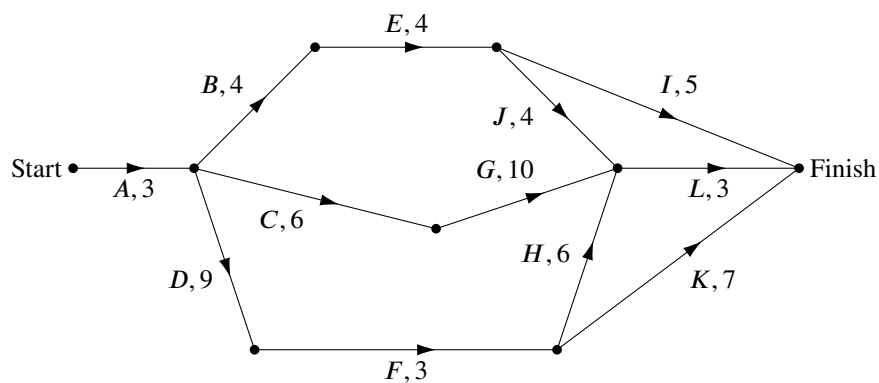
Test results

	Test indicated flu	Test did not indicate flu	Total
People with flu	151	6	157
People without flu	32	392	424
Total	183	398	581

A person is selected at random from the tested group. What is the probability that their test result is accurate?

- A. 26% B. 83% C. 93% D. 97%

- 14 A project involves activities A to L. The network below shows these activities and their completion time in days. The minimum completion time for the project is 24 days.



What is the critical path for this project?

- A. *ABEI* B. *ACGL* C. *ADFK* D. *ADFHL*

- 15 A sphere and a cylinder have the same radius. The height of the cylinder is three times the height of the sphere. What is the ratio of the volume of the cylinder to the volume of the sphere?

- A. 3 : 2 B. 3 : 4 C. 9 : 2 D. 9 : 4

Section II

85 marks

Attempt Questions 16–38

Allow about 2 hours and 5 minutes for this section

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 16 (3 marks)

Andrew is 35 years old, and likes to keep fit by exercising.

- (a) A person's maximum heart rate (MHR) is given by the formula: 1
$$\text{MHR} = 220 - \text{age in years}$$

What is Andrew's maximum heart rate?

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- (b) Andrew will get the most benefit from this exercise if his heart rate is between 2
50% and 80% of his maximum heart rate. Between what two heart rates should Andrew
be aiming for to get the most benefit from his exercise?

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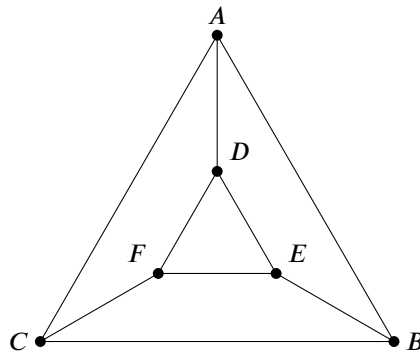
Question 17 (2 marks)

Solve the equation: $\frac{5x - 4}{3} = 4x + 1$ 2

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Question 18 (4 marks)

A network graph is shown below.



- (a) What is the degree of vertex A ? **1**

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- (b) Write down a trail from A to C which uses 4 edges. **1**

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- (c) Explain briefly why $EFCABC$ is not a path. **1**

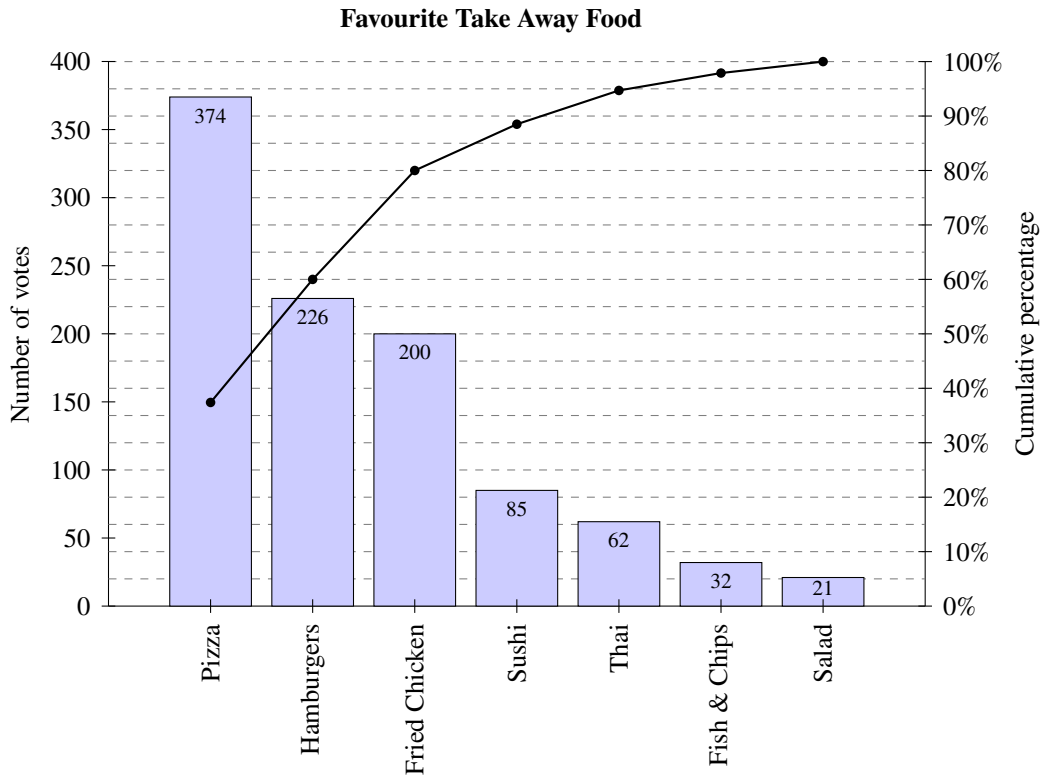
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- (d) Write down a cycle, starting and ending at F which involves three other vertices. **1**

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Question 19 (4 marks)

The Pareto chart below represents the favourite take away food of a randomly selected group of people.



(a) How many people were surveyed? **1**

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(b) What percentage of people chose either Pizza or Hamburgers? **1**

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(c) What percentage of people chose Fried Chicken? **2**

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Question 20 (3 marks)

A popular soft drink comes in bottles which are labelled as containing 600 mL. The actual amounts of soft drink in bottles are normally distributed with a mean of 606 mL and a standard deviation of 2 mL.

- (a) What is the z-score of a bottle with 606 mL of soft drink? **1**

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- (b) A bottle has a z-score of -2 . What amount of soft drink would be in this bottle? **1**

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- (c) What percentage of bottles will contain less than 600 mL? **1**

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Question 21 (3 marks)

A study of the flying foxes in a section of the Royal Botanic Gardens used the capture-recapture technique. In the first stage of the study, 770 flying foxes were caught, tagged and released. Later, in the second stage of the study, some flying foxes were captured from the same section. Of these, 420 were found to be tagged, which was 35% of the total number captured during the second stage.

- (a) How many flying foxes were captured during the second stage of the study? **1**

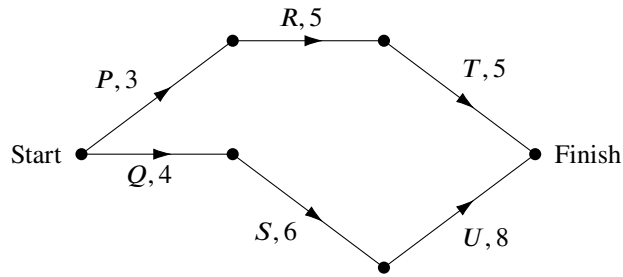
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- (b) Calculate the estimate for the total population of flying foxes in this section of the Royal Botanic Gardens. **2**

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Question 22 (4 marks)

A network diagram with six activities is shown below. The duration of each activity, in hours, is also shown.



- (a) What activities have no predecessor? 1

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- (b) Complete the activity table for this network diagram. 2

Activity	Time (hours)	Predecessor
<i>P</i>		
<i>Q</i>		
<i>R</i>		
<i>S</i>		
<i>T</i>		
<i>U</i>		

- (c) Explain why the minimum completion time of this project is 18 hours. 1

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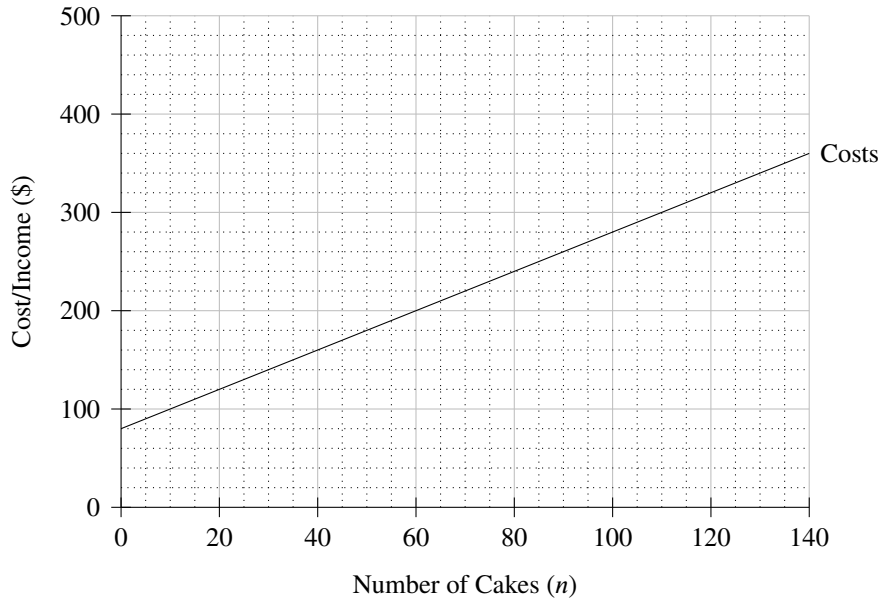
Question 23 (2 marks)

The average rate for domestic electricity in NSW is 28.66¢ per kWh. Calculate the cost of running a 2500-watt air conditioner for 6 hours per day over 90 days. 2

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Question 24 (6 marks)

Kayla runs a bakery. She can bake a maximum of 140 cakes in a day. The fixed daily cost of production is $\$a$ and the variable cost of production is $\$b$ per cake. The diagram below shows a graph of Kayla's daily production costs.



- (a) Determine the values of a and b . **2**

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- (b) Kayla sells her cakes for $\$4$ each. Write an equation representing the income $\$I$ made from selling n pastries. **1**

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- (c) On the diagram above, draw the straight line representing the income equation in (b). **1**

- (d) How many cakes must Kayla bake and sell in order to break even? **1**

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- (e) How many cakes must Kayla sell in a day to make a profit of $\$100$ profit? **1**

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Question 25 (5 marks)

The following table gives the future value of an annuity with a contribution of \$1 at the end of every period.

Future Value Interest Factors

Number of periods	Interest rate per period			
	0.5%	1%	1.5%	2%
6	6.0755	6.1520	6.2296	6.3081
12	12.3356	12.6825	13.0412	13.4121
18	18.7858	19.6147	20.4894	21.4123
24	25.4320	26.9735	28.6335	30.4219
30	32.2800	34.7849	37.5387	40.5681
36	39.3361	43.0769	47.2760	51.9944
42	46.6065	51.8790	57.9231	64.8622
48	54.0978	61.2226	69.5652	79.3535
54	61.8167	71.1410	82.2952	95.6731
60	69.7700	81.6697	96.2147	114.0515

Ayla invests \$450 at the end of every month in an annuity where interest is earned at 6% p.a., compounded monthly.

- (a) Use this table to determine the future value of Ayla’s investment after 3 years. **1**

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- (b) How much interest does Ayla earn on her investment? **2**

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- (c) Rather than making monthly investments, what single amount could Ayla invest at the start of the 3 years at the same interest rate to reach the same future value? **2**

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Question 28 (2 marks)

What is the future value of \$3900 if it is invested for 5 years at 6% p.a. interest, compounded quarterly? **2**

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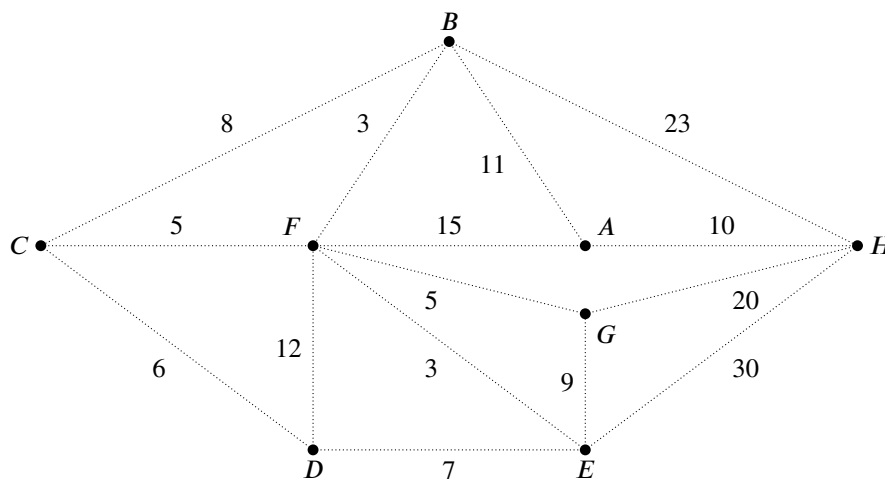
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Question 29 (3 marks)

The diagram shows roads connecting some places of interest in Berlin. The numbers represent the times taken, in minutes, to walk along the roads.



(a) On the diagram above, draw a minimum spanning tree for this network. **2**

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(b) Calculate the weight of your minimum spanning tree. **1**

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Question 30 (5 marks)

Mia has a gross annual income of \$81 797. She has allowable deductions of \$6947. She must pay tax on all taxable income. Her employer has deducted \$14 100 in PAYG tax throughout the financial year.

- (a) Show that Mia’s taxable income is \$74 850. **1**

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- (b) Using the tax table below, calculate the income tax that Mia must pay. **2**

Taxable Income	Tax on Taxable Income
\$1 – \$10 000	Nil
\$10 001 – \$35 000	10c for each \$1 over \$10 000
\$35 001 – \$60 000	\$2500 plus 20c for each \$1 over \$35 000
\$60 001 – \$100 000	\$7500 plus 30c for each \$1 over \$60 000
\$100 001 and over	\$19 500 plus 40c for each \$1 over \$100 000

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- (c) The Medicare levy is charged at 2% of taxable income. Calculate Mia’s Medicare levy. **1**

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- (d) Will Mia receive a refund or will she need to pay an additional amount in tax? What is the amount of her refund or tax bill? **1**

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Question 31 (5 marks)

At a certain small restaurant, the waiting time is defined as the time between sitting down at a table and a waiter first arriving at the table. This waiting time depends upon the number of other customers already seated in the restaurant.

Amelia is a customer who visited the restaurant on 10 separate days. The table below shows the number of customers already seated in the restaurant (x) and her waiting time (y minutes).

Number of customers (x)	9	3	4	10	8	12	7	11	2	6
Waiting time (y minutes)	11	7	5	11	9	14	9	12	2	8

- (a) Calculate the value of r , the correlation coefficient for this set of data. Give your answer correct to 3 decimal places. **1**

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- (b) Describe the direction and strength of the relationship between the waiting time and the number of customers. **1**

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- (c) Using your calculator, determine the equation of the least squares regression line of best fit. Give your answer in the form $y = mx + c$, with values of m and c correct to 2 decimal places. **2**

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- (d) Explain the significance of the value of c in the context of this scenario. **1**

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**2 UNIT STANDARD MATHEMATICS
2023 TRIAL HSC EXAMINATION**

SECTION I

1 $3(x - 2) - 2(x - 1) = 3x - 6 - 2x + 2$ 1 **C**
 $= x - 4$

2 2 **C**

3 3 **D**

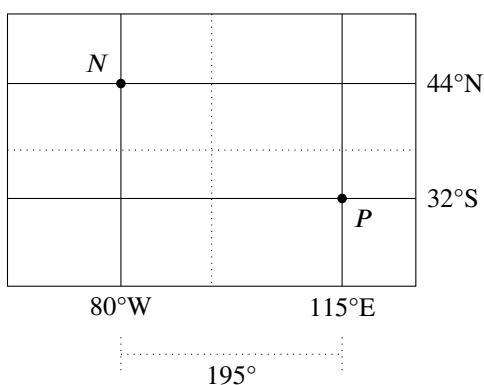
4 Normal hours on Thursday = 6 hours 4 **B**
 Normal hours on Friday = 7 hours
 Normal hours on Saturday = $4\frac{1}{2} \times 1.5$
 $= 6.75$
 Normal hours on Sunday = 3×2
 $= 6$

5 Precision = 1 cm 5 **C**

Absolute error = $\frac{1}{2} \times 1$
 $= 0.5 \text{ cm}$

Percentage error = $\frac{0.5}{174} \times \frac{100}{1}$
 $= 0.2873563218$
 $\approx 0.3\%$

6 6 **B**



Angular distance = $80 + 115$
 $= 195^\circ$

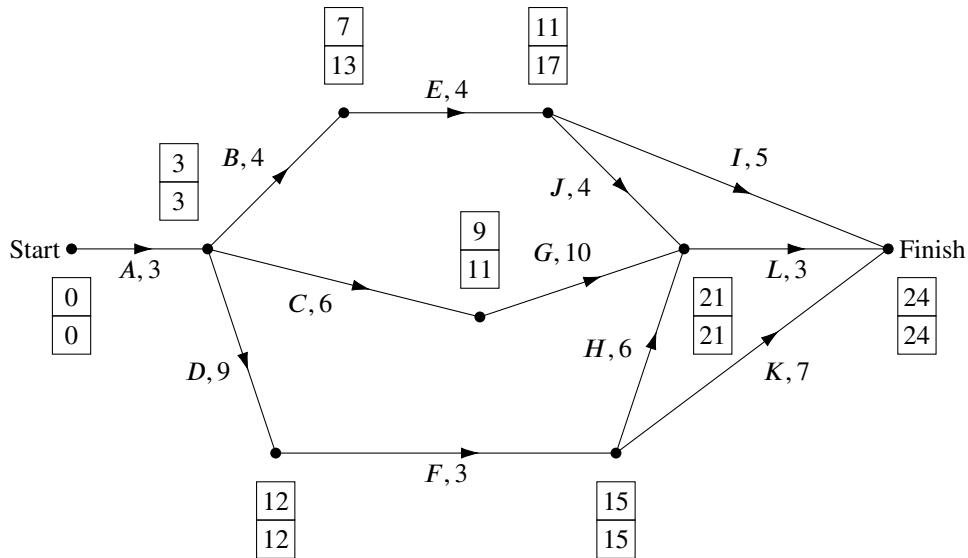
Time difference = $\frac{195}{15}$
 $= 13 \text{ hours}$

\therefore Time in Perth = 10 pm Wednesday + 13 h
 $= 22 \text{ h} + 13 \text{ h}$
 $= 35 \text{ h}$
 $= 11 \text{ h next day}$
 $= 11 \text{ am Thursday}$

- 7 Pay for 4 weeks = 760×4 7 **D**
= \$3040
- Holiday loading = $17\frac{1}{2}\%$ of \$3040
= 0.175×3040
= \$532
- \therefore Total Pay = $3040 + 532$
= \$3572
- 8 Value of shares = 500×6.82 8 **B**
= \$3410
- Dividend = 3.5% of \$3410
= 0.035×3410
= \$119.35
- 9 9 **C**
- 10 10 **A**
- 11 11 **A**
- 12 Number of days = $9 + 31$ 12 **D**
= 40
- \therefore Amount to repay = $P(1 + r)^n$
= $420 \left(1 + \frac{0.204}{365}\right)^{40}$
= \$429.49
- 13 Number of accurate results = $151 + 392$ 13 **C**
= 543
- $\therefore P(\text{accurate}) = \frac{543}{581} \times \frac{100}{1}$
= 93.4595525
 $\approx 93\%$

14 We can complete a forward and backward scan to find ESTs and LSTs.

14 D



Now we can check float times to find the critical path.

$$\text{Float time for activity } A = 3 - 0 - 3 = 0$$

$$\text{Float time for activity } B = 13 - 3 - 4 = 6$$

$$\text{Float time for activity } C = 11 - 3 - 6 = 2$$

$$\text{Float time for activity } D = 12 - 3 - 9 = 0$$

$$\text{Float time for activity } F = 15 - 12 - 3 = 0$$

$$\text{Float time for activity } H = 21 - 15 - 6 = 0$$

$$\text{Float time for activity } L = 24 - 21 - 3 = 0$$

∴ The critical path is *ADFHL*.

15 The radius of the sphere is r units, so its diameter is $2r$ units. Therefore the height of the sphere is $2r$ units.

15 C

The radius of the cylinder is r units. The height of the cylinder is three times the height of the sphere, so the height of the cylinder is $3 \times 2r = 6r$ units.

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h \\ &= \pi \times r^2 \times 6r \\ &= 6\pi r^3 \end{aligned}$$

$$\begin{aligned} \therefore \text{Ratio} &= 6 : \frac{4}{3} \\ &= 18 : 4 \\ &= 9 : 2 \end{aligned}$$

SECTION II

QUESTION 16

(a) Maximum heart rate = $220 - \text{Age}$
 $= 220 - 35$
 $= 185 \text{ bpm}$

(b) Lower rate = 50% of 185
 $= 0.50 \times 185$
 $= 92.5 \text{ bpm}$

Upper rate = 80% of 185
 $= 0.85 \times 185$
 $= 148 \text{ bpm}$

\therefore Andrew should aim for a heart rate between 92.5 and 148 bpm.

QUESTION 17

$$\frac{5x - 4}{3} = 4x + 1$$

$$5x - 4 = 3(4x + 1)$$

$$5x - 4 = 12x + 3$$

$$5x = 12x + 7$$

$$-7x = 7$$

$$x = -1$$

QUESTION 18

- (a) Degree of vertex $A = 3$
- (b) A trail is a walk in which no edges are repeated. Possible trails are $ABEFC$, $ADEBC$ and $ADEFC$.
- (c) A path is a walk where no vertex is visited more than once (except possibly the start and finish). In the walk $EFCABC$, the vertex C is visited twice.
- (d) A cycle is a closed path. A cycle starts and ends at the same vertex and no other vertices are visited more than once. Possible cycles are $FCADF$, $FCBEF$, $FDACF$ and $FEBCF$.

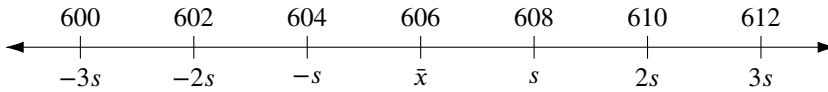
QUESTION 19

(a) Number of people surveyed = $374 + 226 + 200 + 85 + 62 + 32 + 21$
 $= 1000$

(b) Percentage = 60%

(c) Percentage = $80 - 60$
 $= 20\%$

QUESTION 20



- (a) $z\text{-score} = 0$
- (b) Amount = 602 mL
- (c) Percentage = $\frac{1}{2}(100 - 99.7)$
= 0.15%

QUESTION 21

- (a) Using the unitary method:
 35% of number captured = 420
 1% of number captured = $420 \div 35$
 = 12
 100% of number captured = 12×100
 = 1200
 \therefore In the second stage 1200 flying foxes were captured.

- (b) Let N be the number of flying foxes. Therefore:
 $\frac{770}{N} = \frac{420}{1200}$
 $420N = 924000$
 $N = 2200$
 \therefore There are approximately 2200 flying foxes in this section.

QUESTION 22

- (a) Activities P and Q have no predecessor.

(b)

Activity	Time (hours)	Predecessor
P	3	–
Q	4	–
R	5	P
S	6	Q
T	5	R
U	8	S

- (c) Path PRT will take 13 hours to complete. Path QSU will take 18 hours to complete, so path PRT can be completed within this time. The entire project won't be completed until all activities are completed, so will take 18 hours.

QUESTION 23

$$\begin{aligned}\text{Daily energy use} &= 2500 \times 6 \\ &= 15\,000 \text{ Wh} \\ &= 15 \text{ kWh}\end{aligned}$$

$$\begin{aligned}\text{Annual energy use} &= 15 \times 90 \\ &= 1350 \text{ kWh}\end{aligned}$$

$$\begin{aligned}\therefore \text{Cost} &= 1350 \times 0.2866 \\ &= \$386.91\end{aligned}$$

QUESTION 24

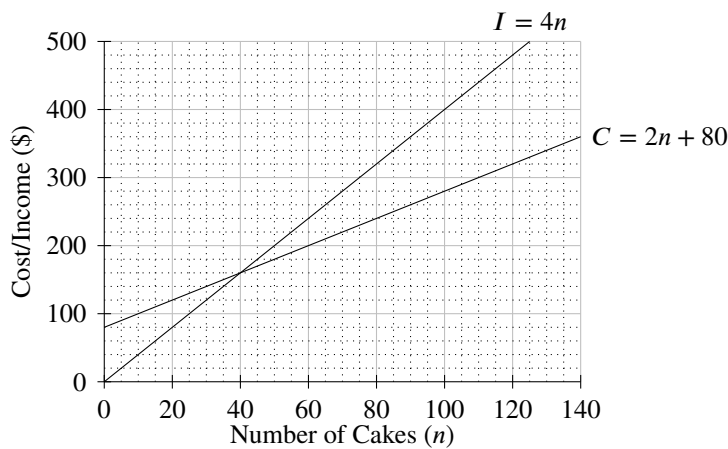
(a) $a = 80$

$$\begin{aligned}b &= \frac{\text{change in } C}{\text{change in } n} \\ &= \frac{360 - 80}{140 - 0} \\ &= \frac{280}{140} \\ &= 2\end{aligned}$$

\therefore Equation for production costs is $C = 2n + 80$.

(b) Equation for production costs is $I = 4n$.

(c)



(d) 40 pastries must be made and sold to break even.

(e) From the graph, 90 pastries must be made and sold to make a profit of \$100.

or

$$\begin{aligned}\text{Profit} &= \text{Income} - \text{Cost} \\ &= 4n - (2n + 80) \\ &= 4n - 2n - 80 \\ &= 2n - 80\end{aligned}$$

$$\begin{aligned}\text{For a profit of } \$100, \\ 2n - 80 &= 100 \\ 2n &= 180 \\ n &= 90\end{aligned}$$

\therefore 90 pastries must be made and sold to make a profit of \$100.

QUESTION 25

(a) $r = 6 \div 12$
 $= 0.5\%$ per month
 $n = 3 \times 12$
 $= 36$ months

$$\begin{aligned}\text{Future value} &= 39.3361 \times 450 \\ &= \$17\,701.25\end{aligned}$$

(b) Total payments = $450 \times 3 \times 12$
 $= \$16\,200$

$$\begin{aligned}\therefore \text{Interest} &= 17701.25 - 16200 \\ &= \$1501.25\end{aligned}$$

(c) Using the compound interest formula:

$$\begin{aligned}A &= P(1 + r)^n \\ 17701.25 &= P(1 + 0.005)^{36} \\ 17701.25 &= P(1.005)^{36} \\ 1.196680525P &= 17701.25 \\ P &= 14791.95962\end{aligned}$$

\therefore Ayla should invest \$14 791.96 at the start of the 3 years to reach the same future value.

QUESTION 26

(a) Mean = 15.7

(b) $Q_1 = 14$
 $Q_3 = 16$
Interquartile range = $16 - 14$
 $= 2$

$$\begin{aligned}\text{Outlier} &< Q_1 - 1.5 \times \text{IQR} \\ &< 14 - 1.5 \times 2 \\ &< 11\end{aligned}$$

$$\begin{aligned}\text{Outlier} &> Q_3 + 1.5 \times \text{IQR} \\ &< 16 + 1.5 \times 2 \\ &< 19\end{aligned}$$

\therefore The score of 12 may not be regarded as an outlier, while the score of 22 may be regarded as an outlier.

(c) When the outlier is removed, the mean will decrease but the median remains unchanged.

QUESTION 27

$$\begin{aligned}\text{Total distance travelled} &= 13.7 \times 2 \times 5 \\ &= 137 \text{ km}\end{aligned}$$

$$\begin{aligned}\text{Amount of fuel required} &= 137 \times \frac{9.6}{100} \\ &= 13.152 \text{ L}\end{aligned}$$

$$\begin{aligned}\therefore \text{Cost} &= 13.152 \times 1.57 \\ &= 20.64864 \\ &= \$20.65\end{aligned}$$

QUESTION 28

$$\begin{aligned}r &= 6 \div 4 \\ &= 1.5\% \text{ per quarter} \\ &= 0.015\end{aligned}$$

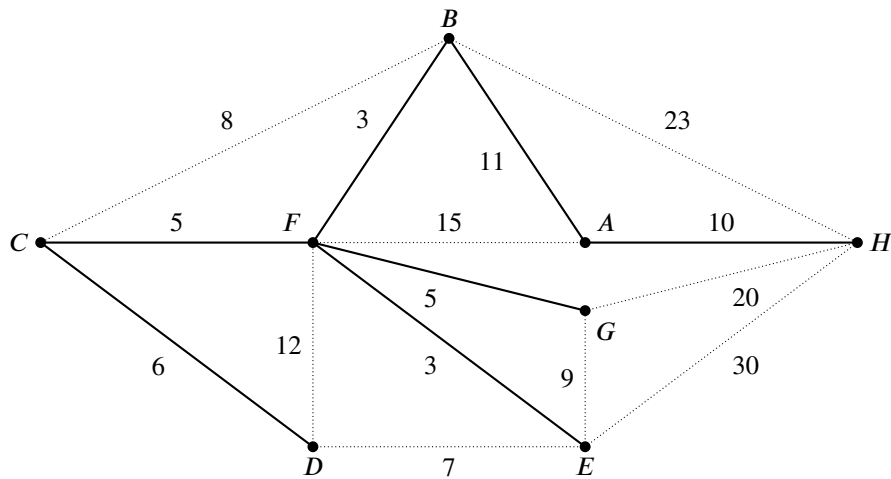
$$\begin{aligned}n &= 5 \times 4 \\ &= 20 \text{ quarters}\end{aligned}$$

$$\begin{aligned}\therefore \text{Future value} &= P(1 + r)^n \\ &= 3900(1 + 0.015)^{20} \\ &= 3900(1.015)^{20} \\ &= \$5252.73\end{aligned}$$

QUESTION 29

(a) Using Kruskal's method:

<i>AB</i>	11	✓
<i>AF</i>	15	
<i>AH</i>	10	✓
<i>BC</i>	8	(This edge cannot be used as it would create a cycle.)
<i>BF</i>	3	✓
<i>BH</i>	23	
<i>CD</i>	6	✓
<i>CF</i>	5	✓
<i>DE</i>	7	(This edge cannot be used as it would create a cycle.)
<i>DF</i>	12	
<i>EF</i>	3	✓
<i>EG</i>	9	(This edge cannot be used as it would create a cycle.)
<i>EH</i>	30	
<i>FG</i>	5	✓
<i>GH</i>	20	



(b) Minimum weight = $11 + 10 + 3 + 6 + 5 + 3 + 5$
 $= 43$ minutes

QUESTION 30

(a) Taxable income = $81797 - 6947$
 $= \$74\,850$

(b) Income tax = $7500 + 0.30 \times (74850 - 60000)$
 $= \$11\,955$

(c) Medicare levy = 2% of $\$74\,850$
 $= 0.02 \times 74850$
 $= \$1497$

(d) Total tax = $11955 + 1497$
 $= \$13\,452$

\therefore Mia's refund = $14100 - 13452$
 $= \$648$

QUESTION 31

(a) $r = 0.952481354$
 ≈ 0.952

(b) There is a strong positive correlation between the waiting time and the number of customers.

(c) $m = 0.979166667$
 $c = 1.75$

\therefore Equation is $y = 0.98x + 1.75$

(d) The value of c , the vertical intercept indicates an expected waiting time of 1.75 minutes (1 min 45 s) if Amelia sits down in an empty restaurant.

QUESTION 32

Using the declining balance method,

$$\begin{aligned}
 S &= V_0(1 - r)^n \\
 2023 &= V_0(1 - 0.10)^3 \\
 2023 &= V_0(0.90)^3 \\
 0.729V_0 &= 2023 \\
 V_0 &= 2775.034294
 \end{aligned}$$

∴ The purchase price of the computer was \$2775.

QUESTION 33

For Sofia, $BAC = 0.05$, $M = 59$ and $N = 4$.

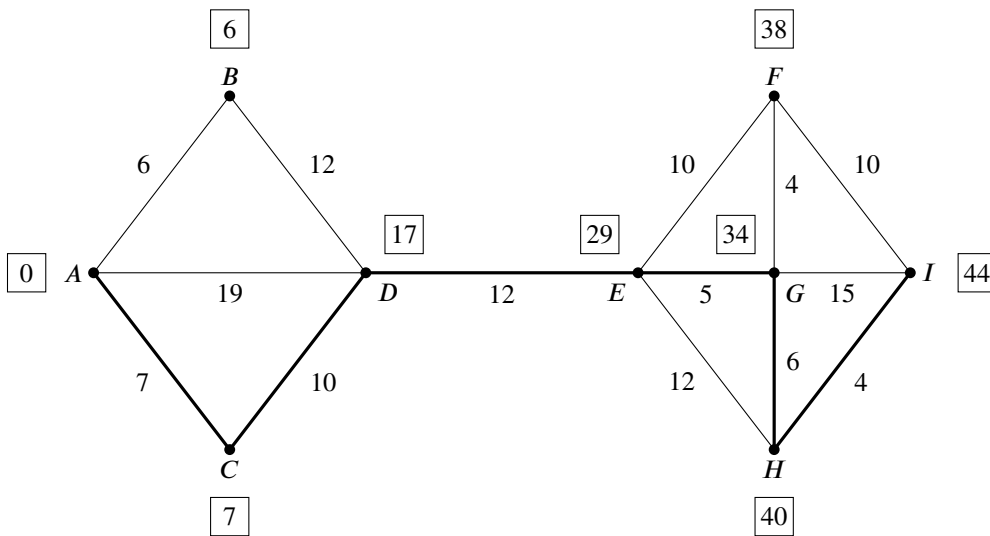
Using the BAC equation,

$$\begin{aligned}
 BAC_{\text{female}} &= \frac{10N - 7.5H}{5.5M} \\
 0.05 &= \frac{10(4) - 7.5H}{5.5(59)} \\
 0.05 &= \frac{40 - 7.5H}{324.5} \\
 16.225 &= 40 - 7.5H \\
 -7.5H &= -23.775 \\
 H &= 3.17 \\
 &\approx 3 \text{ h } 10 \text{ min}
 \end{aligned}$$

∴ Sofia will be able to leave at approximately 10:10 pm.

QUESTION 34

(a)



∴ The shortest time from A to I is 44 minutes.

(b) The route which gives the shortest driving time is $ACDEGHI$.

(c) Road length = ST

$$\begin{aligned}
 &= 90 \times \frac{44}{60} \\
 &= 66 \text{ km}
 \end{aligned}$$

QUESTION 35

$$\begin{aligned} \text{(a)} \quad r &= 7.2 \div 12 \\ &= 0.6\% \text{ per month} \\ n &= 30 \times 12 \\ &= 360 \text{ months} \end{aligned}$$

$$\begin{aligned} \text{Monthly repayment} &= 350000 \div 147.321 \\ &= 2375.764487 \\ &= \$2375.76 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{Total amount repaid} &= 2375.76 \times 12 \times 30 \\ &= \$855\,273.60 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \text{Sophie's estimated monthly repayment} &= 2375.76 + 350 \\ &= \$2725.76 \end{aligned}$$

$$\begin{aligned} r &= 7.2 \div 12 \\ &= 0.6\% \text{ per month} \\ n &= 20 \times 12 \\ &= 240 \text{ months} \end{aligned}$$

$$\begin{aligned} \text{Monthly repayment} &= 350000 \div 127.008 \\ &= 2755.731922 \\ &= \$2755.73 \end{aligned}$$

\therefore Sophie's new monthly repayment is less than the repayment required. She will not be able to repay the loan within 20 years.

or

$$\begin{aligned} \text{Sophie's estimated monthly repayment} &= 2375.76 + 350 \\ &= \$2725.76 \end{aligned}$$

$$\begin{aligned} r &= 7.2 \div 12 \\ &= 0.6\% \text{ per month} \\ n &= 20 \times 12 \\ &= 240 \text{ months} \end{aligned}$$

$$\begin{aligned} \text{Maximum amount borrowed} &= 2725.76 \times 127.008 \\ &= 346193.3261 \\ &= \$346\,193.33 \end{aligned}$$

\therefore Sophie will not be able to repay the loan within 20 years. The most that Sophie will be able to borrow over 20 years is approximately \$356 193.

QUESTION 36

$$\begin{aligned} \text{Area of front} &\approx \frac{h}{2}[d_f + d_l] + \frac{h}{2}[d_f + d_l] + \frac{h}{2}[d_f + d_l] \\ &\approx \frac{5}{2}[5 + 4.1] + \frac{3}{2}[4.1 + 2.7] + \frac{7}{2}[2.7 + 0] \\ &\approx 22.75 + 10.2 + 9.45 \\ &\approx 42.4 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of base} &= lb \\ &= 15 \times 12 \\ &= 180 \text{ cm}^2 \end{aligned}$$

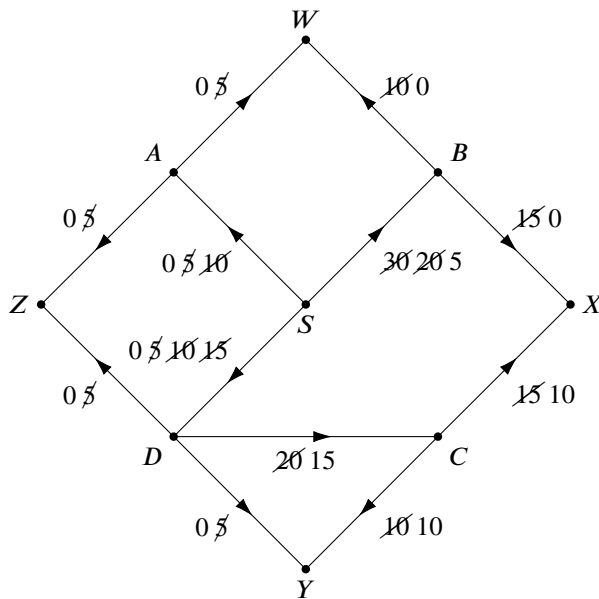
$$\begin{aligned} \text{Area of left side} &= lb \\ &= 5 \times 12 \\ &= 60 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of curved surface} &\approx 564.8 - 60 - 180 - (2 \times 42.4) \\ &\approx 240 \text{ cm}^2 \end{aligned}$$

QUESTION 37

(a) Capacity of cut = 15 + 30 + 10 + 15 + 10
= 80 people per minute

(b)



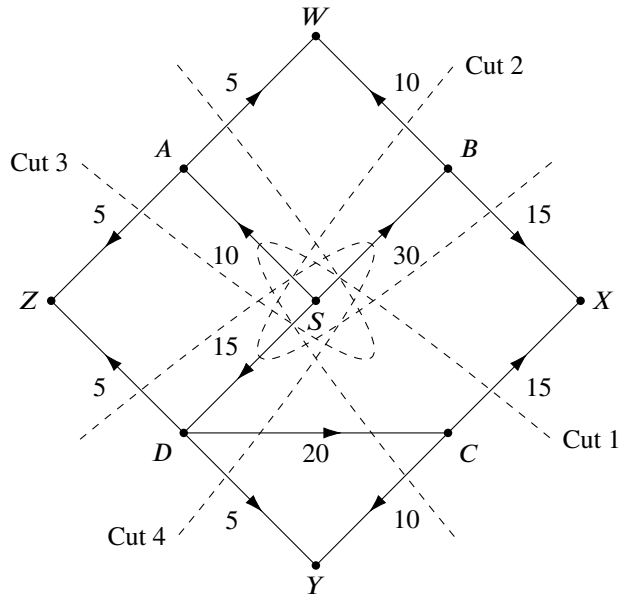
Possible routes are:

<i>SAW</i>	5
<i>SAZ</i>	5
<i>SBW</i>	10
<i>SBX</i>	15
<i>SDZ</i>	5
<i>SDY</i>	5
<i>SDCX</i>	5
<i>SDCY</i>	0

Maximum flow: 50

∴ The maximum flow through this network is 50 people per minute.

or



$$\begin{aligned} \text{Capacity of cut 1} &= 15 + 30 + 10 + 15 + 10 \\ &= 80 \text{ people per minute} \end{aligned}$$

$$\begin{aligned} \text{Capacity of cut 2} &= 10 + 10 + 15 + 15 \\ &= 50 \text{ people per minute} \end{aligned}$$

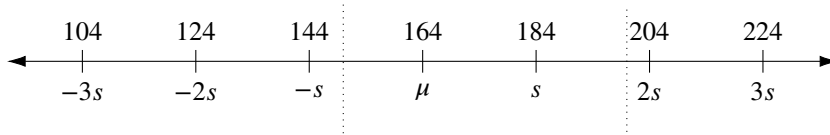
$$\begin{aligned} \text{Capacity of cut 3} &= 5 + 15 + 30 + 5 \\ &= 55 \text{ people per minute} \end{aligned}$$

$$\begin{aligned} \text{Capacity of cut 4} &= 5 + 10 + 30 + 5 \\ &= 50 \text{ people per minute} \end{aligned}$$

\therefore Using the minimum flow-maximum cut theorem, the maximum flow through this network is 50 people per minute.

- (c) The edge SB is below capacity, while edges BW and BX are at capacity. Either of these two edges could be increased by 5 people per minute to improve the maximum flow. Alternatively, the outflow at D is 30 people per minute while the inflow from S is only 15 people per minute. Increasing the capacity of edge SD by up to 15 people per minute will also increase the overall flow of the network.

QUESTION 38



For a distance of 150 km:

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &= \frac{150 - 164}{20} \\ &= -0.7 \end{aligned}$$

For a distance of 200 km:

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &= \frac{200 - 164}{20} \\ &= 1.8 \end{aligned}$$

$$\begin{aligned} \therefore P(\text{distance between 150 km and 200 km}) &= P(z < 1.8) - P(z < -0.7) \\ &= 0.964 - 0.242 \\ &= 0.722 \end{aligned}$$