Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

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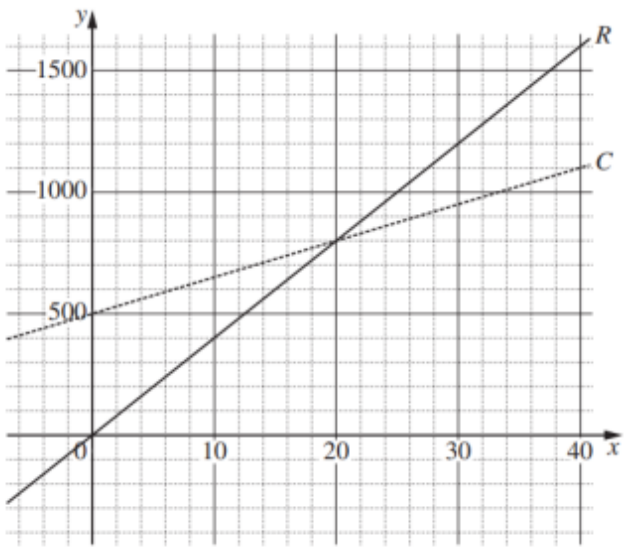
1 A small business makes and sells bird houses.

Technology was used to draw straight-line graphs to represent the cost of making bird houses (C) and the revenue from selling bird houses (R). The x – axis displays the number of bird houses and the y – axis displays the cost/revenue in dollars.



How many bird houses need to be sold to break even?

A. 10

B. 20

C. 30

D. 40

2 The relationship between number of beers consumed (x) and blood alcohol content (y) was studied in 18 male college students by using least squares regression. The following regression equation was obtained from this study $y = -0.0127 + 0.0180x$.

With regards to the study and the regression equation, which of the following statements is correct?

A. Each beer consumed increases blood alcohol content by 1.27%.

B. On average it takes 1.8 beers to increase blood alcohol content by 1%.

C. Each beer consumed increases blood alcohol content by 0.0180.

D. Each beer consumed decreases blood alcohol content by 0.0127.

3 The table below shows the future value of a $1 annuity for varying interest rates over different time periods.

Time period	*Interest rate*				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.526
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

What is the present value of an annuity, correct to the nearest dollar, that would provide a future value of $62 000 after 3 years at 2% per annum, compounded half-yearly?

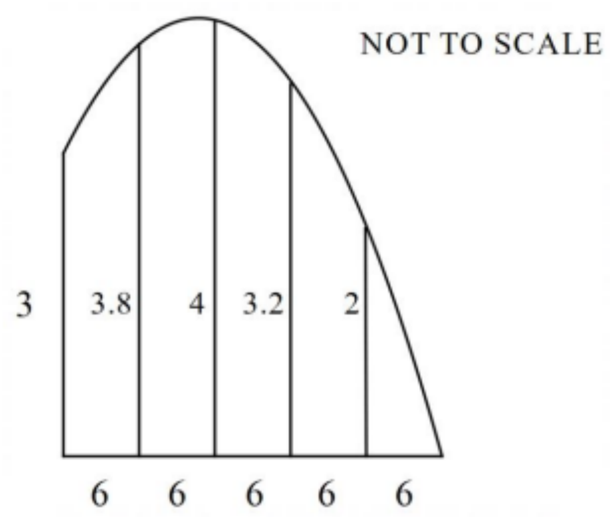
A. $9, 828.63

B. $10, 078.02

C. $20, 258.79

D. $20, 461.37

4 The diagram below shows a poppy field. All measurements are in metres.



What is an approximate value for the area of the poppy field using the trapezoidal rule with 5 sub-intervals?

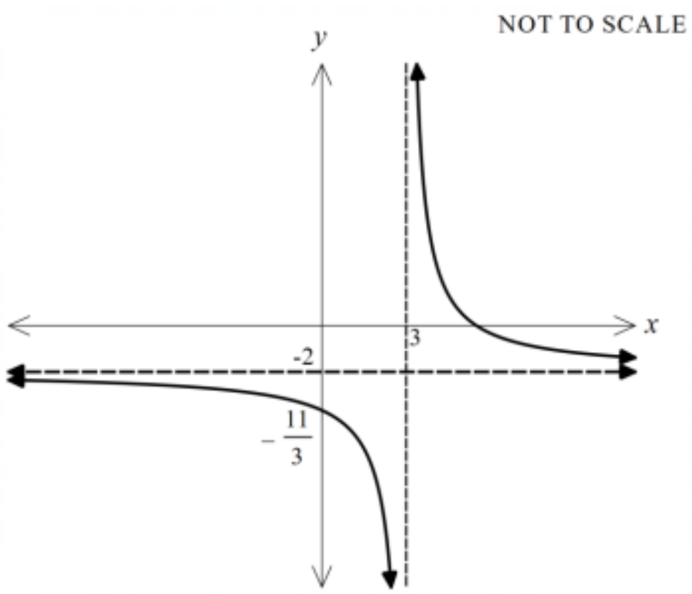
A. 67.5m^2

B. 72.5m^2

C. 81m^2

D. 87m^2

5 Consider the graph of $y = \dfrac{k}{x+b} + c$ below, with y – intercept at $\left(0, -\dfrac{11}{3}\right)$.



Which of the following statements is true?

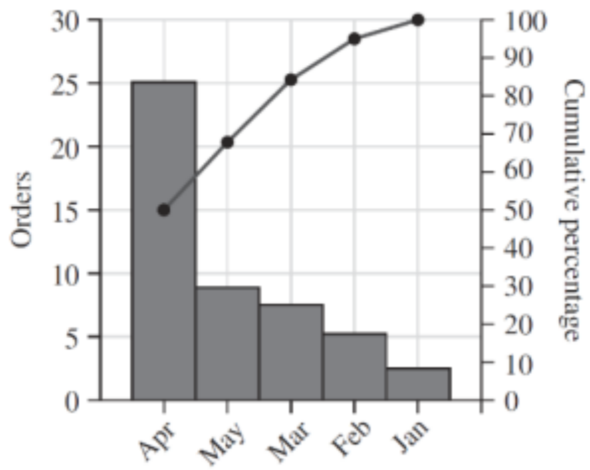
A. $k < b < c$

B. $b < k < c$

C. $k < c < b$

D. $b < c < k$

6 The Pareto chart below shows the orders received by a business for five months.



What percentage of orders were received in May?

A. 18%

B. 30%

C. 45%

D. 69%

7 Let $g(x)=1-f(x)$. If $\int_1^4 f(x)\,dx=7$, what is the value of $\int_1^4 g(x)\,dx$?

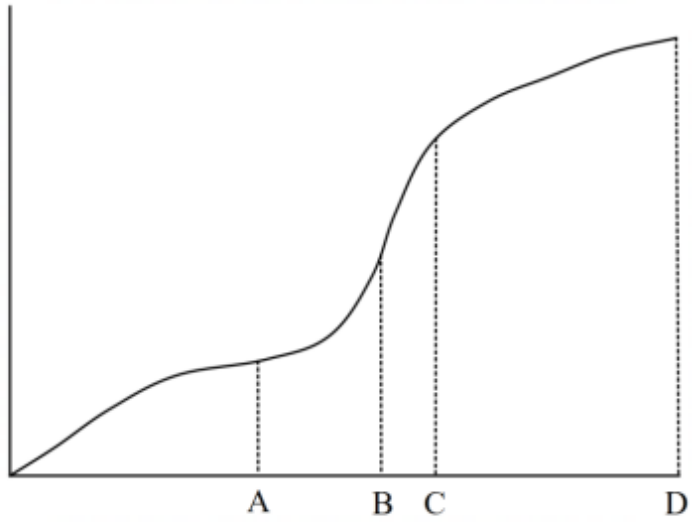
A. -6

B. -4

C. 8

D. 10

8 The cumulative distribution function for the continuous random variable X is graphed below.



What is the mode of the probability density function?

A. A

B. B

C. C

D. D

9 It is known that $f(x)$ is an odd function and $g(x)$ is an even function.

Given that $f(2)=2$ and $g(2)=-2$, what is the value of $f(g(-2))+g(f(-2))$?

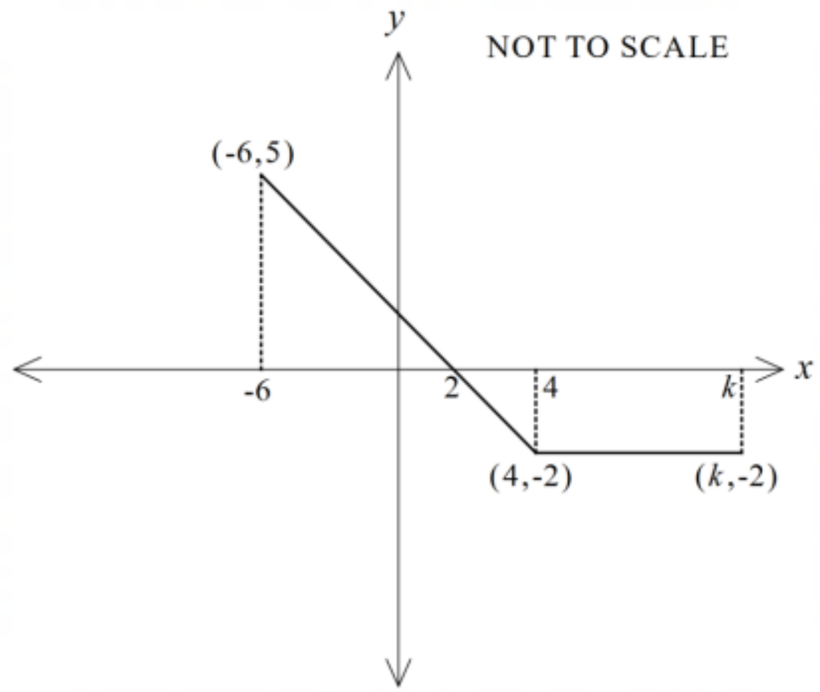
A. -4

B. -2

C. 0

D. 4

10 By using the graph below, which value of k satisfies $\int_{-6}^{k} f(x)\,dx = 0$?



A. 6

B. 11

C. 12

D. 13

End of Section I

Section II

90 marks
Attempt Questions 11-32
Allow about 2 hours and 45 minutes for this section
Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning.

Question 11 (2 marks)

Simplify $\frac{2}{x-1}-\frac{6}{(x-1)(2x+1)}$. **2**

Question 12 (3 marks)

Solve the equation $2\sec\theta-1=9,\ 0\le\theta\le 2\pi$. Give your answer correct to two decimal places. **3**

Question 13 (5 marks)

The probability distribution of a discrete random variable X is shown in the table below, where a, b and c are constants.

x	1	2	3	4
$P(X=x)$	a	b	b	c

The cumulative distribution function of X is shown in the table below where d and e are constants.

x	1	2	3	4
$F(x)$	$\frac{1}{6}$	d	$\frac{2}{3}$	e

a) Determine the value of each of the constants a, b, c, d and e. **3**

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b) The discrete random variable Y is defined as $Y = 10 - 3X$.

Find the value of $P(Y > X)$. **2**

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Question 14 (7 marks)

Differentiate each of the following:

a) $y = x^4 + 2x^{\frac{5}{2}}$. **1**

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b) $y = e^{2x} \tan x$. **2**

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c) $y = \left(x^3 - 3\ln x\right)^5$. **2**

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d) $y = \dfrac{4x}{\cos 3x}$. **2**

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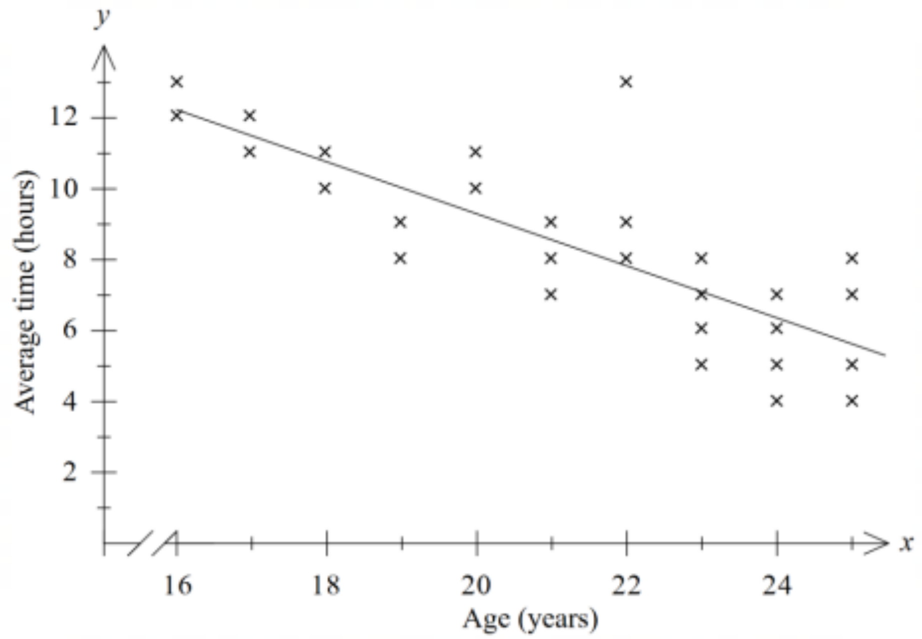
Question 15 (4 marks)

In a training scheme for young people, the average time taken for each age group to reach a certain level of proficiency was measured.

After collecting the data, it is found that the correlation coefficient is -0.804.

A scatterplot showing the data is drawn. The line of best fit with equation $y = 23.96 - 0.73x$, is also drawn.



Describe and interpret the data and other information provided, with reference to the context given. **4**

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Question 16 (3 marks)

Seats in a theatre are arranged in rows. The number of seats in this theatre form the terms of an arithmetic series.

The sixth row has 23 seats, and the fifteenth row has 50 seats. If the theatre has 20 rows, how many seats are in the theatre? **3**

Question 17 (2 marks)

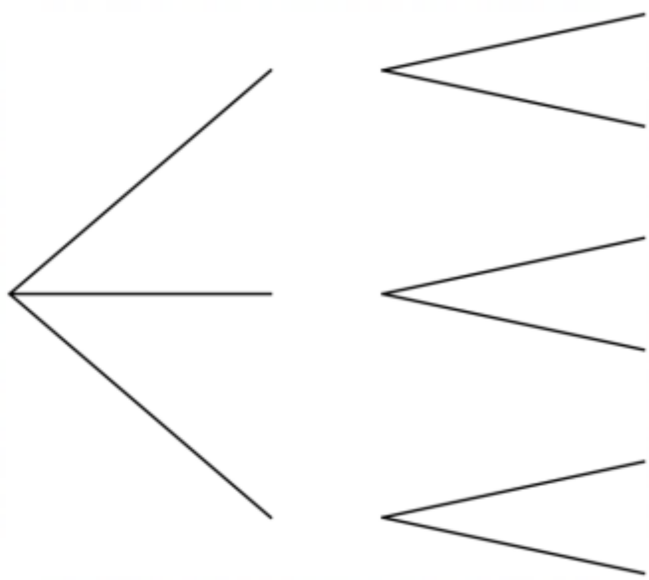
Prove $\dfrac{1}{\sec x - \tan x} + \dfrac{1}{\sec x + \tan x} = 2\sec x$. **2**

Question 18 (5 marks)

During the winter, Andrew attends weekly business meetings in Queenstown and always travels to these meetings by car.

The probability of being dry, raining or snowing during his travel to these meetings is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$, respectively. The respective probabilities of Andrew arriving on time when it is dry, raining or snowing are $\frac{4}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$.

a) Complete the probability tree diagram below, showing all outcomes. **2**



b) Determine the probability that Andrew will arrive late to his next winter business meeting. **1**

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c) Andrew arrived late for his meeting last week. Find the probability that it was raining on that day. **2**

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Question 19 (4 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table provided on page 33 gives the probability that this random variable lies below z for some positive values of z.

The weekly mileages covered by a sales rep, M miles, are normally distributed with a mean of 365 and variance of 600.

Find the probability that in a random week the rep will cover a mileage between 380 and 410. **4**

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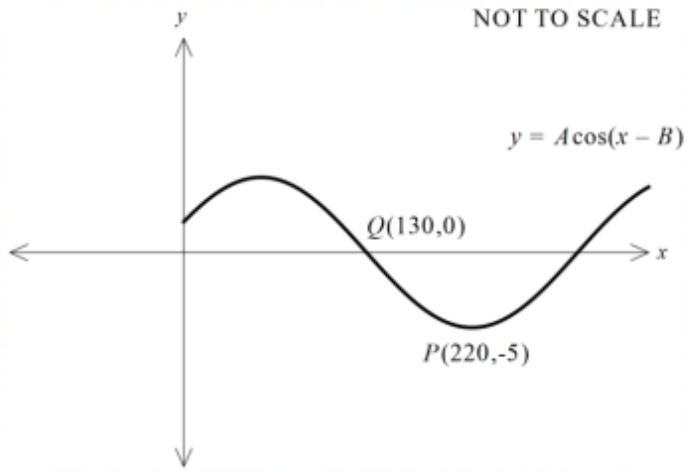
Question 20 (2 marks)

The figure below shows the graph of the curve with equation $y = A\cos(x - B)$,

$0° \leq x \leq 360°$, where A and B are positive constants with $0° < B < 90°$.



The graph meets the x axis at point $Q(130,0)$ and the point $P(220,-5)$ is the minimum point of the curve. State the value of A and the value of B. **2**

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Question 21 (6 marks)

Find the following integrals:

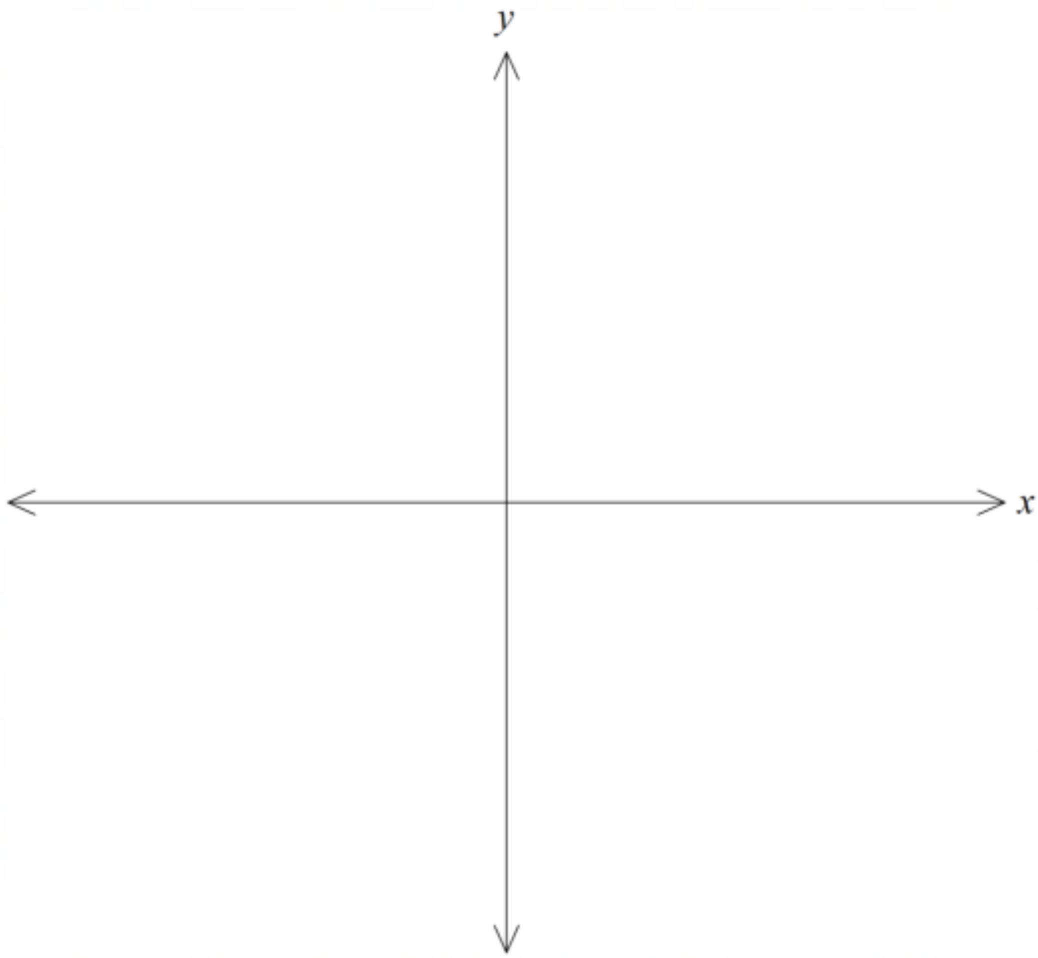
a) $\int \sqrt{x} - 4\cos 2x\, dx$ **2**

b) $\int 3^{2x+1}\, dx$ **2**

c) $\int \frac{3}{x} + \frac{4}{x^2} - \frac{2}{x^3}\, dx$ **2**

Question 22 (4 marks)

a) Sketch the graphs of the functions $f(x) = x - 2$ and $g(x) = (2x-1)(2-x)$ showing the intercepts. **2**



b) Hence, or otherwise, solve the inequality $x - 2 > (2x-1)(2-x)$. **2**

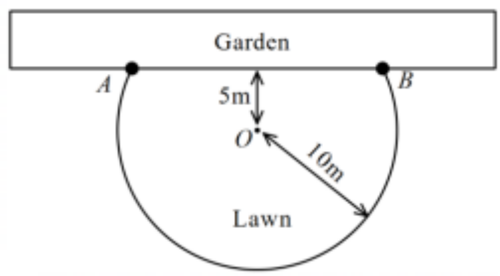
Question 23 (4 marks)

Find the global maximum and minimum values of $y = x^3 - 3x^2 + 5$, where $-\frac{1}{2} \le x \le 4$. **4**

Question 24 (6 marks)

A water sprinkler covers a circular lawn area of radius 10 metres. The sprinkler (O) is placed 5 metres from a rectangular garden bed. Garden stakes are placed at A and B, as shown in the diagram below.

NOT TO SCALE



a) Show that $\angle AOB = \dfrac{2\pi}{3}$. **1**

b) Show that the major arc length of the lawn area covered by the sprinkler is $\dfrac{40\pi}{3}$ m. **2**

c) Find the exact area of the lawn that the sprinkler will cover. **3**

Question 25 (3 marks)

a) Differentiate $y = \frac{\ln x}{x}$. **1**

b) Hence, evaluate $\int_{e}^{e^2} \frac{2 - \ln x^2}{x^2} \, dx$. **2**

Question 26 (4 marks)

A particle P is moving on the x axis and its velocity $v\,\text{ms}^{-1}$, t seconds after a given instant, is given by $v = t^2(3-t), t \geq 0.$

When $t = 2,$ P is observed to be $4\,\text{m}$ from the origin $O,$ in the positive direction of the $x-$axis.

a) Find the acceleration of P when $t = 2.$ **1**

b) The particle is initially at rest and is also at rest when $t = T.$ Determine the distance of P from O when $t = T.$ **3**

Question 27 (5 marks)

A car dealership has a car for sale for a cash price of \$20 000. It can also be bought on terms over three years. The first six months are interest free. After the first six months, reducible interest is charged at the rate of 1% per month, calculated at the end of each month. Repayments are to be made in equal monthly instalments beginning at the end of the first month.

A customer buys the car on these terms and agrees to monthly repayments of $\$M$. Let $\$A_n$ be the amount owing at the end of the n^{th} month.

a) Find an expression for A_6. **1**

b) Show that $A_8 = (20\,000 - 6M)1.01^2 - M(1 + 1.01)$. **1**

c) Find the value of M. **3**

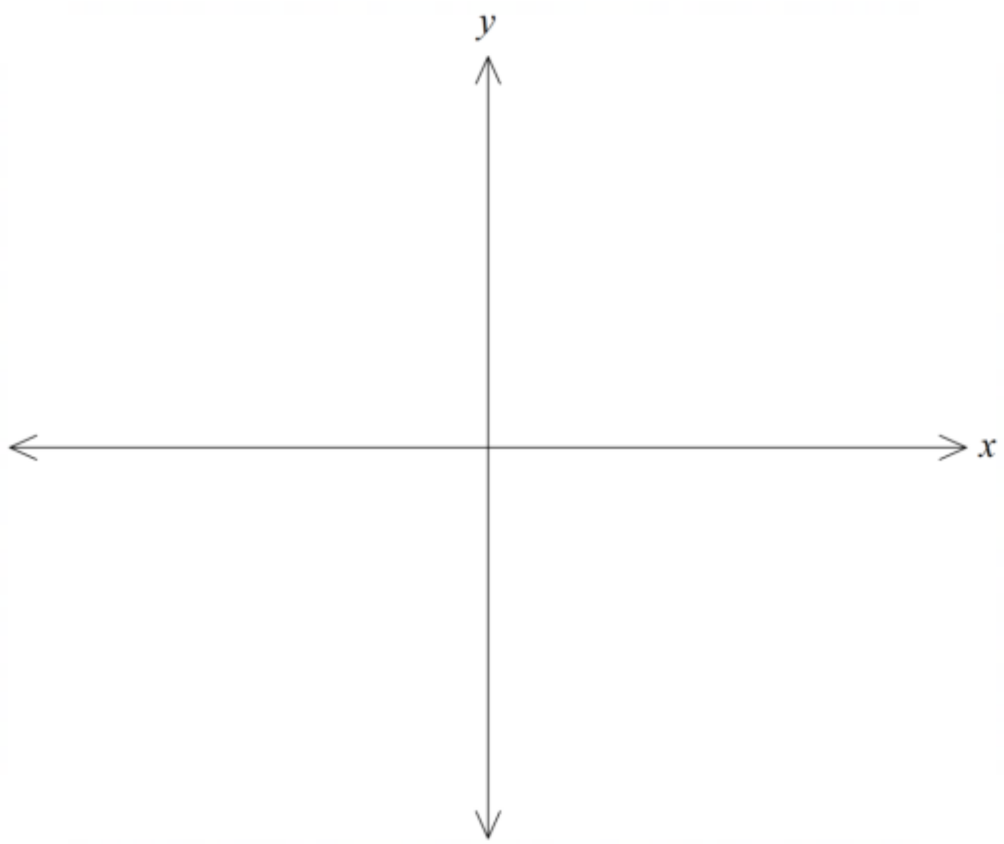
Question 28(8 marks)

Consider $f(x) = e^{2x+1}x^4$.

a) Find the stationary point(s) and determine their nature. **3**

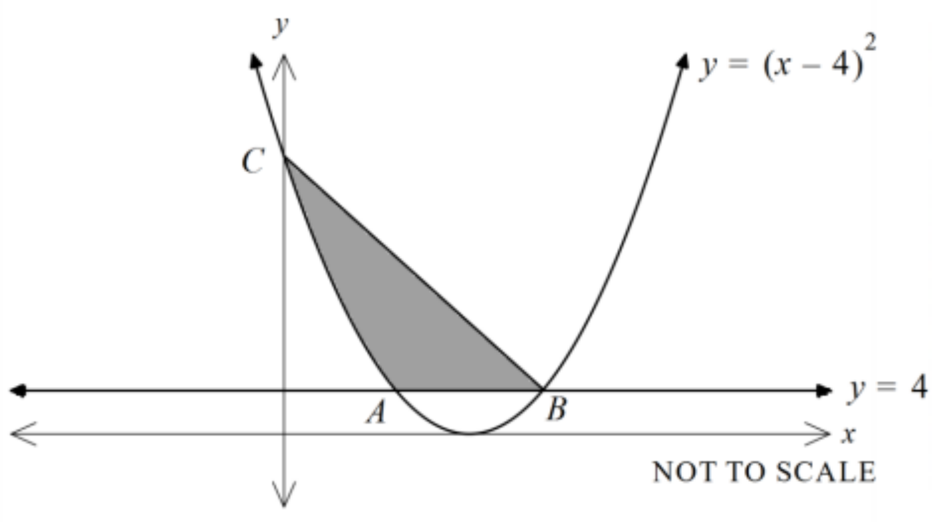
b) Find the point(s) of inflection. **2**

c) Sketch the graph of $f(x)$ showing all features. **3**



Question 29 (3 marks)

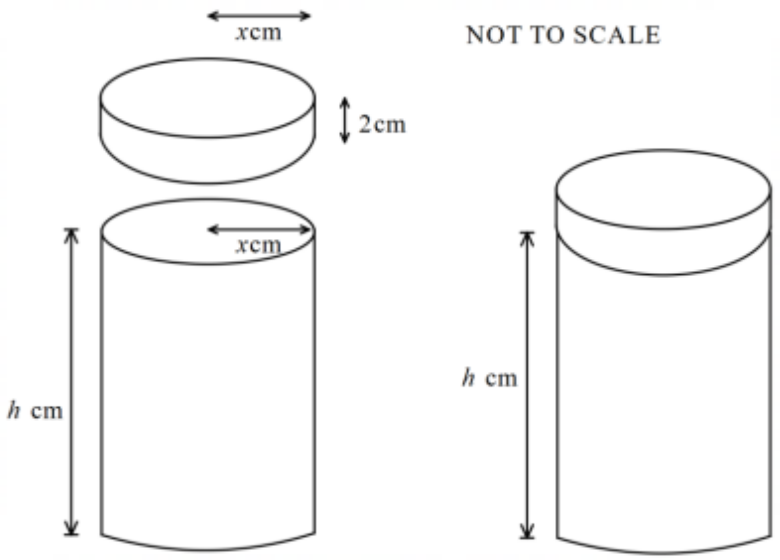
The diagram below shows the curve with equation $y = (x-4)^2$, intersected by the straight line with equation $y = 4$, at the points A and B. The curve meets the y axis at the point C.



Calculate the exact area of the shaded region, bounded by the curve and the straight-line segments AB and BC. **3**

Question 30 (5 marks)

The figure below shows the design of coffee jar with a lid sitting directly on top.



The jar is in the shape of a right circular cylinder of radius x cm. It is fitted with a lid of height 2 cm, which fits tightly on the top of the jar, so it may be assumed that it has the same radius as the jar. The jar and its lid are made of thin sheet metal and there is no wastage. The total metal used to make the jar and its lid is 190π cm^2.

a) Show that the volume of the jar, V cm^3, is given by $V = \pi(95x - 2x^2 - x^3)$. **2**

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b) Find the maximum volume of the jar, leaving your answer in exact form. **3**

Question 31 (3 marks)

Given $f(x)=\dfrac{x+1}{2x-1}$ and $f(g(x))=\dfrac{3x+2}{3x-5}$, find $g(x)$. **3**

Question 32 (2 marks)

Find $\int \frac{1}{\sqrt{x^{\frac{3}{2}} + 5x}} dx$. **2**

End of Exam

Standard Normal Probabilities

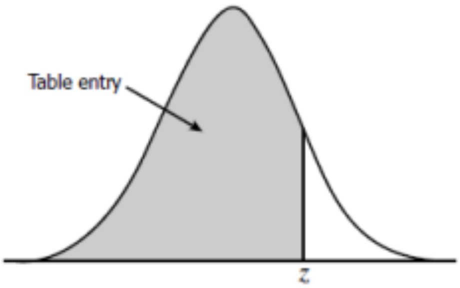


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Baulkham Hills High School

Task 4 Trials Examination 2024

Marking Guideline- Yr 12 Mathematics Advanced

Section I (10 marks)

Award 1 marks to each correct answer.

Answers:

1. B 2. C 3. B 4. D 5. D 6. A 7. B 8. B 9. A 10. D

Question	Suggested solutions	Answer
1	Break even point, occurs at the point of intersection. $x = 20$	B
2	The gradient is 0.018, so the average amount is 1.8%.	C
3	$\text{Present value } = \frac{62000}{6.1520}$ $= \$10,078.02$	B
4	$\frac{6}{2}(3+0+2(3.8+4+3.2+2)) = 87\text{cm}^2$	D
5	$y = \frac{k}{x+b} + c$ hyperbola translated down by 2 and right by 3 $b = -3, c = -2$ $y = \frac{k}{x-3} - 2$ sub $\left(0, -\frac{11}{3}\right)$ $-\frac{11}{3} = \frac{k}{0-3} - 2$ $k = 5$ $\therefore b < c < k$	D
6	$68\% - 50\% = 18\%$ *or* $\frac{9}{25+9+8+5+3} = 18\%$	A
7	$\int_1^4 g(x)\,dx = \int_1^4 1 - f(x)\,dx$ $= \int_1^4 1\,dx - \int_1^4 f(x)\,dx$ $= [x]_1^4 - 7$ $= 4 - 1 - 7$ $= -4$	**B**
8	Look for the point where the steepest gradient is. So when differentiated it becomes the maximum. This happens at point B	B

9	$g(-2) = -2$ (even) $f(-2) = -2$ (odd) $f(g(-2)) + g(f(-2)) = f(-2) + g(-2)$ $= -2 + -2$ $= -4$	A
10	$\int_{-6}^{k} f(x)\,dx = 0$ $\int_{-6}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx + \int_{4}^{k} f(x)\,dx = 0$ $\left(\frac{1}{2}\times 8\times 5\right) - \left(\frac{1}{2}\times 2\times 2\right) - 2(k-4) = 0$ $20 - 2 - 2k + 8 = 0$ $26 - 2k = 0$ $k = 13$	D

Section II (90 marks)

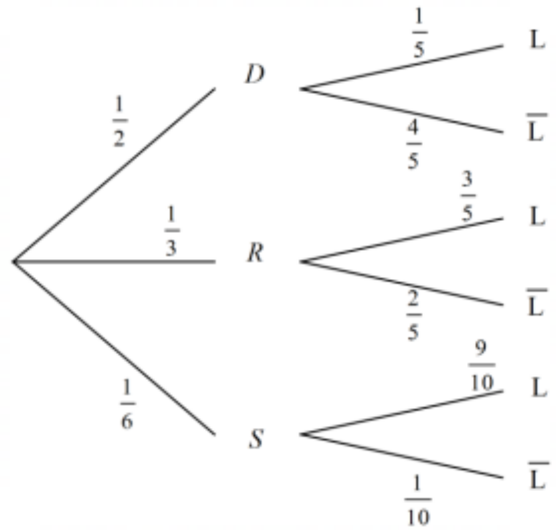
In all questions, award full marks for correct answers with necessary working.

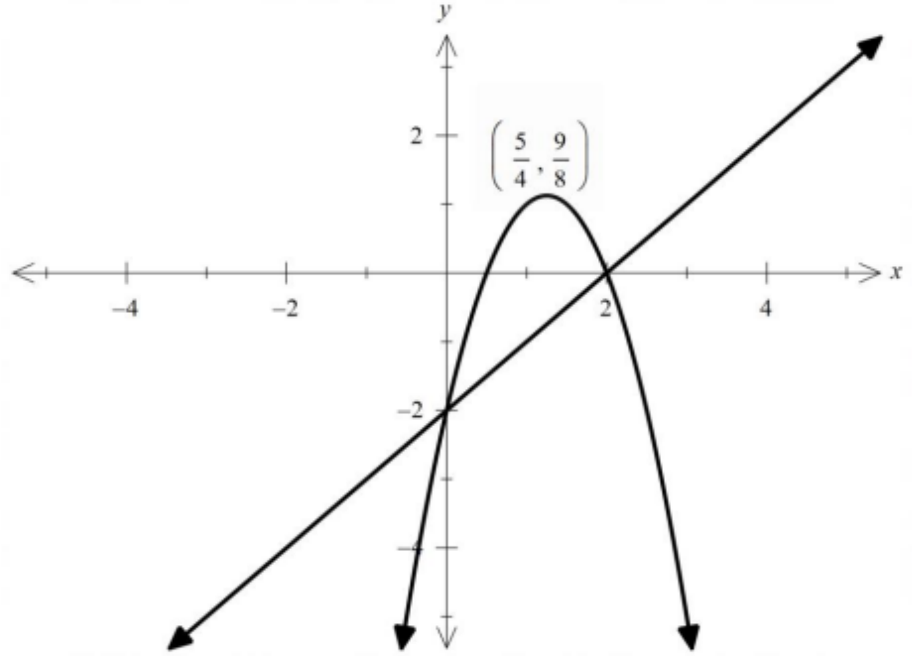
Use the suggested solutions in conjunction with the marking criteria.

Q	Suggested solutions	Marking Criteria
11	$\frac{2}{x-1} - \frac{6}{(x-1)(2x+1)} = \frac{2(2x+1)}{(x-1)(2x+1)} - \frac{6}{(x-1)(2x+1)}$ $= \frac{4x+2-6}{(x-1)(2x+1)}$ $= \frac{4x-4}{(x-1)(2x+1)}$ $= \frac{4(x-1)}{(x-1)(2x+1)}$ $= \frac{4}{2x+1}$	2 correct solution 1 correctly forms equivalent fraction
12	$2\sec\theta - 1 = 9$ $\sec\theta = 5$ $\frac{1}{\cos\theta} = 5$ $\cos\theta = \frac{1}{5}$ $\theta = 1.37,\ 4.91\ (\text{2dp})$	3 correct solution 2 correctly find $\theta = 1.37$ 1 correctly simplifies the equation in terms of $\cos\theta$
13a	$F(4) = 1$ $\therefore e = 1$ $P(X=1) = F(1)$ $\therefore a = \frac{1}{6}$ $P(X=1) + P(X=2) = F(2)$ $a + b = d$ $\frac{1}{6} + b = d$	3 correct solution 2 correctly finds 3 values 1 correctly finds 2 values

	$P(X=1)+P(X=2)+P(X=3)=F(3)$ $a+2b=\frac{2}{3}$ $\frac{1}{6}+2b=\frac{2}{3}$ $2b=\frac{1}{2}$ $b=\frac{1}{4}$ $\frac{1}{6}+b=d$ $\therefore d=\frac{5}{12}$ $a+2b+c=1$ $\frac{1}{6}+\frac{1}{2}+c=1$ $\therefore c=\frac{1}{3}$	
13b	$P(Y>X)=P(10-3X>X)$ $=P(-4X>-10)$ $=P(X<2.5)$ $=P(X=1,2)$ $=a+b$ $=d$ $=\frac{5}{12}$	2 correct solution 1 recognises $P(X<2.5)$, or equivalent merit
14a	$y=x^4+2x^{\frac{5}{2}}$ $y'=4x^3+5x^{\frac{3}{2}}$	1 correct solution
14b	$y=e^{2x}\tan x$ $y'=u'v+v'u$ $=2e^{2x}\tan x+e^{2x}\sec^2 x$ $u=e^{2x}$ $v=\tan x$ $u'=2e^{2x}$ $v'=\sec^2 x$	2 correct solution 1 correctly attempts to use product rule or equivalent merit
14c	$y=\left(x^3-3\ln x\right)^5$ $y'=5\times\left(3x^2-\frac{3}{x}\right)\left(x^3-3\ln x\right)^4$ $=\left(15x^2-\frac{15}{x}\right)\left(x^3-3\ln x\right)^4$	2 correct solution 1 correctly attempts to use chain rule, or equivalent merit
14d	$y=\frac{4x}{\cos 3x}$ $y'=\frac{u'v-v'u}{v^2}$ $y'=\frac{4\cos 3x+12x\sin 3x}{(\cos 3x)^2}$ $u=4x$ $v=\cos 3x$ $u'=4$ $v'=-3\sin 3x$	2 correct solution 1 correctly attempts to use the quotient rule, or equivalent merit
15	Suggestions • The relationship is linear and negative/decreasing. The correlation of -0.804 is moderate/strong. • As age increases by 1 year, the average time decreases by almost 1 year • The older the people were getting, the less time was needed to reach proficiency level • There seems to be an outlier. A 22-year old took 13 hours to complete the course • There are more 21 to 25 year olds than the younger group.	4 marks Provides a comprehensive description and interpretation of the data and other information, in the given context

		3 marks Provides a sound description and interpretation of the data and other information, in the given context 2 marks Provides some description and interpretation of the data and/or other Information 1 mark Provides some relevant information
16	$T_n = a + (n-1)d$ $23 = a + 5d \quad 50 = a + 14d$ Solve simultaneously $27 = 9d$ $d = 3$ $a = 8$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $S_{20} = \frac{20}{2}\left[2 \times 8 + 19 \times 3\right]$ $S_{20} = 730$ $\therefore$ There are a total of 730 seats	3 correct solution 2 correctly finds a and d 1 correctly forms a pair of simultaneous equations
17	$\frac{1}{\sec x - \tan x} + \frac{1}{\sec x + \tan x} = 2\sec x$ $LHS = \frac{1}{\sec x - \tan x} + \frac{1}{\sec x + \tan x}$ $= \frac{\sec x + \tan x + \sec x - \tan x}{\sec^2 x - \tan^2 x}$ $= \frac{2\sec x}{1}$ $= 2\sec x$ $= RHS$	2 correct proof 1 correctly forms equivalent fraction

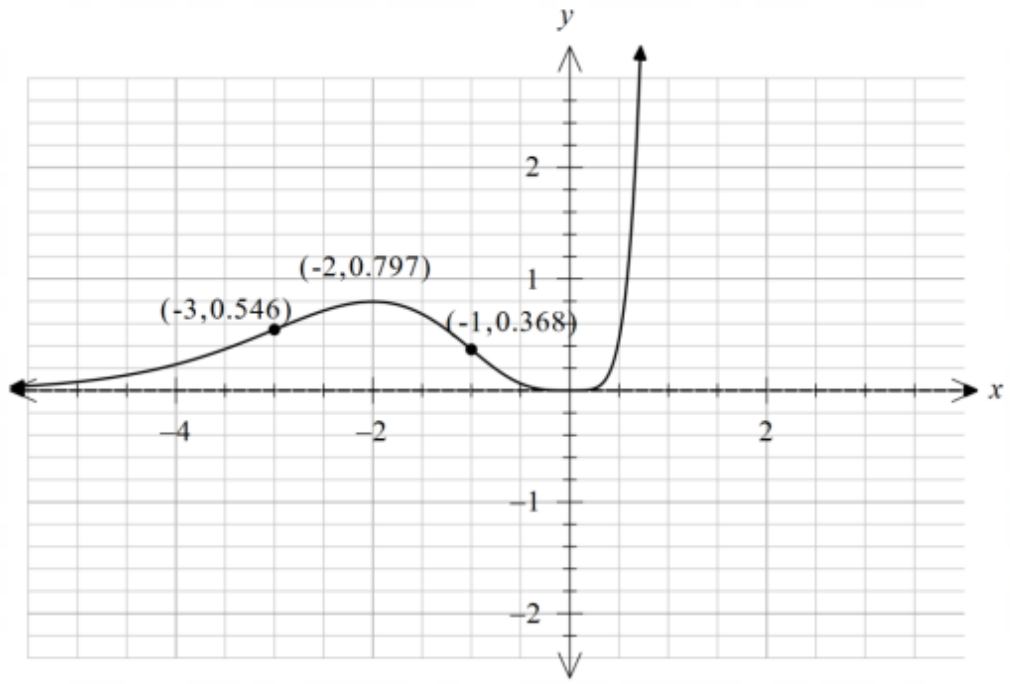
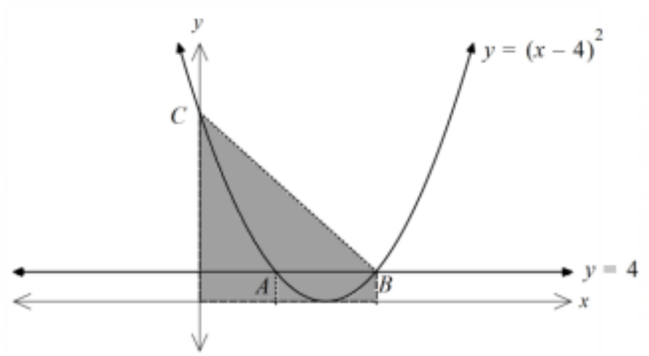
18a		2 correct diagram 1 some significant progress Note: they do not need to list the outcomes on the right for full marks
18b	$P(\text{dry, late}) + P(\text{rain, late}) + P(\text{snow, late})$ $= \frac{1}{2} \times \frac{1}{5} + \frac{1}{3} \times \frac{3}{5} + \frac{1}{6} \times \frac{9}{10}$ $= \frac{9}{20}$	1 correct solution
18c	$P(\text{rain} \mid \text{late}) = \frac{P(\text{rain and late})}{P(\text{late})}$ $= \frac{\frac{1}{3} \times \frac{3}{5}}{\frac{9}{20}}$ $= \frac{4}{9}$	2 correct solution 1 attempts to use conditional probability
19	$P(380 < M < 410) = P(M < 410) - P(M < 380)$ $= P\left(z < \frac{410-365}{\sqrt{600}}\right) - P\left(z < \frac{380-365}{\sqrt{600}}\right)$ $= P(z < 1.83712...) - P(z < 0.61237...)$ $= 0.9671 - 0.7291$ $= 0.238$	4 correct solution 3 finds the correct probability for both mileages from the provided table, or equivalent merit 2 finds the correct z score for both mileages, or equivalent merit 1 correctly identifies to subtract in the correct order
20	Looking at the original $\cos x$ graph, the minimum turning point is at $(180, -1)$. The minimum turning point is now at $(220, -5)$, which means the graph was translated to the right by 40 and a vertical stretch by a scale factor of -5. $\therefore A = 5$ and $B = 40$.	2 correct solution 1 correctly identifies A or B
21a	$\int \sqrt{x} - 4\cos 2x \, dx = \frac{2x^{\frac{3}{2}}}{3} - 2\sin 2x + C$	2 correct solution, 1 attempts to integrate by correctly

		integrating $\sqrt{x}$ or $\cos 2x$
21b	$\int 3^{2x+1}\,dx = \dfrac{3^{2x+1}}{2\ln 3} + C$	2 correct solution including $+C$ 1 obtains $\dfrac{3^{2x+1}}{\ln 3}$ or equivalent merit
21c	$\int \dfrac{3}{x} + \dfrac{4}{x^2} - \dfrac{2}{x^3}\,dx = 3\ln\lvert x\rvert - \dfrac{4}{x} + \dfrac{1}{x^2} + C$	2 correct solution 1 correctly integrates $\dfrac{3}{x}$, or equivalent merit
22a		2 correct graphs 1 provides correct graph of $g(x)$, or $f(x)$
22b	The graphs intersect at $(0,-2)$. The line is above the parabola when $x < 0$ or $x > 2$.	2 correct solution 1 correctly identifies one of the solutions
23	$y = x^3 - 3x^2 + 5$ $\dfrac{dy}{dx} = 3x^2 - 6x$ $3x^2 - 6x = 0$ $3x(x-2) = 0$ Stationary points $(0,5)$ and $(2,1)$ $x = 0$ or $x = 2$ End points of interval When $x = -0.5$, $y = 4.125$ When $x = 4$, $y = 21$ Global minimum value $= 1$ Global maximum value $= 21$	4 correct solution 3 finds the y values for at least 3 relevant points 2 solves the quadratic to find x coordinates of stationary points 1 finds the derivative, or equivalent merit

24a	$\cos\theta = \frac{5}{10}$ $\theta = \frac{\pi}{3}$ $2\theta = 2\times\frac{\pi}{3}$ $= \frac{2\pi}{3}$	1 correct solution
24b	major $\angle AOB = 2\pi - \frac{2\pi}{3}$ $= \frac{4\pi}{3}$ Arc length $= r\theta$ $= 10\times\frac{4\pi}{3}$ $= \frac{40\pi}{3}$	2 correct solution 1 finds major $\angle AOB$, or equivalent merit
24c	Area circle - Area minor segment $= \pi r^2 - \left[\frac{1}{2}r^2(\theta - \sin\theta)\right]$ $= \pi\times 10^2 - \left[\frac{1}{2}\times 10^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)\right]$ $= 100\pi - \frac{100\pi}{3} + 50\sin\frac{2\pi}{3}$ $= \frac{200\pi}{3} + 50\times\frac{\sqrt{3}}{2}$ $= \frac{200\pi}{3} + 25\sqrt{3}$	3 correct solution 2 correctly finds the area of the minor segment, or equivalent merit 1 correct substitution into formula to find area of minor segment, or equivalent merit
25a	$y = \frac{\ln x}{x}$ $y' = \frac{x\times\frac{1}{x} - \ln x}{x^2}$ $y' = \frac{1-\ln x}{x^2}$	1 correct solution

25b	$\int_e^{e^2} \frac{2-\ln x^2}{x^2}\,dx = \int_e^{e^2} \frac{2-2\ln x}{x^2}dx$ $= 2\int_e^{e^2} \frac{1-\ln x}{x^2}dx$ $= 2\left[\frac{\ln x}{x}\right]_e^{e^2}$ $= 2\left[\frac{\ln e^2}{e^2} - \frac{\ln e}{e}\right]$ $= 2\left[\frac{2}{e^2} - \frac{1}{e}\right]$	2 correct solution 1 correctly manipulating integral to obtain a)
26a	$v = 3t^2 - t^3$ $a = 6t - 3t^2$ $a = 6(2) - 3(2)^2$ $a = 12 - 12$ $a = 0$	1 correct solution
26b	$v = t^2(3-t)$ $0 = t^2(3-t)$ $t = 0$ or 3 $x = \int 3t^2 - t^3$ $x = t^3 - \frac{1}{4}t^4 + C$ When $t = 2, x = 4$ $4 = 2^3 - \frac{1}{4} \times 2^4 + C$ $C = 0$ $\therefore x = t^3 - \frac{1}{4}t^4$ When $t = T = 3$ $x = 3^3 - \frac{1}{4} \times 3^4$ $x = 6.75\,\text{m}$	3 correct solution 2 correctly integrates and finds the constant to find the displacement equation 1 correctly identifies $T = 3$
27a	$A_1 = 20\,000 - M$ $A_2 = 20\,000 - 2M$ $\vdots$ $A_6 = 20\,000 - 6M$	1 correct solution

<table>
<tr><td>27b</td><td>$A_7 = A_6(1.01) - M$
$= (20\ 000 - 6M)(1.01) - M$
$A_8 = A_2(1.01) - M$
$= \left[(20\ 000 - 6M)(1.01) - M\right](1.01) - M$
$= (20\ 000 - 6M)(1.01)^2 - M(1.01) - M$
$= (20\ 000 - 6M)(1.01)^2 - M(1 + 1.01)$</td><td>1 correct solution</td></tr>
<tr><td>27c</td><td>$A_{36} = (20\ 000 - 6M)(1.01)^{30} - M(1 + 1.01 + \ldots + 1.01^{29})$
$0 = (20\ 000 - 6M)(1.01)^{30} - M\left(\frac{1.01^{30} - 1}{0.01}\right)$
$0 = 20\ 000(1.01)^{30} - 6M(1.01)^{30} - M\left(\frac{1.01^{30} - 1}{0.01}\right)$
$0 = 20\ 000(1.01)^{30} - M\left[6(1.01)^{30} + \left(\frac{1.01^{30} - 1}{0.01}\right)\right]$
$M = \frac{20\ 000(1.01)^{30}}{6(1.01)^{30} + \left(\frac{1.01^{30} - 1}{0.01}\right)}$
$M = \$628.78$</td><td>3 correct solution
2 express A_{36} using the sum of a gp
1 attempts to solve $A_{36} = 0$</td></tr>
<tr><td>28a</td><td>$f(x) = e^{2x+1}x^4$
$f'(x) = 2e^{2x+1}x^4 + 4x^3e^{2x+1}$
$f''(x) = 4e^{2x+1}x^4 + 8x^3e^{2x+1} + 12x^2e^{2x+1} + 8x^3e^{2x+1}$
$f''(x) = 4e^{2x+1}x^4 + 16x^3e^{2x+1} + 12x^2e^{2x+1}$
let $f'(x) = 0$
$0 = 2x^3e^{2x+1}(x + 2)$
$x = 0, -2$
$f''(0) = 0 \Rightarrow$ potential point of inflection
$f''(-2) = -0.79.. < 0 \Rightarrow$ maximum point
$\therefore$ maximum point at $(-2, 0.79..)$
Test (0,0)
<table><tr><td>x</td><td>-0.5</td><td>0</td><td>1</td></tr><tr><td>$f''(x)$</td><td>1.25..</td><td>0</td><td>642.7</td></tr></table>$\therefore$ (0,0) is a minimum stationary point</td><td>3 correct solution
2 correctly obtains $x = 0, -2$
1 correctly finds the first derivative</td></tr>
<tr><td>28b</td><td>$f''(x) = 4e^{2x+1}x^4 + 16x^3e^{2x+1} + 12x^2e^{2x+1}$
let $f''(x) = 0$
$0 = 4e^{2x+1}x^4 + 16x^3e^{2x+1} + 12x^2e^{2x+1}$
$0 = 4x^2e^{2x+1}(x^2 + 4x + 3)$
$x = -3, -1, 0$
<table><tr><td>x</td><td>-4</td><td>-3</td><td>-2.5</td><td>-1</td><td>-0.5</td><td>0</td><td>1</td></tr><tr><td>$f''(x)$</td><td>0.175..</td><td>0</td><td>$-343..$</td><td>0</td><td>1.25..</td><td>0</td><td>642.73..</td></tr></table>$\therefore (-3, 0.545..)$ and $(-1, 0.367..)$ are points of inflection</td><td>2 finds the coordinates and checks the concavity
1 finds $x = -3, -1$</td></tr>
</table>

28c		3 correct graph 2 sketches the curve showing most of the main features 1 provides a sketch showing the stationary point and 1 other feature
29	Solve simultaneously to find the coordinates of A and B $(x-4)^2 = 4$ $x = 6$ and $x = 2$ $A(2,4)$ and $B = (6,4)$ Since $y = (x-4)^2$ When $x = 0, y = 16$ $\therefore C(0,16)$ 	3 correct solution 2 correctly finds $\int_0^2 (x-4)^2 dx$ and the points of A, B and C, or equivalent merit 1 correctly finds $\int_0^2 (x-4)^2 dx$, or equivalent merit

	Find the area of the whole trapezium, subtract the area of the rectangle and subtract the area under the curve from $x = 0$ to $x = 2$ $\text{Trapezium} = \frac{16+4}{2} \times 6 = 60$ $\text{Rectangle} = 4 \times 4 = 16$ $\text{Area under the curve} = \int_0^2 (x-4)^2\,dx$ $= \int_0^2 x^2 - 8x + 16\,dx$ $= \left[\frac{1}{3}x^3 - 4x^2 + 16x\right]_0^2$ $= \frac{8}{3} - 16 + 32$ $= \frac{56}{3}$ $\therefore \text{shaded area} = 60 - 16 - \frac{56}{3}$ $= \frac{76}{3}$	
30a	$A = 190\pi$ $2\pi x(h+2) + 2(\pi x^2) = 190\pi$ $2x(h+2) + 2x^2 = 190$ $x(h+2) + x^2 = 95$ $xh + 2x + x^2 = 95$ $xh = 95 - 2x - x^2$ $V = \pi r^2 h$ $V = \pi x^2 h$ $V = \pi x(xh)$ $V = \pi x(95 - 2x - x^2)$ $V = \pi(95x - 2x^2 - x^3)$	2 correct solution 1 finds an expression for xh, or equivalent merit

30b	$V = \pi(95x - 2x^2 - x^3)$ $\frac{dV}{dx} = \pi(95 - 4x - 3x^2)$ $0 = \pi(95 - 4x - 3x^2)$ $3x^2 + 4x - 95 = 0$ $(3x+19)(x-5) = 0$ $x = 5$ or $-\frac{19}{3}$ $\therefore x = 5$ $\frac{dV}{dx} = \pi(95 - 4x - 3x^2)$ $\frac{d^2V}{dx^2} = \pi(-4 - 6x)$ when $x = 5$, $\frac{d^2V}{dx^2} = -34\pi < 0$ $\therefore$ maximum occurs when $x = 5$ $V = \pi(95x - 2x^2 - x^3)$ $V = \pi(95(5) - 2(5)^2 - (5)^3)$ $V = 300\pi$	3 correct solution 2 finds $x = 5$ 1 correctly finds $\frac{dV}{dx}$
31	$f(x) = \frac{x+1}{2x-1}$ and $f(g(x)) = \frac{3x+2}{3x-5}$ $f(g(x)) = \frac{g(x)+1}{2g(x)-1}$ $\frac{g(x)+1}{2g(x)-1} = \frac{3x+2}{3x-5}$ $(g(x)+1)(3x-5) = (3x+2)(2g(x)-1)$ $3xg(x) - 5g(x) + 3x - 5 = 6xg(x) - 3x + 4g(x) - 2$ $9g(x) + 3xg(x) = 6x - 3$ $g(x)(3x+9) = 6x - 3$ $g(x) = \frac{6x-3}{3x+9}$ $\therefore g(x) = \frac{2x-1}{x+3}$	3 correct solution 2 some progress 1 rewrites $f(g(x))$ in terms of $g(x)$, or equivalent merit

32	$\int \frac{1}{\sqrt{x^{\frac{3}{2}}+5x}}\,dx = \int \frac{1}{\sqrt{x\left(x^{\frac{1}{2}}+5\right)}}\,dx$ $= \int \frac{1}{\sqrt{x}\sqrt{x^{\frac{1}{2}}+5}}\,dx$ $= \int x^{-\frac{1}{2}}\left(x^{\frac{1}{2}}+5\right)^{-\frac{1}{2}}\,dx$ $= 4\left(x^{\frac{1}{2}}+5\right)^{\frac{1}{2}}+C$	2 correct solution 1 attempts to use the reverse chain rule, or equivalent