Ryde SECONDARY COLLEGE
INSPIRE ACHIEVE SUCCEED

NESA Student Number:

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 32)

- Attempt Questions 11 – 32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. An infinite geometric series has a first term of 10 and a limiting sum of 30.

What is the common ratio?

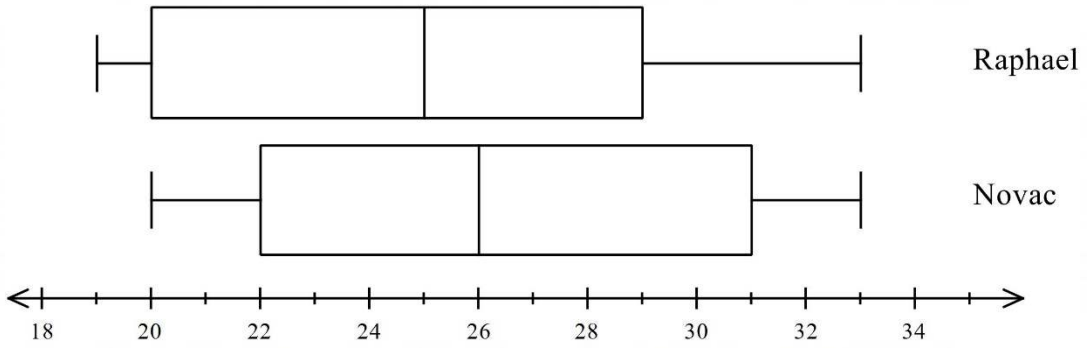
A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

2. The parallel box plots below are used to compare the time taken to complete a set of tennis (to the nearest minute), by two players over their last 40 games.



Which is a true statement about the two sets of data?

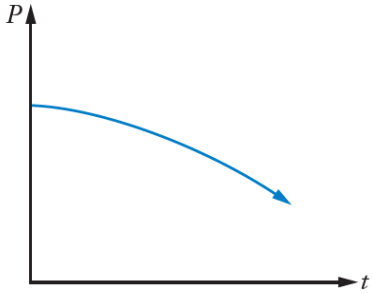
A. They have the same interquartile range.

B. They have the same median.

C. They have the same range.

D. They have the same upper quartile.

3. The population (P) of Marvel, a super villain's town is shown over time (t) by the graph below:



Which statement best describes the population of Marvel.

A. The population is increasing at an increasing rate.

B. The population is increasing at a decreasing rate.

C. The population is decreasing at an increasing rate.

D. The population is decreasing at a decreasing rate.

4. The curve $y = \ln x$ is translated to the left by π units and then dilated horizontally by a scale factor of 3.

What is the equation which describes this new curve?

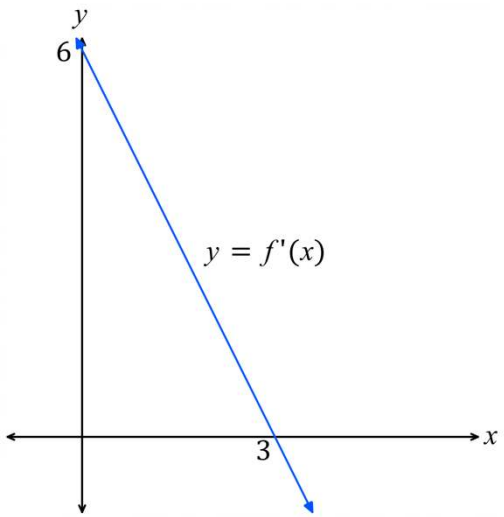
A. $y = \ln\left(\frac{x}{3} + \pi\right)$

B. $y = \ln\left(\frac{x}{3} - \pi\right)$

C. $y = 3\ln(x + \pi)$

D. $y = \ln\left(\frac{x+\pi}{3}\right)$

5. The graph of $y = f'(x)$ is shown below.



Not to scale

The curve $y = f'(x)$ has a maximum value of 20.
What is the equation of the curve?

A. $y = x^2 - 6x + 11$
B. $y = x^2 - 6x + 20$
C. $y = 20 + 6x - x^2$
D. $y = 11 + 6x - x^2$

6. $\int \left(\frac{1}{5}\right)^x dx$

A. $\frac{1}{\ln 5 \cdot 5^x} + C$

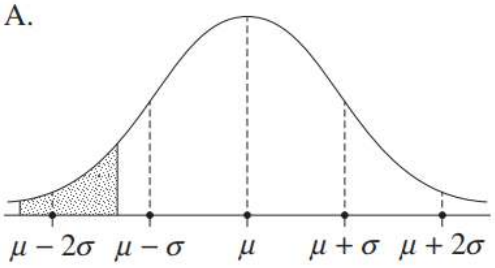
B. $\frac{-1}{\ln 5 \cdot 5^x} + C$

C. $\frac{-1}{\ln \frac{1}{5} \cdot \frac{1}{5}x} + C$

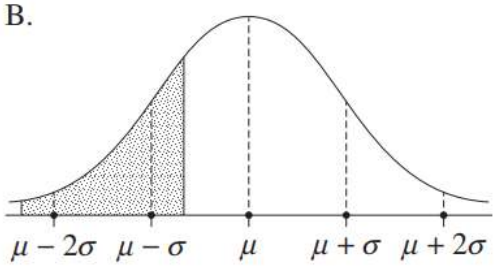
D. $\frac{1}{5} \ln x + C$

7. Assume the weights of newborn babies are normally distributed with a mean of μ and a standard deviation of σ. A particular baby has a weight below the lower quartile of the distribution but not in the bottom 10% of the distribution. Which of the following best represents the region in which this baby's weight lies?

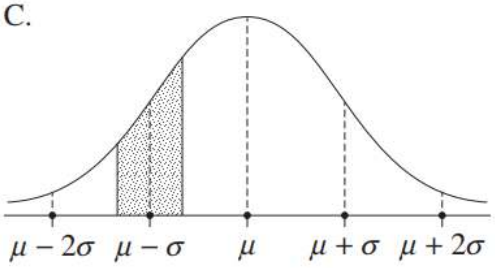
A.



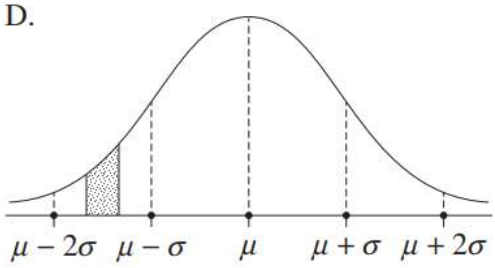
B.



C.



D.



8. Differentiate $x \sin\left(\frac{1}{x}\right)$

A. $\sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right)$

B. $\cos\left(\frac{1}{x}\right) - \frac{1}{x}\sin\left(\frac{1}{x}\right)$

C. $\sin\left(\frac{1}{x}\right) + \frac{1}{x}\cos\left(\frac{1}{x}\right)$

D. $x\cos\left(\frac{1}{x}\right)$

9. Solve $\tan\left(x + \frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}}$ for $[-\pi \leq x \leq \pi]$

A. $-\frac{4\pi}{3}, \frac{\pi}{3}$

B. $-\frac{5\pi}{6}, \frac{\pi}{6}$

C. $-\frac{2\pi}{3}, \frac{\pi}{3}$

D. $\frac{5\pi}{6}, -\frac{\pi}{6}$

10. The probability density function of a random variable is given by:

$$f(x) = \begin{cases} \frac{1}{50} e^{-\frac{x}{50}} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

What is the median to 1 decimal place?

A. -1.0

B. 0.02

C. 0.5

D. 34.7

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

NESA Student Number:

Mathematics Advanced

Section II Answer Booklet 1

90 marks
Attempt Questions 11 – 23

Allow about 2 hours and 45 minutes for this section

Booklet 1 — Attempt Questions 11–23 (46 marks)
Booklet 2 — Attempt Questions 24–31 (44 marks)

Instructions

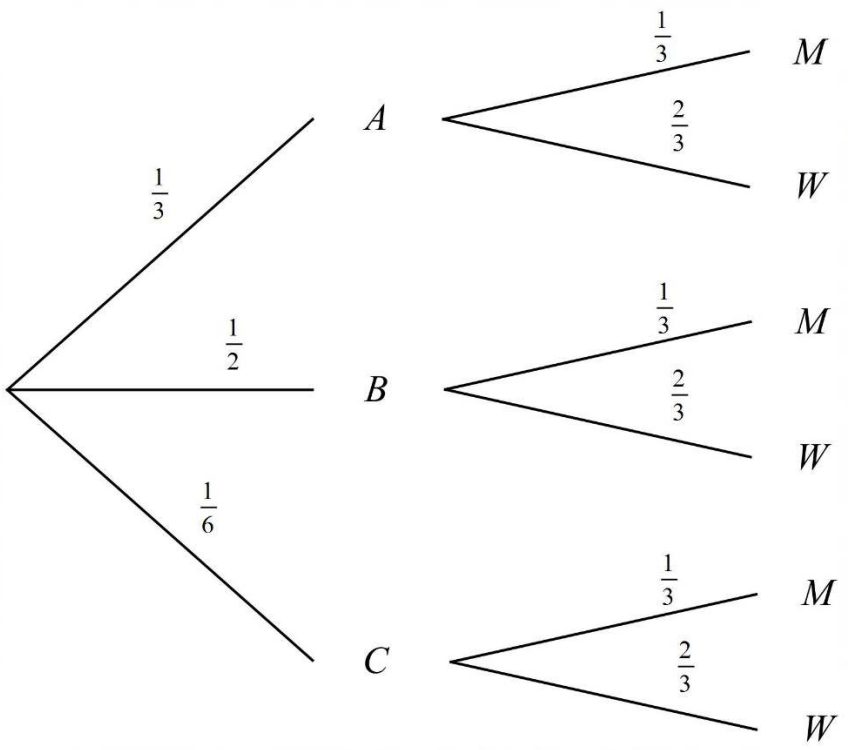
- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of Booklet 1. If you use this space, clearly indicate which question you are answering.

Question 11 (3 marks)

There are three candidates for team leader on a camp, Andie, Bec and Cat.

There are two candidates for team mascot at the camp, Miao and Woof.

Based on a survey of camp attendees, the probability tree shows the likelihood of the candidates being chosen for each position.



(a) What is the probability that Andie will be leader with Miao as mascot? **1**

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(b) What is the probability that Andie will be the leader or Miao will be the mascot, but not both? **2**

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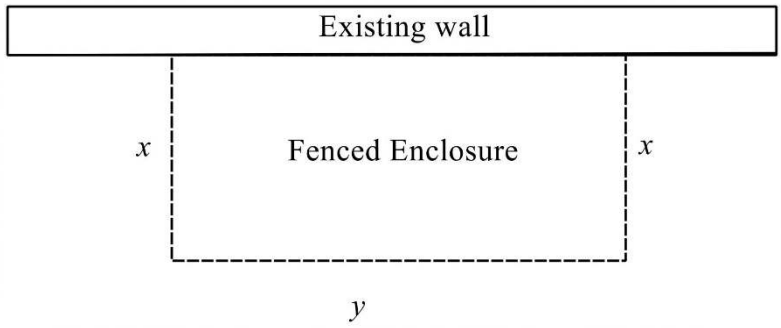
Question 12 (4 marks)

Chloe plans to build a rectangular enclosure for her new chickens against an existing wall, using 16 metres of wire fencing as shown below. She calls the dimensions of the enclosure x and y.



(a) She reasons that $2x + y = 16$. Write y in terms of x and hence, show that the area of the enclosure can be written as $A = 16x - 2x^2$. **1**

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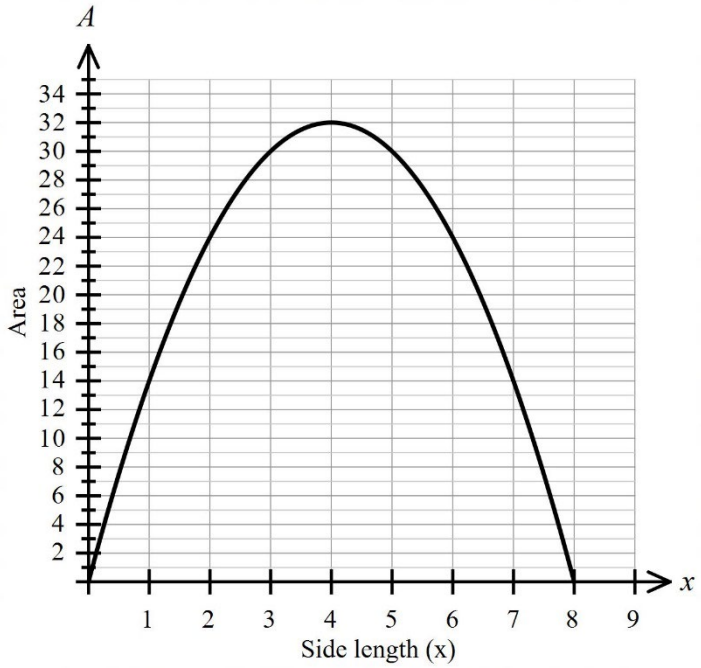
(b) The graph of $A = 16x - 2x^2$ is shown below. **2**



Use the graph to determine the value(s) of x which would give the enclosure an area of 24 m^2 and give the dimensions of the enclosure in each case.

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Question 12 continues on page 10.

(c) Determine the dimensions (both x and y) that will achieve the maximum possible area for the enclosure. **1**

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Question 13 (4 marks)

Differentiate the following with respect to x:

(a) $\dfrac{3e}{2x^2}$ **1**

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(b) $\dfrac{\log_e x}{x^2}$ **2**

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(c) $\sqrt{\sin x}$ **1**

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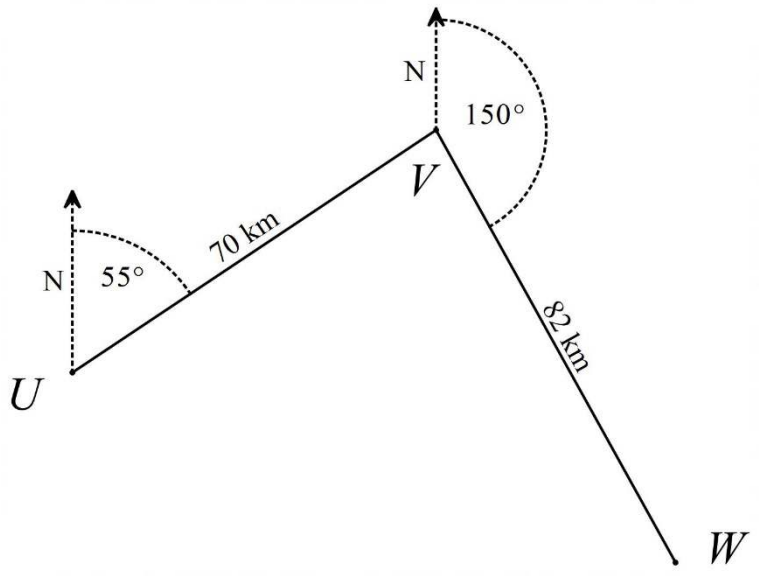
Question 14 (4 marks)

A helicopter leaves Underwood and flies 70 km on a bearing of 055° to Vanna Beach.
It then flies 82 km on a bearing of 150° to Weston.

The diagram below illustrates the journey.



(a) It then plans to fly directly back to Underwood. **2**
Calculate, to the nearest km, the distance that it will fly.

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(b) Determine the bearing (from Weston) on which it should fly to return to Underwood. **2**
Give your answer to the nearest degree.

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Question 15 (3 marks)

The displacement of a particle at time (t) seconds is given by: **3**

$$x = 3e^{-2t} + 4e^{-t} + 2t \ .$$

Find the exact time at which the particle comes to rest.

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Question 16 (3 marks)

(a) Show $\dfrac{3\sec^2 3x}{\tan 3x} = 3\sec 3x \ \operatorname{cosec} 3x$ **2**

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(b) Hence or otherwise find $\int 3\sec 3x \ \operatorname{cosec} 3x \ dx$ **1**

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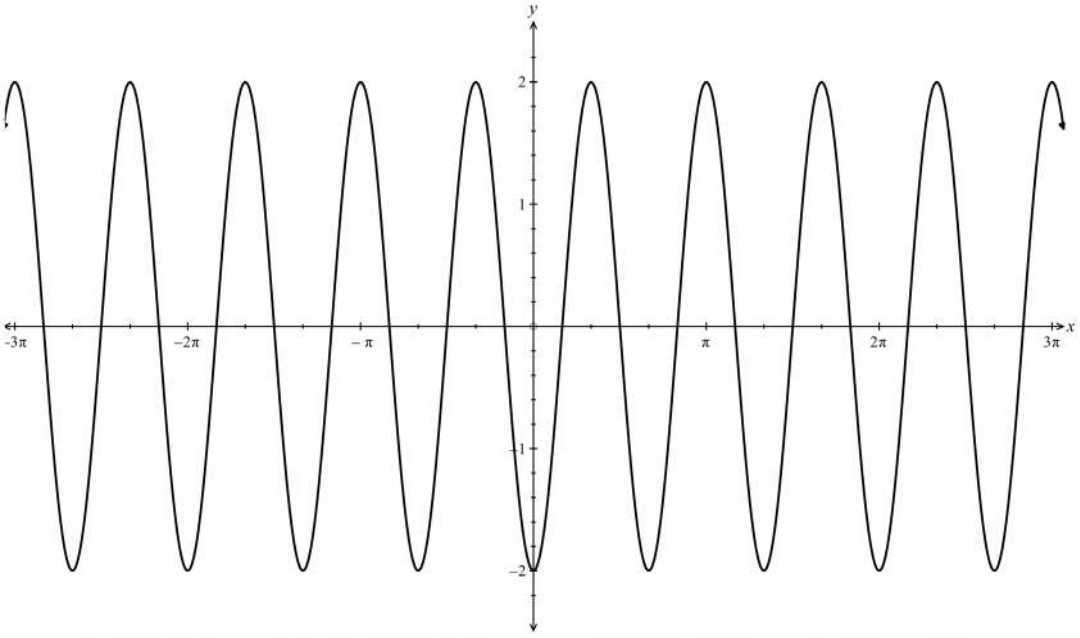
Question 17 (3 marks)

James sketched the curve shown below to model a tidal experiment in his Science class. James knows the curve is in the form $y = k\cos(ax + b)$ where $k > 0$, $a > 0$ and $0 \leq b \leq \pi$. **3**

By finding k, a and b, write the equation for James' tidal experiment.



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Question 18 (3 marks)

The gradient of a curve is given by $y' = 1 - 6\sin 3x$. **3**

The curve passes through the point (0, 7).

What is the equation of the curve?

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Question 19 (4 marks)

For an arithmetic series the third term (T_3) is 32 and the sixth term (T_6) is 17.

(a) Find the common difference. **2**

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(b) Find the sum of the terms from T_{10} to T_{20} inclusive. **2**

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Question 20 (5 marks)

At 7 pm on a Wednesday evening after a storm, Jack's water tank was full. The capacity of the tank was 3 000 litres. The tap on the tank, however, was leaking such that the change in volume at any time (t) hours was proportional to the volume (V) of the tank and can be modelled by the equation $V = V_0 e^{kt}$.

(a) Find V_0, the initial volume of the tank. **1**

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(b) Given that the volume of the tank after 3 hours is 1 900 litres, find the value of k correct to 4 decimal places. **2**

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(c) By the time Jack discovered that the tank was leaking, there were only 250 litres of water remaining. At what time and on which day did Jack discover the leak (correct to the nearest minute)? **2**

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Question 21 (4 marks)

The table below gives the future value of an annuity of $1 per period for various periods and interest rates.

Table of Future Value Interest Factors								
	Interest rate per period							
Number of Periods	0.25%	0.30%	0.35%	0.40%	0.45%	0.50%	0.55%	0.60%
53	56.5961	57.3530	58.1230	58.9063	59.7033	60.5141	61.3391	62.1785
54	57.7376	58.5250	59.3264	60.1419	60.9719	61.8167	62.6765	63.5516
55	58.8819	59.7006	60.5340	61.3825	62.2463	63.1258	64.0212	64.9329
56	60.0291	60.8797	61.7459	62.6280	63.5264	64.4414	65.3733	66.3225
57	61.1792	62.0624	62.9620	63.8786	64.8123	65.7636	66.7329	67.7204
58	62.3322	63.2485	64.1824	65.1341	66.1040	67.0924	68.0999	69.1267
59	63.4880	64.4383	65.4070	66.3946	67.4014	68.4279	69.4744	70.5415
60	64.6467	65.6316	66.6359	67.6602	68.7047	69.7700	70.8565	71.9647
61	65.8083	66.8285	67.8692	68.9308	70.0139	71.1189	72.2463	73.3965
62	66.9729	68.0290	69.1067	70.2065	71.3290	72.4745	73.6436	74.8369
63	68.1403	69.2331	70.3486	71.4874	72.6499	73.8368	75.0487	76.2859
64	69.3106	70.4408	71.5948	72.7733	73.9769	75.2060	76.4614	77.7436
65	70.4839	71.6521	72.8454	74.0644	75.3098	76.5821	77.8820	79.2101
66	71.6601	72.8670	74.1004	75.3607	76.6487	77.9650	79.3103	80.6854

(a) Julia invests $250 per month in an annuity which pays 5.4% p.a. compounding monthly. What will be the value of the annuity after 5 years? **2**

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(b) Carrington finds that he can only get an interest rate of 3.6% p.a. compounding monthly. **2**

If he wants to achieve the same total amount as Julia after the same period, what amount should he invest each month?

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Question 22 (2 marks)

Consider the geometric series below: **2**

$$\left(\frac{4}{x+1}\right) + \left(\frac{4}{x+1}\right)^2 + \left(\frac{4}{x+1}\right)^3 + \ldots\ldots\ldots$$

Show that the series will have a sum to infinity when $x = 4$, but not when $x = -4$.

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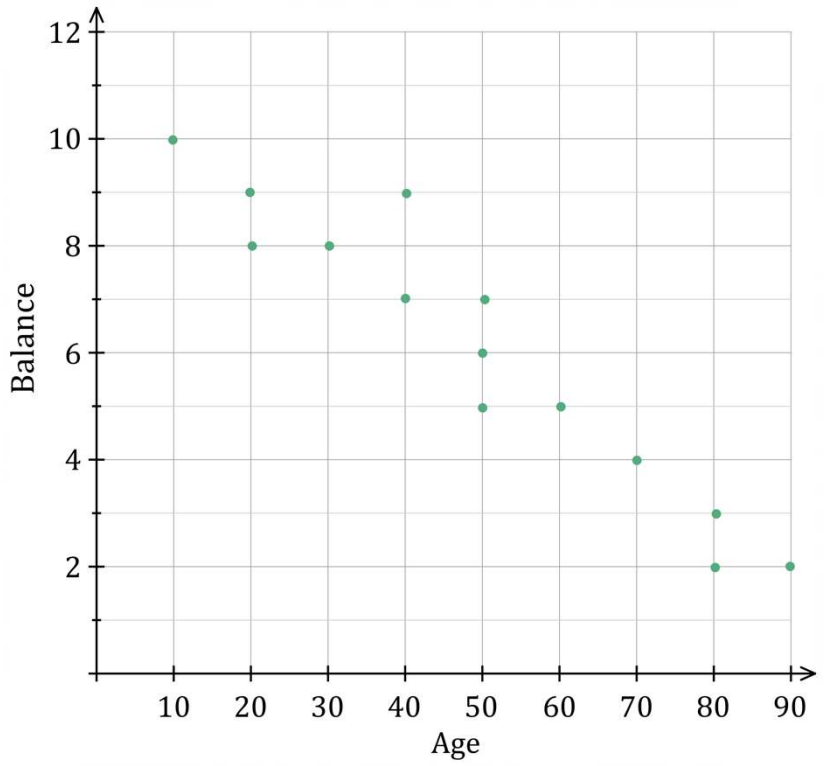
Question 23 (4 marks)

The scatterplot below shows the relationship between age and balance.



(a) Draw a line of best fit on the scatterplot. Find the gradient of this line. **2**

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(b) Hannah is 40 years old. What is her expected balance ? **1**

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(c) Calculate the value of the Pearson's correlation coefficient. Answer correct to two decimal places. **1**

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2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

NESA Student Number:

Mathematics Advanced

SectionII Answer Booklet 2

Booklet 2 — Attempt Questions 24–31 (44 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of Booklet 2. If you use this space, clearly indicate which question you are answering.

Question 24 (6 marks)

A company manufactures light bulbs. The expected life of the light bulb t in days is given by the probability density function.

$$f(t) = \begin{cases} \frac{1}{k} e^{-\frac{t}{4\,000}} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k. **2**

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(b) A light bulb is considered faulty if its lifespan is less than 100 days. **2**

Determine the percentage of light bulbs that are considered faulty and the z-score that 100 days would represent on a normal distribution curve.

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(c) Find the median number of days a light bulb lasts if it is produced by this company. **2**

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Question 25 (3 marks)

Lachie and Emily are enrolled in different schools but are doing the same courses in English and Mathematics.

When they complete their Trial HSC Exams, they compare their results in Mathematics and English from their separate schools.

	English Result	Mathematics Result
Lachie (Bradman College)	68	Forgotten
Emily (Waugh High School)	68	79

The statistical results for the exams at both schools are shown below.

All of the exam results were normally distributed.

School	Mean English	Standard Deviation English	Mean Mathematics	Standard Deviation Mathematics
Bradman College	68	8	62	9
Waugh High School	74	6	64	5

(a) Use *z*-scores to determine which student performed best on their English exam, compared to their school cohort. **1**

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(b) Lachie did not remember his Maths mark but did remember being told he had a z-score of 2 on the Maths exam. **2**

Calculate Lachie's raw Maths mark and determine who performed best on their Maths exam, compared to their school cohort.

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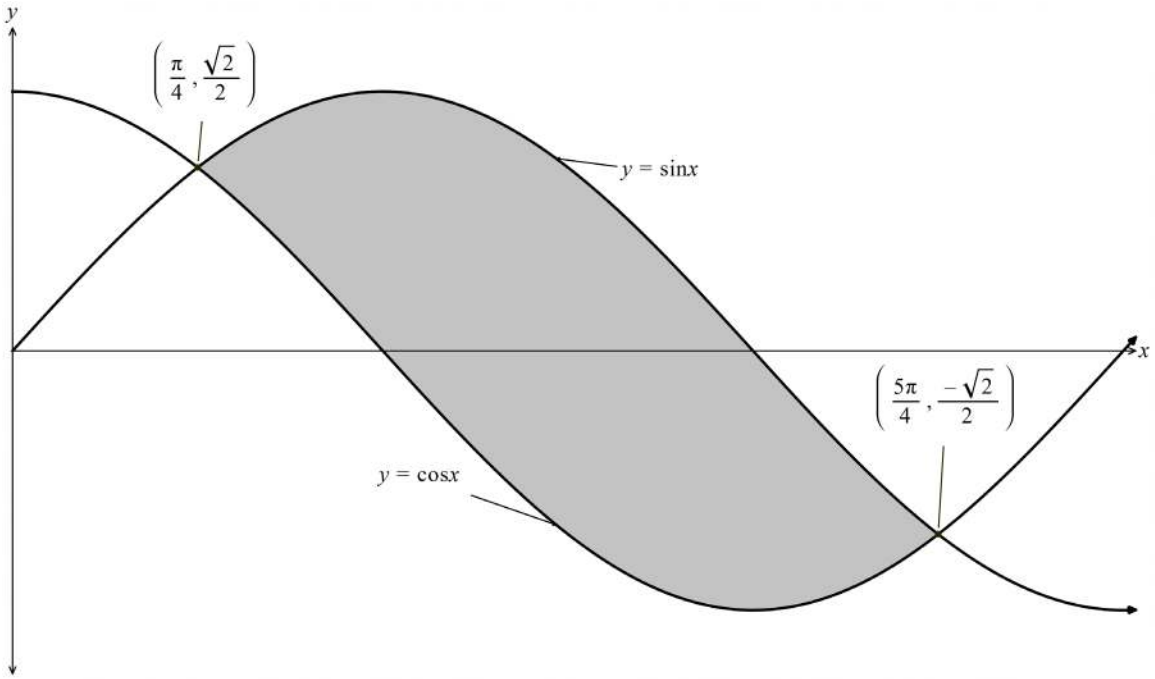
Question 26 (4 marks)

The graph below shows an area enclosed between the curves $y = \sin x$ and $y = \cos x$ **4**



Find an exact value for the shaded area.

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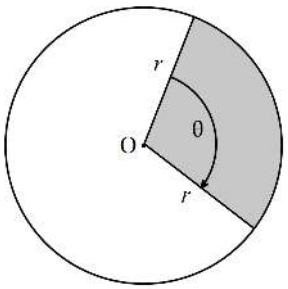
Question 27 (4 marks)

The diagram below shows a shaded sector in a circle with radius r and centre O. **4**

The sector subtends an angle of θ at the centre of the circle. The area of the sector is 49 cm^2.



Show that $\theta = \frac{98}{r^2}$ and hence show that the minimum perimeter of the sector will occur when the arc length of the sector is equal to the diameter of the circle.

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Question 28 (6 marks)

The velocity, in cm/s of a particle moving in a straight line is given by the equation:

$$v = 4t^3 - 24t^2 + 20t$$ where t is the time in seconds.

When $t = 0$ the particle is at $x = 3$. That is, the particle is initially 3cm to the right of the origin.

(a) Find an expression for x, the position of the particle as a function of time. **2**

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(b) What feature will the graph of x as a function of t have at $t = 1$? **2**

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(c) How far will the particle travel in the first 2 seconds? **2**

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Question 29 (4 marks)

The function $f(x) = \cos 3x - 1$ is defined in the interval $0 \leq x \leq \frac{2\pi}{3}$.

(a) What is the amplitude and period of the function? **1**

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(b) What is the range of the function? **1**

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(c) Sketch $f(x) = \cos 3x - 1$ in the interval $0 \leq x \leq \frac{2\pi}{3}$. **2**

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Question 30 (3 marks)

$$A = \int_1^5 \frac{10}{x+1} dx$$

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Use the trapezoidal rule with four sub-intervals to evaluate the above integral.
Answer correct to two decimal places.

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Question 31 (7 marks)

The derivative of a function $y = f(x)$ is given by $f'(x) = 3x^2 - 2x - 1$.

(a) Find the x-values of the two stationary points of $y = f(x)$, and determine the nature of the stationary points. **2**

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(b) The curve passes through the point (0, 3). Find an expression for $f(x)$. **2**

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Questions 31 continues on page 30

(c) Hence sketch the curve, clearly indicating the stationary points. **2**

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(d) For what values of x is the curve concave up? **1**

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Question 32 (7 marks) **Marks**

A rectangular prism has a height h cm, width x cm and length $2.5x$ cm. The surface of area of the rectangular prism is 6480 cm^2.

(a) Find an expression for h in terms of x. **2**

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(b) Show that the volume V cm^3 of the rectangular prism is given by: **1**

$$V = \frac{5x(6480 - 5x^2)}{14}$$

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(c) If the volume of the rectangular prism is to be maximised, find the exact values of x and h. **4**

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Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

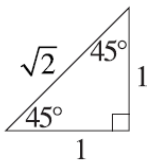
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

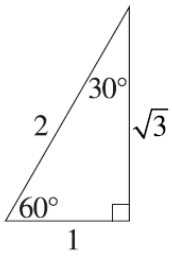
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

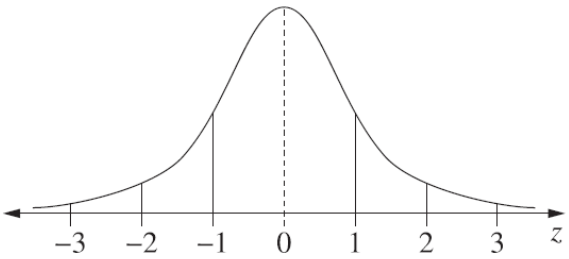
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$|\underset{\sim}{u}| = |x\underset{\sim}{i} + y\underset{\sim}{j}| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

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2023 Trial Higher School Certificate Examination
Mathematics Advanced

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ ← correct C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

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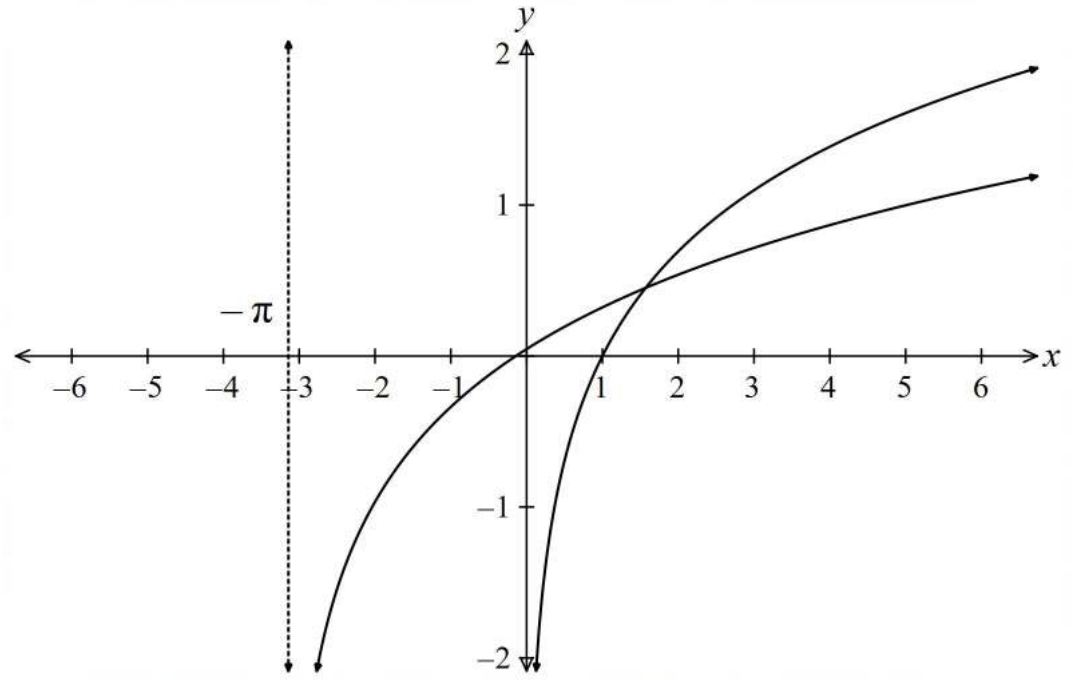
2023 | TRIAL HSC EXAMINATION

Mathematics Advanced Solutions

Section I

No	Working	Answer
1.	a = 10 and S = 15 $$S = \frac{a}{1-r}$$ $$30 = \frac{10}{1-r}$$ $$30 - 30r = 10$$ $$30r = 20$$ $$r = \frac{2}{3}$$	**C**
2.	The medians are 25 and 26 respectively, so are not equal. They have the same upper extreme but different lower extremes, so the ranges are different. For Raphael, Upper Quartile (Q_3) = 29, Lower Quartile (Q_1) = 20, Interquartile Range = 29 – 20 = 9 For Novak, Upper Quartile (Q_3) = 31, Lower Quartile (Q_1) = 22, Interquartile Range = 31 – 22 = 9 So, the Upper quartiles are not equal, but the interquartile range is the same.	**A**
3.	$\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$ Therefore it is decreasing at an increasing rate.	**C**

No	Working	Answer
4.	To shift left by a units, replace x by $x + a$, so x becomes $x + \pi$ To dilate horizontally by a factor of k, replace x by $\frac{x}{k}$, so $x + \pi$ becomes $\frac{x+\pi}{3}$ $y = \ln(x)$ becomes $y = \ln\left(\frac{x+\pi}{3}\right)$  	**D**

No	Working	Answer
5.	$\frac{dy}{dx} = mx + b$ $= -2x + 6$ $y = \int (-2x + 6)\, dx$ $= -x^2 + 6x + C$ Maximum occurs when $f'(x) = 0$ $-2x + 6 = 0$ $x = 3$ $20 = -(3)^2 + 6 \times 3 + C$ $C = 11$ $\therefore$ Equation of the curve is $y = 11 + 6x - x^2$.	**D**

No	Working	Answer
6.	$\int \left(\frac{1}{5}\right)^x dx$ $= \frac{\left(\frac{1}{5}\right)^x}{\ln\left(\frac{1}{5}\right)} + C$ $= \frac{\frac{1}{5^x}}{\ln 1 - \ln 5} + C$ $= \frac{\frac{1}{5^x}}{-\ln 5} + C$ $= -\frac{1}{5^x} \times \frac{1}{\ln 5}$ $= -\frac{1}{\ln 5 \cdot 5^x}$	**B**
7.	The given weight of the Baby is $10\% <$ Baby *weight* $< 25\%$ 25% lies within 0 → -1 standard deviations 10% lies within -1 → -2 standard deviations Only diagram *C* satisfies these conditions	**C**

No	Working	Answer
8.	$x \sin\left(\frac{1}{x}\right) \qquad u = x \qquad v = \sin(x^{-1})$ $u' = 1 \qquad v' = -\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$ $\frac{d}{dx} = \sin\left(\frac{1}{x}\right) - \frac{x}{x^2}\cos\left(\frac{1}{x}\right)$ $= \sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right)$	**A**
9.	$\tan\left(x + \frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}}$ $\tan\theta = \frac{1}{\sqrt{3}}$ $\theta = -\frac{11\pi}{6}, -\frac{5\pi}{6}, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$ $x + \frac{5\pi}{6} = -\frac{11\pi}{6}, -\frac{5\pi}{6}, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$ $x = -\frac{16\pi}{6}, -\frac{10\pi}{6}, -\frac{4\pi}{6}, \frac{2\pi}{6}, \frac{8\pi}{6}$ $x = -\frac{8\pi}{3}, -\frac{5\pi}{3}, -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}$ $x = -\frac{2\pi}{3}, \frac{\pi}{3} \quad [-\pi \le x < \pi]$	**C**

No	Working	Answer
10.	$\int_0^x \frac{1}{50} e^{-\frac{x}{50}} dx = 0.5$ $0.5 = \left[-e^{-\frac{x}{50}} \right]_0^x$ $0.5 = \left[-e^{-\frac{x}{50}} \right] - [-e^0]$ $0.5 = \left[-e^{-\frac{x}{50}} \right] + 1$ $-0.5 = -e^{-\frac{x}{50}}$ $0.5 = e^{-\frac{x}{50}}$ $-\frac{x}{50} = \ln 0.5$ $x = 34.7$	**D**

Ryde Secondary College
Mathematics Advanced Trial HSC

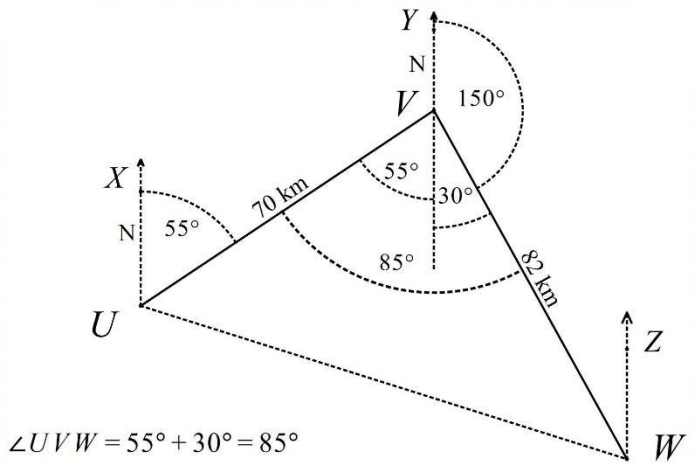
Solutions 2023

Section II

Question		Working and answer	Marks	Mark Allocation
11.	a)	$P(AM) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$	**1**	1 mark for correct solution.
	b)	$P(A \text{ or } M \text{ but not both}) = P(AW) + P(BM) + P(CM)$ $= \frac{1}{3} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{3}$ $= \frac{2}{9} + \frac{1}{6} + \frac{1}{18}$ $= \frac{4}{9}$	**2**	2 marks for correct solution. 1 mark for at least one of the relevant product probabilities calculated correctly from the tree.

12.	a)	$2x + y = 16$ $y = 16 - 2x$ $A = x(16 - 2x)$ $A = 16x - 2x^2$	1	1 mark for correct derivation.
	b)	From the graph, when $A = 24$, $x = 2$ or $x = 6$. (These could also be found by solving $16x - 2x^2 - 24 = 0$) This gives dimensions of 2 m by 12 m or 6 m by 4 m.	2	2 marks for two sets of correct dimensions 1 mark for correct values of x or equivalent merit.
	c)	The maximum area is 32 m^2 which occurs when $x = 4$ m, so the dimensions are width = 4 and length = $16 - 2 \times 4 = 8$. Dimensions are 4 m by 8 m.	1	1 mark for correct dimensions.

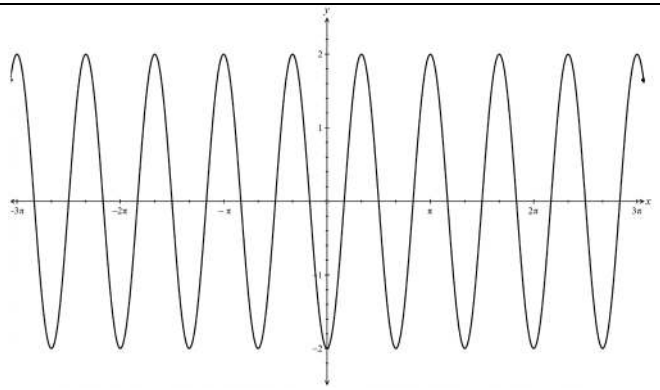
13.	a)	$\frac{d}{dx}\left[\frac{3e}{2x^2}\right]$ $\frac{d}{dx}\left[\frac{3e}{2}x^{-2}\right]$ $= -2 \times \frac{3e}{2}x^{-3}$ $= -\frac{3e}{x^3}$	1	1 mark for correct differentiation.
	b)	$\frac{d}{dx}\left[\frac{\log_e x}{x^2}\right]$ $u = \ln x$ $v = x^2$ $u' = \frac{1}{x}$ $v' = 2x$ $= \frac{\frac{x^2}{x} - 2x \ln x}{x^4}$ $= \frac{x - 2x \ln x}{x^4}$ $= \frac{x(1 - 2 \ln x)}{x^4}$ $= \frac{1 - 2 \ln x}{x^3}$	2	2 marks for correct differentiation. 1 mark for correct use of quotient rule.

	c)	$\frac{d}{dx}\sqrt{\sin x}$ $\frac{d}{dx}(\sin x)^{\frac{1}{2}}$ $= \frac{1}{2}\cos x(\sin x)^{-\frac{1}{2}}$ $= \frac{\cos x}{2\sqrt{\sin x}}$	1	1 mark for correct differentiation.
14.	a)	Let X, Y and Z be points due north of U, V and W respectively  $\angle UVW = 55° + 30° = 85°$ Using the cosine rule: $UW^2 = 70^2 + 82^2 - 2 \times 70 \times 82 \times \cos(85°)$ $= 10\,623.452$ $UW = 103.07$ $UW =$ 103 km (nearest km)	2	2 marks for correct solution 1 mark for finding $\angle UVW$ and applying trig methods to find UW or equivalent merit.

	b)	The sine rule can be used to find Two cases are shown below. $\frac{\sin \angle VWU}{70} = \frac{\sin 85°}{103.07}$ $\sin \angle VWU = \frac{70 \times \sin 85°}{103.07}$ $= 0.67656$ $\angle VWU = \sin^{-1}(0.67656)$ $= 42.575$ $\angle ZWV = 30°$ (alternate angles) $\angle UWZ = 30 + 43 = 73°$ Bearing required $= 360 - 73 = 287°$	The angle inside the tringle *UVW*. $\frac{\sin \angle VUW}{82} = \frac{\sin 85°}{103.7}$ $\sin \angle VUW = \frac{82 \times \sin 85°}{103.7}$ $= 0.7925$ $\angle VUW = \sin^{-1}(0.7925)$ $= 52.4242$ $\angle XUW = 55 + 52 = 107$ $\angle UWZ = 180 - 107 = 73°$ Bearing required $= 360 - 73 = 287°$	2	2 marks for correct solution. 1 mark for finding a correct acute angle by any method but not calculating the bearing. Or 1 mark for incorrectly finding acute angle and applying correctly in finding the bearing or equivalent merit.
		The angles can also be found using the Cosine rule, one example is shown below $\cos(\angle VWU) = \frac{82^2 + 103.07^2 - 70^2}{2 \times 82 \times 103.07}$ $= 0.73638238$ $\angle VWU = \cos^{-1}(0.73638238)$ $= 42.57584516$ $\angle ZWV = 30°$ (alternate angles) $\angle UWZ = 30 + 43 = 73°$ Bearing required $= 360 - 73 = 287°$			

15.		$x = 3e^{-2t} + 4e^{-t} + 2t$ $\frac{dx}{dt} = -6e^{-2t} - 4e^{-t} + 2$ $\frac{dx}{dt} = 0$ at rest $0 = -6e^{-2t} - 4e^{-t} + 2$ Let $x = e^{-t}$ $0 = 6x^2 + 4x - 2$ $0 = 3x^2 + 2x - 1$ $(x+1)(3x-1) = 0$ $\therefore e^{-t} = -1$(No solution exists) or $e^{-t} = \frac{1}{3}$ $-t = \ln\left(\frac{1}{3}\right)$ $= -\ln\left(\frac{1}{3}\right)$ $= -(\ln 1 - \ln 3)$ $= \ln 3$	3	3 marks for correct solution. 2 marks for correct differentiation and progress towards solving $\frac{dx}{dt} = 0$. 1 mark for correct differentiation.

16.	a)	$\frac{3\sec^2 3x}{\tan 3x} = \frac{3}{\frac{\cos^2 3x}{\frac{\sin 3x}{\cos 3x}}}$ $= \frac{3}{\cos^2 3x} \times \frac{\cancel{\cos 3x}}{\sin 3x}$ $= 3 \sec 3x \operatorname{cosec} 3x$	2	2 marks for correct solution. 1 mark for correct equivalent fractions in terms of sine and cosine.
	b)	$\int 3\sec 3x \operatorname{cosec} 3x \, dx$ $= \int \frac{3\sec^2 3x}{\tan 3x} \, dx$ $= \ln(\tan 3x) + C$	1	1 mark correct solution.

17.		 $y = k\cos(ax + b)$ * graph has amplitude = 2 $(k > 0)$ $\therefore k = 2$ * graph completes 3 cycles $0 \le x \le 2\pi$ $\therefore a = 3$ when $x = 0$ $y = -2$ $-2 = 2\cos b$ $\cos b = -1$ $b = \pi$ $(0 \le b \le \pi)$ $\therefore y = 2\cos(3x + \pi)$ or from graph see phase shift left $\frac{\pi}{3}$ units as $(0 \le b \le \pi)$ $\therefore y = 2\cos 3\left(x + \frac{\pi}{3}\right)$	3	3 marks for correct solution in either form. 2 marks for any two correct parts of *k, a,* or *b* or equivalent merit. 1 mark for any correct *k, a*, or *b* or equivalent merit. Solution is also possible treating it as a transformation of the graph of y = cosx, y is dilated by a scale factor of 2, so replace y by $\frac{y}{2}$ x is dilated by a scale factor of $\frac{1}{3}$, so replace x by 3x x is translated by $\frac{\pi}{3}$ to the left , so replace x by x+$\frac{\pi}{3}$ $\frac{y}{2} = \cos 3\left(x + \frac{\pi}{3}\right)$ $y = 2\cos 3(x + \frac{\pi}{3})$

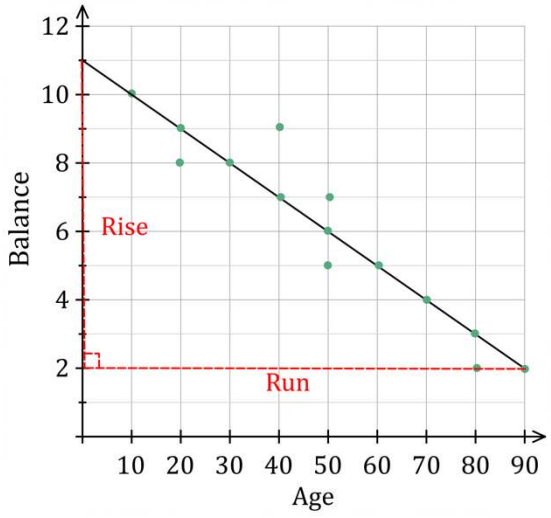
18.		$y' = 1 - 6\sin 3x \quad (0, 7)$ $y = \int 1 - 6\sin 3x \, dx$ $y = x + 2\cos 3x + C$ when $x = 0 \quad y = 7$ $7 = 0 + 2\cos 0 + C$ $7 = 2 + C$ $C = 5$ $\therefore y = x + 2\cos 3x + 5$	2	3 marks for correct solution. 2 mark for correct integration but not finding C or small error in solution or equivalent merit. 1 mark for some attempt to integrate or equivalent merit

19.	a)	$T_n = a + (n-1)d$ $T_3 = a + (3-1)d = 32$ $a + 2d = 32$ ① $T_6 = a + (6-1)d = 17$ $a + 5d = 17$ ② ② – ① $a + 2d = 32$ ① $3d = -15$ $d = -5$ ∴ common difference $d = -5$	2	2 marks for correct value of d 1 mark for setting up the 2 equations or equivalent merit.
	b)	sub d into ① $a - 10 = 32$ $a = 42$ $S_n = \frac{n}{2}[2a + (n-1)d]$ Sum terms $10 \rightarrow 20 = S_{20} - S_9$ $S_{20} = \frac{20}{2}[2 \times 42 + (20-1)(-5)]$ $= -110$ $S_9 = \frac{9}{2}[2 \times 42 + (9-1)(-5)]$ $= 198$ Sum $= S_{20} - S_9$ $= -110 - 198$ $= -308$	2	2 marks for correct solution 1 mark for finding either S_{20} or S_9 or equivalent merit.

20.	a)	$V = V_0 e^{-kt}$ when $t = 0$ $v = 3\,000$ $3\,000 = V_0 e^0$ $V_0 = 3\,000$ litres	1	1 mark for correct solution.
	b)	$V = 3\,000 e^{-kt}$ when $t = 3$ $V = 1\,900$ $1\,900 = 3\,000 e^{-3k}$ $\frac{19}{30} = e^{-3k}$ $\ln\left(\frac{19}{30}\right) = -3k$ $k = 0.1522528008$ $k = 0.1523$	2	2 marks for correct solution. 1 for correct progress toward finding *k* or equivalent merit.
	c)	$250 = 3\,000 e^{-kt}$ $\ln\left(\frac{25}{300}\right) = -kt$ $t = \frac{\ln\left(\frac{25}{300}\right)}{-0.1522528008}$ = 16 hrs 19 mins 7 pm Wednesday + 16 hrs 19 mins = 11:19 am Thursday	2	2 marks for correct solution. 1 for finding *t* or equivalent merit.

21.	a)	5.4% pa compounding monthly is 5.4% ÷ 12 =0.45% per month 5 years is 5×12 = 60 months From the table the $1 factor for 0.45% for 60 months is 68.7047 For $250, the annuity is worth 250 × 68.7047 = $17 176.18	**2**	2 marks for the correct solution. 1 mark for finding the correct interest factor or equivalent merit.
	b)	If annual rate is 3.6% this is 3.6 ÷ 12=0.30% per month From the table the $1 factor for 0.30% for 60 months is 65.6316 If investment is to earn $17176 with a factor of 65.6316 then monthly annuity is 17176 ÷ 65.6316 = $261.70320394 =$261.71 per month	**2**	2 marks for the correct solution. 1 mark for finding the correct interest factor or equivalent merit.

22.		$\left(\frac{4}{x+1}\right)+\left(\frac{4}{x+1}\right)^2+\left(\frac{4}{x+1}\right)^3+\ldots\ldots$ $r=\frac{4}{x+1}$ The sum to infinity exists if $\lvert r\rvert<1$ So we require $\left\lvert\frac{4}{x+1}\right\rvert<1$ When $x=4$ and when $x=-4$ $r=\frac{4}{4+1}=\frac{4}{5}$ $\quad\frac{4}{-4+1}=\frac{4}{-3}=-1\frac{1}{3}$ $\left\lvert\frac{4}{5}\right\rvert=\frac{4}{5}<1$ $\quad\left\lvert-1\frac{1}{3}\right\rvert=1\frac{1}{3}>1$ $\therefore$ limiting sum $\quad\therefore$ no limiting sum	2	2 for correct solution 1 for setting up and solving either equation or equivalent merit.

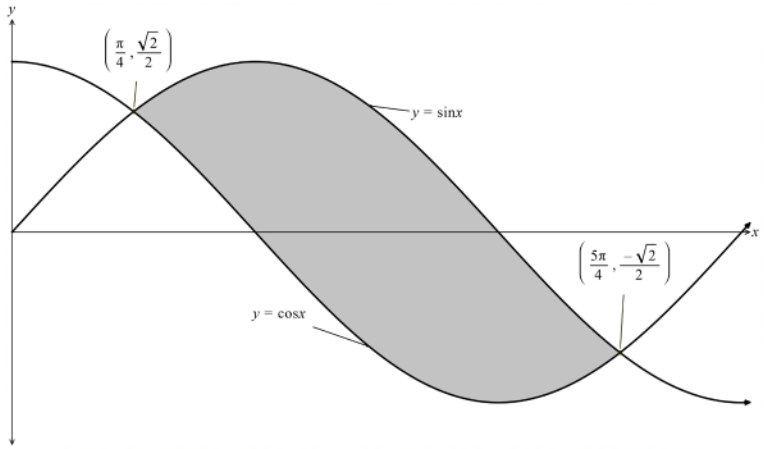
23.	(a)	$m = \frac{\text{Rise}}{\text{Run}}$ $= -\frac{9}{90}$ $= -0.1$  $\therefore$ Gradient is –0.1	2	2 Marks: Correct answer. 1 Mark: Finds the line of best fit or shows some understanding.
	(b)	When age = 40 then balance = 7 (from the scatterplot) $\therefore$ Hannah's balance should be 7.	1	1 Mark: Correct answer.
	(c)	Data: (10,10)(20,8)(20,9)(30,8)(40,7)(40,9)(50,5)(50,6)(50,7) (60,5)(70,4)(80,2)(80,3)(90,2) $r = -0.9552\ldots$ ≈ -0.96	1	1 Mark: Correct answer.

24.	a)	Area under curve = 1 $F(t) = \int_0^t \frac{1}{k} e^{-\frac{t}{4\,000}} = 1$ $\int_0^{\infty} \frac{1}{k} e^{-\frac{t}{4\,000}} = 1$ $\left[-\frac{4\,000}{k} e^{-\frac{t}{4\,000}} \right]_0^{\infty} = 1$ $-\frac{4\,000}{k}\left[e^{-\frac{t}{4\,000}} \right]_0^{\infty} = 1$ $-\frac{4\,000}{k}[0-1] = 1$ $k = 4\,000$	2	2 marks for correct solution for *k*. 1 mark for meaningful progress or with setting correct integral (including area = 1).

	b)	$P(0 \le T \le 100) = \int_0^{100} \frac{1}{4\,000} e^{-\frac{t}{4\,000}}\, dt$ $= \left[-e^{-\frac{t}{4\,000}} \right]_0^{100}$ $= -e^{-\frac{100}{4\,000}} + 1$ $= 1 - e^{-\frac{1}{40}}$ $= 1 - \frac{1}{e^{\frac{1}{40}}}$ $= 0.0246$ $= 2.5\%$ $\therefore$ 2.5% of globes will last less than 100 days and be considered faulty. As 95% of scores lie with 2 standard deviations from the mean this leaves 5% outside 2 standard deviations from the mean So this leaves 2.5% of scores at each end of the distribution $\therefore$ 100 days is approximately a z-score of -2.	2	2 marks for correct solution. 1 mark for finding either the probability or the z-score or equivalent merit.

	c)	$\text{Median} = P(0.5) = \int_0^x \frac{1}{4\,000} e^{-\frac{t}{4\,000}} dt$ $0.5 = \left[-e^{-\frac{t}{4\,000}} \right]_0^x$ $0.5 = \left[-e^{-\frac{x}{4\,000}} \right] - \left[-e^{-\frac{0}{4\,000}} \right]$ $0.5 = \left[-e^{-\frac{x}{4\,000}} + 1 \right]$ $-0.5 = -e^{-\frac{x}{4\,000}}$ $0.5 = e^{-\frac{x}{4\,000}}$ $\ln 0.5 = -\frac{x}{4\,000}$ $x = -4\,000 \times \ln 0.5$ $x = 2\,772.55$ $x = 2\,773 \text{ days}$	2	2 marks for correct solution 1 mark for setting the integral to = 0.5 and some progress made to finding a value for *x* in days or equivalent merit.

25.	a)	On English Exam Lachie's z-score = $\frac{68-68}{8} = 0$ Emily's z-score = $\frac{68-74}{6} = -1$ Lachie performed best, compared to his cohort. .	**1**	1 mark for correct conclusion with z-scores shown
	b)	On Maths Exam Lachie's z-score = $\frac{x-62}{9} = 2$ $x - 62 = 2 \times 9 = 18$ $x = 18 + 62 = 80$ Lachie's raw score was 80 Emily's z-score = $\frac{79-64}{5} = 3$ Emily had the higher z-score, so she performed better on the maths exam	**2**	1 mark for finding the correct raw score for Lachie. 1 mark for correct z-score for Emily and conclusion.

26.		 From the graph, the required region is found by: $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x - \cos x \, dx$ $= [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$ $= \left[-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4}\right] - \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right]$ $= \left[-\left(-\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}\right)\right] - \left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$ $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$ $= \frac{4}{\sqrt{2}}$ $= 2\sqrt{2} \ units^2$	4	4 marks for correct solution in exact value. 3 marks for correct working with an arithmetic error or equivalent merit 2 marks for error in integration and in evaluation or equivalent merit 1 mark for writing the Area as an integral or equivalent merit.

27.		$A = 49 \text{ cm}^2$ $A = \frac{1}{2} r^2\theta$ $49 = \frac{1}{2} r^2\theta$ $98 = r^2\theta$ $\theta = \frac{98}{r^2}$ $l = r\theta$ $l = r \times \frac{98}{r^2}$ $l = \frac{98}{r}$ $P = r + r + \frac{98}{r}$ $= 2r + \frac{98}{r}$ $= 2r + 98r^{-1}$ $\frac{dP}{dr} = 2 - 98r^{-2} = 2 - \frac{98}{r^2}$ $\frac{d^2p}{dr^2} = 196r^{-3} = \frac{196}{r^3}$ stat pt when $\frac{dP}{dr} = 0$ Continued	4	4 marks for correctly showing θ then showing minimum occurs when r = 7 and that this gives required result or equivalent merit 3 marks for correctly showing θ and substituting to get expression for P and differentiation to find minimum when r= 7 or equivalent merit 2 marks for correctly showing θ and for finding an equation for the perimeter and differentiating or equivalent merit. 1 mark for showing the equation for θ or equivalent merit.

		$2 - \frac{98}{r^2} = 0$ $\frac{98}{r^2} = 2$ $\frac{r^2}{98} = \frac{1}{2}$ $r^2 = 49$ $r = 7$ as $r > 0$ Find nature of $T\ P$ when $r = 7$ $\frac{d^2P}{dr^2} = \frac{196}{7^3} > 0 \therefore$ min value $\therefore$ minimum perimeter occurs when $r = 7$ cm $d = 14$ cm $l = r\ \theta$ where $r = 7$ and $\theta = \frac{98}{r^2} = \frac{98}{7^2}$ $l = 7 \times \frac{98}{7^2}$ $l = \frac{98}{7}$ $l = 14$ cm $= 2 \times r =$ diameter So the minimum perimeter occurs when the arc length is equal to the diameter.		

28.	a)	$v = 4t^3 - 24t^2 + 20t$ $x = \int 4t^3 - 24t^2 + 20t$ $x = \frac{4t^4}{4} - \frac{24t^3}{3} + \frac{20t^2}{2} + C$ $= t^4 - 8t^3 + 10t^2 + C$ when $t = 0$ $x = 3$ $3 = 0 + C$ $c = 3$ $\therefore x = t^4 - 8t^3 + 10t^2 + 3$	2	2 marks for correct solution. 1 mark for integration to find *x* or equivalent merit.
	b)	when $t = 1$ $v = 4 \times 1^3 - 24 \times 1^2 + 20 \times 1$ $= 0$ $\therefore$ stat pt at $t = 1$ $\frac{d^2x}{dt^2} = 12t^2 - 48t + 20$ when $t = 1$ $\frac{d^2x}{dt^2} = 12 - 48 + 20 < 0 \therefore$ max turning pt	2	2 marks for correct solution 1 mark for showing *v*=0 and recognising this is a stat pt or equivalent merit.

<table>
<tr>
<td></td>
<td>c)</td>
<td>$x = t^4 - 8t^3 + 10t^2 + 3$
$\therefore$ Total distance $= 3 + 11$
$= 14$ cm
<table>
<tr><td>t</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>x</td><td>3</td><td>6</td><td>-5</td></tr>
</table>
</td>
<td>2</td>
<td>2 marks for correct solution.

1 mark for realising the particle changes direction or equivalent merit.</td>
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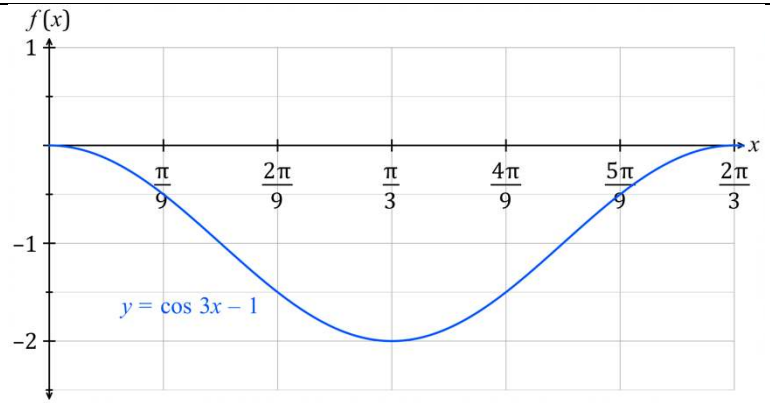
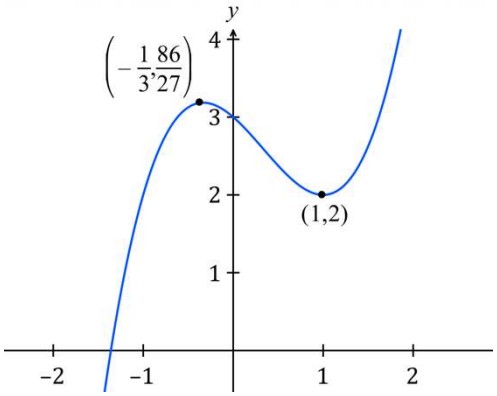
29.	a)	$f(x) = \cos 3x - 1$ $\text{Amplitude} = 1 \text{ and Period} = \frac{2\pi}{3}$	1	1 Mark

	b)	Maximum value of $f(x) = \cos 3x - 1$ is 0 Minimum value $f(x) = \cos 3x - 1$ occurs when $x = \frac{\pi}{3}$ $f(x) = \cos 3x - 1$ $= \cos 3 \times \left(\frac{\pi}{3}\right) - 1$ $= -2$ $\therefore$ Range is 2 or $-2 \leq f(x) \leq 0$	2	1 Marks: Correct answers.

	(c)		2	2 Marks: Correct answer. 1 Mark: Shows some understanding.
30		$y = \frac{10}{x+1}$ Width of the strip or trapezia. $h = \frac{b-a}{n} = \frac{5-1}{4} = 1$ $A \approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + y_2 + y_3)]$ $\approx \frac{1}{2}\left[5 + \frac{5}{3} + 2\left(\frac{10}{3} + 2.5 + 2\right)\right]$ $\approx 11\frac{1}{6}$ square units.	3	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Finds the width of the strip or equivalent merit.

x	1	2	3	4	5
y	5	$\frac{10}{3}$	2.5	2	$\frac{5}{3}$

31	(a)	Stationary points $f'(x) = 0$ $3x^2 - 2x - 1 = 0$ $(x-1)(3x+1) = 0$ $x = 1$ and $x = -\frac{1}{3}$ Nature of stationary points $f''(x) = 6x - 2$ At $x = 1$ $f"(1) = 4 > 0$ $\therefore$ Minimum turning point at $x = -1$ At $x = -\frac{1}{3}$ $f"(-\frac{1}{3}) = -4 < 0$ $\therefore$ Maximum turning point at $x = 0.\dot{3}$	2	2 Marks: Correct answer. 1 Mark: Finds both *x*-values or determines the nature of one stationary point, or equivalent merit.
	(b)	$f(x) = \frac{3x^3}{3} - \frac{2x^2}{2} - x + C$ At the point (0, 3) $3 = 0^3 - 0^2 - 0 + C$ $C = 3$ $\therefore f(x) = x^3 - x^2 - x + 3$	2	2 Marks: Correct answer. 1 Mark: Finds the correct primitive or equivalent merit.
	(c)	$x = 1$ then $y = 2$ $x = -\frac{1}{3}$ then $y = \frac{86}{27}$  	2	2 Marks: Correct answer. 1 Mark: Provides curve with correct shape or equivalent merit.

	(d)	Concave up then $f''(x) > 0$ $6x - 2 > 0$ $6x > 2$ $x > \frac{1}{3}$	1	1 Mark: Correct answer.
32	(a)	$SA = 2x \times \frac{5x}{2} + 2hx + 2h \times \frac{5x}{2}$ $6480 = 5x^2 + 2hx + 5hx$ $h = \frac{6480 - 5x^2}{7x}$	2	2 Marks: Correct answer. 1 Mark: Applies the formula for the SA of a rectangular prism.
32	(b)	$V = \frac{5x}{2} \times x \times h$ $= \frac{5x^2}{2} \times \frac{6480 - 5x^2}{7x}$ $= \frac{5x(6480 - 5x^2)}{14}$	1	1 Mark: Correct answer.

32	(c)	$V = \frac{16\,200x}{7} - \frac{25x^3}{14}$ $\frac{dV}{dx} = \frac{16\,200}{7} - \frac{75x^2}{14}$ Maximum volume when $\frac{dV}{dx} = 0$ $\frac{16\,200}{7} - \frac{75x^2}{14} = 0$ $32\,400 - 75x^2 = 0$ $75x^2 = 32\,400$ $x^2 = \frac{32\,400}{75}$ $x = \pm\sqrt{432}$ $= 12\sqrt{3} \quad (x > 0)$ To find the value of *h* use part(a) $h = \frac{6480 - 5x^2}{7x}$ $= \frac{6480 - 5\left(12\sqrt{3}\right)^2}{7 \times 12\sqrt{3}}$ $= \frac{4320}{84\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{120\sqrt{3}}{7}$ Test whether $x = 12\sqrt{3}$ is a maxima. $\frac{d^2V}{dx^2} = -\frac{75x}{7} = -\frac{75 \times 12\sqrt{3}}{7} < 0$ at $x = 12\sqrt{3}$ Hence maximum volume when $x = 12\sqrt{3}$ cm. $\therefore\ x = 12\sqrt{3}$ and $h = \frac{120\sqrt{3}}{7}$ for maximum volume.	4	