Student Number: ________________

GIRRAWEEN HIGH SCHOOL

2024 | **TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen.
- NESA approved calculators may be used.
- A reference sheet is provided.
- In section II, show relevant mathematical reasoning and/or calculations.

Total marks: 70

Section I – 10 marks

- Attempt questions 1 - 10
- Allow about 15 minutes for this section.

Section II – 60 marks

- Attempt questions 11 - 16.
- Allow about 1 hour and 45 minutes for this section.
- Start each question on a new page.

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10

1. Given that
   $\sin A = \dfrac{5}{13}$ and $\sin B = \dfrac{8}{17}$, find the value of $\sin(A + B)$. A and B are acute angles.

   A. $-\dfrac{107}{140}$

   B. $-\dfrac{21}{221}$

   C. $\dfrac{107}{140}$

   D. $\dfrac{171}{221}$

2. The angle between the vectors $\underset{\sim}{u} = -2\underset{\sim}{i} + 6\underset{\sim}{j}$ and $\underset{\sim}{v} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$ is closest to:

   A. $45°$

   B. $60°$

   C. $135°$

   D. $120°$

3. What is the exact value of $\displaystyle\int_0^{\frac{\pi}{8}} \cos^2 x \, dx$?

   A. $\dfrac{\pi - 2\sqrt{2}}{8}$

   B. $\dfrac{\pi - 2\sqrt{2}}{16}$

   C. $\dfrac{\pi + 2\sqrt{2}}{8}$

   D. $\dfrac{\pi + 2\sqrt{2}}{16}$

4. What is the value of $\int_{0}^{1} x^2 \sqrt{1 - x^3}\, dx$, using the substitution $u = 1 - x^3$?

A. $-\frac{2}{9}$

B. $-\frac{1}{9}$

C. $\frac{1}{9}$

D. $\frac{2}{9}$

5. A course regulation requires that the same number of students achieve each grade from A to E where possible.
What is the smallest number of students required to ensure that at least one particular grade is awarded 7 times?

A. 30

B. 31

C. 35

D. 36

6. A circular disc is heated so that the face is increasing in area at a constant rate of $10\ \text{cm}^2$/s. What is the rate at which the radius of the disc is increasing when the radius is 6 cm?

A. 0.26 cm/s

B. 0.27cm/s

C. 3.76 cm/s

D. 3.77 cm/s

7. What is the multiplicity of the root $x = 1$ of the equation

$f(x) = 3x^5 - 5x^4 + 5x - 3$?

A. 1

B. 2

C. 3

D. 4

8. What is the inverse function of $f(x) = e^{x^3}$?

A. $f^{-1}(x) = 3e^x$

B. $f^{-1}(x) = 3\ln x$

C. $f^{-1}(x) = \sqrt[3]{\ln x}$

D. $f^{-1}(x) = \sqrt{3\ln x}$

9. A curve is represented by the following parametric equations:

$x = 2\cos\theta, y = 2\sin\theta$

Which of the following is the cartesian equation of the curve?

A. $x + y = 2$

B. $x + y = 4$

C. $x^2 + y^2 = 2$

D. $x^2 + y^2 = 4$

10. The polynomial $f(x) = 2x^2 + kx + 4$ can be expressed as $f(x) = (x - 2)g(x) + 6$. Which of the following is the correct expression for $g(x)$?

A. $2x - 1$

B. $2x + 1$

C. $2x - 3$

D. $2x + 3$

Section II

60 marks
Attempt all questions
Allow about 1 hour and 45 minutes for this section.

Start each question on a new page in the answer booklet provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing paper is available on request.

Question 11 (10 marks)

a. Solve $\dfrac{1}{2x-1} - x > 0$ [3]

b. The polynomial $4x^3 - 12x^2 + 5x + 6 = 0$ has roots α, β and γ.

Find α, β and γ given that one of the roots is the sum of the other two. [3]

c. (i) Write $3\cos x - \sqrt{3}\sin x$ in the form $R\cos(x + \alpha)$. [2]

(ii) Hence, solve $3\cos x - \sqrt{3}\sin x = \sqrt{3}$ for $0 \leq x \leq 2\pi$. [2]

Question 12 (10 marks)

a. Given that $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{v} = -2\underset{\sim}{i} + 4\underset{\sim}{j}$, find $\text{proj}_{\underset{\sim}{u}}\underset{\sim}{v}$. [2]

b. Use the substitution $t = \tan\dfrac{x}{2}$ to solve

$\cos x - 3\sin x + 3 = 0$ for $0 \leq x \leq 2\pi$. [4]

c. Use the substitution $u = \sin x$ to find $\displaystyle\int -\cos x\,\sin^{\frac{3}{2}} x\; dx$. [2]

d. In how many ways can 8 people be placed in groups of 2? [2]

Question 13 (10 marks)

a. Factorise $P(x) = 4x^3 - 3x^2 - 25x - 6$ [3]

b. Prove that $\sec A\,(1 - \sin A)(\sec A + \tan A) = 1$ [2]

c. Use mathematical Induction to prove that $3^{3n} + 2^{n+2}$ is divisible by 5 [3]

for $n \geq 1$.

d. Find $\displaystyle\int \frac{1}{9 + 16x^2}\; dx$ [2]

Question 14 (10 marks)

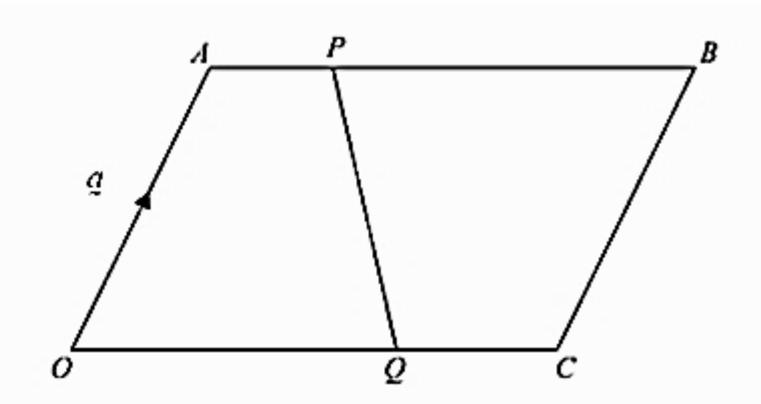
a. In the diagram below, $OABC$ is a parallelogram. [3]

$\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

P is the point on AB such that $AP = \frac{1}{4} AB$.

Q is the point on OC such that $OQ = \frac{2}{3} OC$.

Find $\overrightarrow{PQ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$, giving your answer in simplest surd form.



b. Newton's law of cooling states that the rate of change of the temperature θ, of a body at any time t, is proportional to the difference in the temperature of the body and the temperature m, of the surrounding medium, i.e.

$\frac{d\theta}{dt} = k(\theta - m)$, where k is a constant.

(i) Show that $\theta = m + Ae^{kt}$ where A is a constant, satisfies this equation. [1]

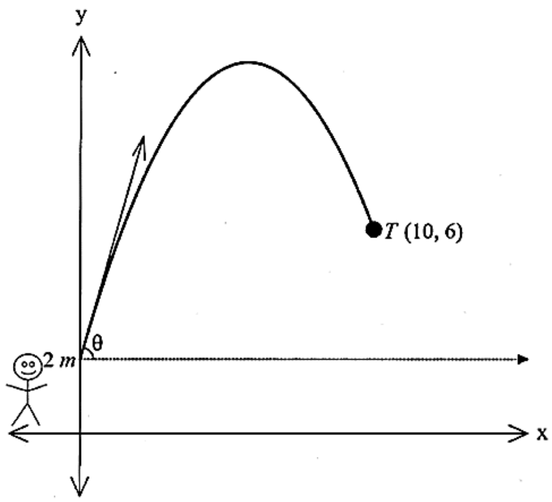
(ii) If the temperature of the surrounding air is $40°C$ and the temperature of the body drops from $170°C$ to $105°C$ in 45 minutes, find the temperature of the body in another 90 minutes (to 2 decimal places). [2]

(iii) Find the time taken for the temperature of the body to drop to $80°C$ (to the nearest minute). [2]

c. Find the coefficient of x in the expansion of $\left(x + \frac{2}{x^2}\right)^{10}$ [2]

Question 15 (10 marks)

a. David throws a pebble from a height of 2 metres at an angle of θ to the horizontal, with a velocity of 30m/s.



(i) Show that the expression for the horizontal and vertical displacements at t seconds after projection are
$x = 30t \cos \theta$ and $y = -5t^2 + 30t \sin\theta + 2$ respectively. [2]
(Take g = 10m/s and take the origin to be the ground directly below David)

(ii) Show that the equation of the path of the particle is:
$y = \frac{-x^2}{180}(1 + \tan^2 \theta) + x \tan \theta + 2$ [2]

(iii) If David manages to hit a target at point T, which is 10 metres away horizontally and 6 metres high, find the two possible angles of projection. [2]

b. For the function $y = 2\sin^{-1}(2x - 1) + \frac{\pi}{2}$

(i) Find the domain and range. [2]

(ii) Sketch the curve, clearly showing the coordinates of the endpoints. [2]

Question 16 (10 marks)

a. Solve the equation $\sin 3x - \sin x + \cos 2x = 0$ for $0 \leq x \leq 2\pi$. [3]

b. (i) Prove that $8\cos^4 x = 3 + 4\cos 2x + \cos 4x$. [2]

(ii) Hence, find the volume generated when the area bounded by the curve $y = \cos^2 x$, the x-axis and $x = 0$ and $x = \dfrac{\pi}{4}$ is rotated about the x-axis. [2]

c. A committee of 5 is to be chosen from six men and seven women.

(i) How many committees are possible if there are no restrictions? [1]

(ii) How many committees are possible if there are more women than men? [2]

End of Examination

Mathematics Assessment Solutions
Girraween High School

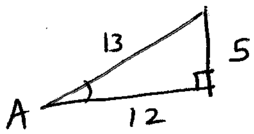
Task: Trial HSC Extension 1	Year: 2024

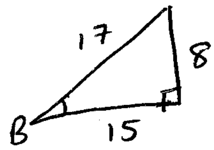
Suggested Solutions

Question 1–2

1. D 2. C 3. D 4. D 5. B 6. B 7. C 8. C 9. D 10. B

1. A and B lie in Q1





$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$= \frac{5}{13} \times \frac{15}{17} + \frac{12}{13} \times \frac{8}{17}$$
$$= \frac{171}{221} \quad [D]$$

2. $$\cos\theta = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{u}||\underset{\sim}{v}|}$$
$$= \frac{(-2) \times 4 + 6 \times (-2)}{\sqrt{40} \times \sqrt{20}}$$
$$= \frac{-20}{20\sqrt{2}}$$
$$\theta = 135^\circ \quad [C]$$

Marker's Feedback

Task: Trial HSC Extension 1	Year: 2024

Suggested Solutions

Question 3

3. $\displaystyle\int_0^{\pi/8} \cos^2 x \, dx$

$$= \int_0^{\pi/8} \frac{1}{2}(1+\cos 2x)\, dx$$

$$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\pi/8}$$

$$= \frac{1}{2}\left[\frac{\pi}{8} + \frac{1}{2}\sin\left(\frac{\pi}{4}\right) - 0\right]$$

$$= \frac{\pi}{16} + \frac{\sqrt{2}}{8}$$

$$= \frac{\pi + 2\sqrt{2}}{16} \qquad [D]$$

Marker's Feedback

Task:	Year: 2024

Suggested Solutions

Question 4

$$\int_0^1 x^2\sqrt{1-x^3}\,dx$$

$u = 1-x^3$

$\frac{du}{dx} = -3x^2$

$dx = \frac{-1}{3x^2}\,du$

when $x=0$, $u=1$

when $x=1$, $u=0$

$$= \frac{1}{3}\int_1^0 \sqrt{u}\,du$$

$$= \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_0^1$$

$$= \frac{2}{9} \qquad [D]$$

Question 5

$5\times 6+1 = 31 \qquad [B]$

Marker's Feedback

Suggested Solutions

Question 6

$\frac{dA}{dt} = 10\,cm^2/s$; $\frac{dr}{dt} = ?$; $r=6$; $A = \pi r^2$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$10 = 2\pi r \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{10}{2\pi r}$$

$$= \frac{10}{2\pi \times 6} \quad (\text{when } r=6)$$

$$\doteqdot 0.27\,cm/s$$

[B]

Question 7

$P(x) = 3x^5 - 5x^4 + 5x - 3$; $P(1) = 0$

$P'(x) = 15x^4 - 20x^3 + 5$; $P'(1) = 0$

$P''(x) = 60x^3 - 60x^2$; $P''(1) = 0$

$P'''(x) = 180x^2 - 120x$; $P(1) = 60 \neq 0$

$\therefore$ Multiplicity $= 3$

[C]

Marker's Feedback

Suggested Solutions

Question 8

$f(x): y = e^{x^3}$

$f^{-1}(x): \quad x = e^{y^3}$

$\ln x = \ln e^{y^3}$

$\ln x = y^3$

$y = \sqrt[3]{\ln x}$ [C]

Question 9

$x^2+y^2 = (2\cos\theta)^2 + (2\sin\theta)^2$

$= 4(\sin^2\theta+\cos^2\theta)$

$x^2+y^2 = 4$ [D]

Marker's Feedback

Suggested Solutions

Question 10

$f(x) = (x-2)g(x) + 6 \Rightarrow f(2) = 6$ (Remainder theorem)

$f(2) = 2 \times 2^2 + 2k + 4 = 6$

$12 + 2k = 6$

$2k = -6$

$k = -3$

$2x^2 - 3x + 4 = (x-2)g(x) + 6$

$2x^2 - 3x - 2 = (x-2)g(x)$

$(x-2)(2x+1) = (x-2)g(x)$

$\therefore g(x) = 2x+1$ [B]

Marker's Feedback

Suggested Solutions

Question 11

a) $\dfrac{1}{2x-1} - x > 0 \quad , \quad x \neq \dfrac{1}{2}$

Solve

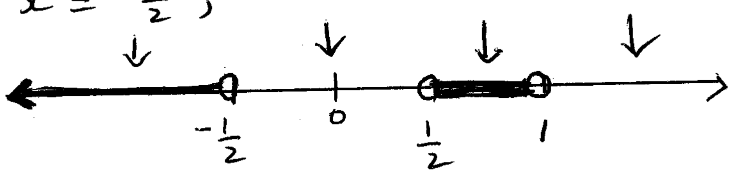
$$\frac{1}{2x-1} = x$$

$$2x^2 - x = 1$$

$$2x^2 - x - 1 = 0$$

$$(2x+1)(x-1) = 0 \qquad 1$$

$$x = -\frac{1}{2}, 1$$



Test $x = -1$

$$\frac{1}{2(-1)-1} - (-1) > 0$$

✓

Test $x = 0$

$$\frac{1}{-1} - 0 > 0$$

×

Test $x = 0.8 \qquad 1$

$$\frac{1}{2(0.8)-1} - 0.8 > 0$$

✓

Test $x = 2$

$$\frac{1}{2(2)-1} - 2 > 0$$

×

Solution:

$x < -\frac{1}{2}$, $\frac{1}{2} < x < 1 \qquad 1$

$\boxed{3}$

Marker's Feedback

Suggested Solutions

Question 11

a)

$$\frac{1}{2x-1} - x > 0 \quad ; \quad x \neq \frac{1}{2}$$

$$\frac{1}{2x-1} > x$$

$\left(\times (2x-1)^2\right)$

$$2x-1 > x(2x-1)^2$$ 1

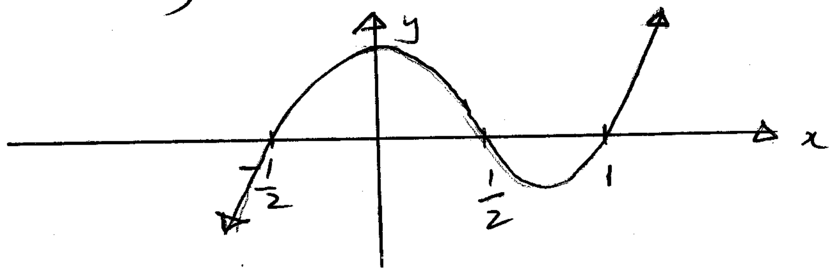
$$x(2x-1)^2 < 2x-1$$

$$x(2x-1)^2 - (2x-1) < 0$$

$$(2x-1)\left[x(2x-1) - 1\right] < 0$$

$$(2x-1)(2x^2 - x - 1) < 0$$ 1

$$(2x-1)(2x+1)(x-1) < 0$$



1

Solution : $x < -\frac{1}{2}$, $\frac{1}{2} < x < 1$ $\boxed{3}$

Marker's Feedback

Suggested Solutions

Question 11

b) $4x^3 - 12x^2 + 5x + 6$, roots α, β, γ

$\Rightarrow \alpha, \beta, \alpha+\beta$

Sum of roots : $\alpha + \beta + \alpha + \beta = 3$

$\alpha + \beta = \frac{3}{2}$

ie. $\gamma = \frac{3}{2} \Rightarrow \alpha + \beta = \frac{3}{2}$ 1

product of root : $\frac{3}{2}\alpha\beta = -\frac{3}{2}$

$\alpha\beta = -1$; $\beta = \frac{3}{2} - \alpha$

Substitute $\beta = \frac{3}{2} - \alpha$ into $\alpha\beta = -1$ 1

$\alpha\left(\frac{3}{2} - \alpha\right) = -1$

$\frac{3}{2}\alpha - \alpha^2 = -1$

$2\alpha^2 - 3\alpha - 2 = 0$

$(\alpha - 2)(2\alpha + 1) = 0$

$\alpha = 2, -\frac{1}{2}$

Roots : $\alpha = 2, \beta = -\frac{1}{2}, \gamma = \frac{3}{2}$ 1

$\boxed{3}$

Marker's Feedback

Suggested Solutions

Question 11 c)

i) $3\cos x - \sqrt{3}\sin x \equiv R\cos(x+\alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

Equating like terms,

$R\cos\alpha = 3$ — ① ; $R\sin\alpha = \sqrt{3}$ — ②

Squaring and adding ① and ②

$R^2(\sin^2\alpha + \cos^2\alpha) = 12$

$R = 2\sqrt{3}$ $(R>0)$ 1

$\sin\alpha = \frac{1}{2}$; $\cos\alpha = \frac{\sqrt{3}}{2}$ $\Rightarrow$ α in Q1

$\therefore \alpha = \frac{\pi}{6}$ 1

$\therefore 3\cos x - \sqrt{3}\sin x \equiv 2\sqrt{3}\cos(x+\pi/6)$ $\boxed{2}$

ii) $3\cos x - \sqrt{3}\sin x = \sqrt{3}$

$2\sqrt{3}\cos(x+\pi/6) = \sqrt{3}$

$\cos(x+\pi/6) = \frac{1}{2}$ 1

$x + \pi/6 = \pi/3$ or $5\pi/3$

$x = \pi/6, 3\pi/2$ 1 $\boxed{2}$

Marker's Feedback

Suggested Solutions

Question 12

a) $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$; $\underset{\sim}{v} = -2\underset{\sim}{i} + 4\underset{\sim}{j}$

$$\text{proj}_{\underset{\sim}{u}}\,\underset{\sim}{v} = \frac{\underset{\sim}{u}\cdot\underset{\sim}{v}}{\underset{\sim}{u}\cdot\underset{\sim}{u}}\cdot\underset{\sim}{u} \qquad 1$$

$$= \frac{2\times(-2) + 3\times 4}{2^2 + 3^2}\cdot 2\underset{\sim}{i} + 3\underset{\sim}{j}$$

$$= \frac{8}{13}\left(2\underset{\sim}{i} + 3\underset{\sim}{j}\right) \qquad 1$$

$\boxed{2}$

Marker's Feedback

Suggested Solutions

Question 12

b) $t = \tan \frac{x}{2}$

$\cos x - 3\sin x + 3 = 0$

$\frac{1-t^2}{1+t^2} - 3\left(\frac{2t}{1+t^2}\right) + 3 = 0$ $\frac{1}{2}$

$1 - t^2 - 6t + 3(1+t^2) = 0$

$1 - t^2 - 6t + 3 + 3t^2 = 0$

$2t^2 - 6t + 4 = 0$

$t^2 - 3t + 2 = 0$

$(t-1)(t-2) = 0$ $\frac{1}{2}$

$t = 1, 2$

ie. $\tan \frac{x}{2} = 1$ or $\tan \frac{x}{2} = 2$

$\frac{x}{2} = \frac{\pi}{4}$ $\qquad$ $\frac{x}{2} = 1.107$

$x = 2.21$ (to 2dp) $\frac{1}{2}$

$x = \frac{\pi}{2}$

Test for $x = \pi$ $\qquad$ $\therefore x = \frac{\pi}{2}, 2.21$

$\cos \pi - 3\sin \pi + 3$ $\frac{1}{2}$

$= -1 + 3 \neq 0$

$\therefore x = \pi$ is not a solution $\boxed{4}$

Marker's Feedback

Suggested Solutions

Question 12

c) $\int -\cos x \sin^{3/2} x \, dx$; $u = \sin x$

$du = \cos x \, dx$

$= -\int u^{3/2} \, du$ 1

$= -\dfrac{u^{5/2}}{5/2} + c$

$= -\frac{2}{5} \sin^{5/2} x + c$ 1

$\boxed{2}$

d) $\dfrac{{}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2}{4!}$ 1

$= 105$ 1

Marker's Feedback

Suggested Solutions

Question 13

a) $P(x) = 4x^3 - 3x^2 - 25x - 6$

Try : Factors of $-6 = \pm 1, \pm 2, \pm 3, \pm 6$

$P(-2) = 4(-2)^3 - 3(-2)^2 - 25(-2) - 6$

$= 0$

$\therefore (x+2)$ is a factor

$$
\begin{array}{r l}
 & 4x^2 - 11x - 3 \\
x+2 \,\big) & 4x^3 - 3x^2 - 25x - 6 \\
- & 4x^3 + 8x^2 \\
\hline
 & -11x^2 - 25x \\
- & -11x^2 - 22x \\
\hline
 & -3x - 6 \\
 & -3x - 6 \\
\hline
\end{array}
$$

$\therefore P(x) = (x+2)(4x^2 - 11x - 3)$

$= (x+2)(x-3)(4x+1)$

$\boxed{3}$

Marker's Feedback

Suggested Solutions

Question 13

b) $\sec A(1-\sin A)(\sec A+\tan A)=1$

$$LHS = \sec A\,(1-\sin A)(\sec A+\tan A)$$

$$= \frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$$

$$= \frac{1}{\cos A}(1-\sin A)\frac{1}{\cos A}(1+\sin A)$$ 1

$$= \frac{1}{\cos^2 A}(1-\sin^2 A)$$

$$= \frac{1}{\cos^2 A}(\cos^2 A)$$ 1

$$= 1$$

$$= RHS$$

$\boxed{2}$

Marker's Feedback

Suggested Solutions

Question 13

c) $3^{3n} + 2^{n+2}$ is divisible by 5 ; $n \geq 1$

Step 1 : Show true for $n=1$

$3^3 + 2^3 = 35$ which is divisible by 5.

Step 2 : Assume true for $n=k$ for integer $k>0$

ie. $3^{3k} + 2^{k+2} = 5M$; M is a positive integer

1

Step 3 : Prove true for $n=k+1$

ie. $3^{3k+3} + 2^{k+3} = 5N$, N is a positive integer

$$
\begin{aligned}
LHS &= 3^{3k+3} + 2^{k+3} \\
&= 3^{3k} \times 3^3 + 2^{k+2} \times 2 \\
&= (5M - 2^{k+2}) \times 27 + 2^{k+2} \times 2 \quad \text{(from assumption)} \\
&= 135M - 27 \times 2^{k+2} + 2 \times 2^{k+2} \\
&= 135M - 25 \times 2^{k+2} \\
&= 5(27M - 5 \times 2^{k+2}) \\
&= 5N
\end{aligned}
$$

1

$\therefore$ If true for $n=k$, then also true for $n=k+1$

Step 4 : By the principle of Mathematical Induction, the result is true for all $n \geq 1$

1

$\boxed{3}$

Marker's Feedback

Suggested Solutions

Question 13

d) $$\int \frac{1}{9+16x^2}\,dx$$

$$= \int \frac{1}{3^2+(4x)^2}\,dx$$

$$= \frac{1}{4}\int \frac{4}{3^2+(4x)^2}\,dx \qquad 1$$

$$= \frac{1}{4} \times \frac{1}{3} \tan^{-1}\left(\frac{4x}{3}\right) + c$$

$$= \frac{1}{12} \tan^{-1}\left(\frac{4x}{3}\right) + C \qquad 1$$

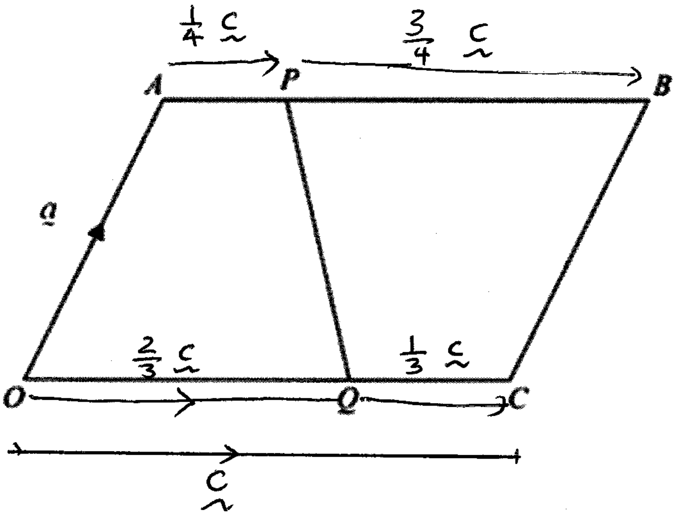
$\boxed{2}$

Marker's Feedback

Suggested Solutions

Question 14

a)



$$\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AB} + \overrightarrow{OQ}$$ 1

$$= -\frac{1}{4}\underset{\sim}{c} - \underset{\sim}{a} + \frac{2}{3}\underset{\sim}{c}$$ 1

$$= \left(\frac{2}{3} - \frac{1}{4}\right)\underset{\sim}{c} - \underset{\sim}{a}$$

$$= \frac{5}{12}\underset{\sim}{c} - \underset{\sim}{a}$$ 1

$\boxed{3}$

Marker's Feedback

Suggested Solutions

Question 14

b) i) $\theta = m + Ae^{kt}$ $\Rightarrow$ $Ae^{kt} = \theta - m$

$$\frac{d\theta}{dt} = kAe^{kt}$$

$$= k(\theta - m) \quad (\text{since } Ae^{kt} = \theta - m)$$ 1

$\therefore$ $\theta = m + Ae^{kt}$ satisfies the equation.

ii) $\theta = m + Ae^{kt}$

At $t = 0$,

$170 = 40 + Ae^{0}$

$\therefore A = 130$

$\theta = 40 + 130e^{kt}$

At $t = 45$

$105 = 40 + 130e^{45k}$

$65 = 130e^{45k}$

$45k \ln e = \ln \frac{65}{130}$

$45k = \ln \frac{65}{130}$ 1

$k \doteqdot -0.0154$

con't ---> P19

Marker's Feedback

Suggested Solutions

Question 14

b (ii) con't :

At $t = 135$

$$\theta = 40 + 130e^{135k}$$

$$= 56 \cdot 25°C \quad (\text{to } 2dp)$$ 1

iii) $\theta = 80°C$, $t = ?$

$$80 = 40 + 130e^{kt}$$ 1

$$40 = 130e^{kt}$$

$$t = \ln \frac{4}{13} \div k$$

$$= 76.53$$ 1

$$= 77 \text{ minutes}$$

$\boxed{2}$

Marker's Feedback

Suggested Solutions

Question 14

c) $\left(x + \frac{2}{x^2}\right)^{10}$

$$T_{k+1} = {}^nC_k\ a^{n-k}\ b^k$$

$$= {}^{10}C_k \cdot x^{10-k} \cdot \left(2x^{-2}\right)^k$$

$$= {}^{10}C_k \cdot x^{10-k} \cdot 2^k\ x^{-2k}$$

$$= {}^{10}C_k\ 2^k \cdot x^{10-3k}$$

$\underline{\underline{1}}$

coefficient of $x \Rightarrow 10-3k=1$

$k = 3$

$\therefore$ coefficient of $x = {}^{10}C_3 \cdot 2^3$

$= 960$

$\underline{\underline{1}}$

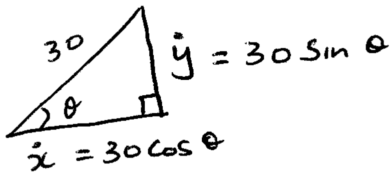
$\boxed{2}$

Marker's Feedback

Suggested Solutions

Question 15

a) i) Initially,



Resolving Forces,

Horizontally

$\ddot{x} = 0$

$\dot{x} = c_1$

when $t = 0$, $\dot{x} = 30\cos\theta$

$c_1 = 30\cos\theta$

$\therefore \dot{x} = 30\cos\theta$

$x = 30t\cos\theta + c_2$

when $t = 0$, $x = 0$

$\therefore c_2 = 0$ 1

$x = 30t\cos\theta$ — ①

Vertically

$\ddot{y} = -10$

$\dot{y} = -10t + c_1$

When $t = 0$, $\dot{y} = 30\sin\theta$

$c_1 = 30\sin\theta$

$\therefore \dot{y} = -10t + 30\sin\theta$

$y = -5t^2 + 30t\sin\theta + c_2$

when $t = 0$, $y = 2$

$\therefore c_2 = 2$ 1

$y = -5t^2 + 30t\sin\theta + 2$ — ②.

$\boxed{2}$

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Suggested Solutions

Question 15

a) (ii) From (1)

$$t = \frac{x}{30\cos\theta} \quad \text{--- (3)}$$

substituting (3) into (2)

1

$$y = -5\left(\frac{x}{30\cos\theta}\right)^2 + 30\sin\theta\left(\frac{x}{30\cos\theta}\right) + 2$$

$$= \frac{-5}{30^2}x^2\sec^2\theta + x\tan\theta + 2$$

$$= \frac{-x^2}{180}(1+\tan^2\theta) + x\tan\theta + 2$$

1

$\boxed{2}$

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Suggested Solutions

Question 15

a) (iii)

when $x = 10$, $y = 6$

$$\therefore \quad 6 = \frac{-10^2}{180}(1 + \tan^2\theta) + 10\tan\theta + 2$$

$$6 = \frac{-5}{9}(1 + \tan^2\theta) + 10\tan\theta + 2$$

$$54 = -5 - 5\tan^2\theta + 90\tan\theta + 18$$ 1

$$5\tan^2\theta - 90\tan\theta + 41 = 0$$

$$\tan\theta = \frac{90 \pm \sqrt{90^2 - 4 \times 5 \times 45}}{10}$$

$$= \frac{90 \pm \sqrt{7280}}{10}$$ 1

$\theta = 87^\circ, 25^\circ$ (θ is acute)

$\boxed{2}$

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Suggested Solutions

Question 15

b) $y = 2\sin^{-1}(2x-1) + \frac{\pi}{2}$

$$\frac{y - \frac{\pi}{2}}{2} = \sin^{-1}(2x-1)$$

Domain : $-1 \le 2x-1 \le 1$

$0 \le x \le 1$ $\underline{\underline{1}}$

Range :

$$-\frac{\pi}{2} \le \frac{y-\frac{\pi}{2}}{2} \le \frac{\pi}{2}$$

$$-\pi \le y - \frac{\pi}{2} \le \pi$$

$$-\frac{\pi}{2} \le y \le \frac{3\pi}{2}$$ $\underline{\underline{1}}$

$\boxed{2}$

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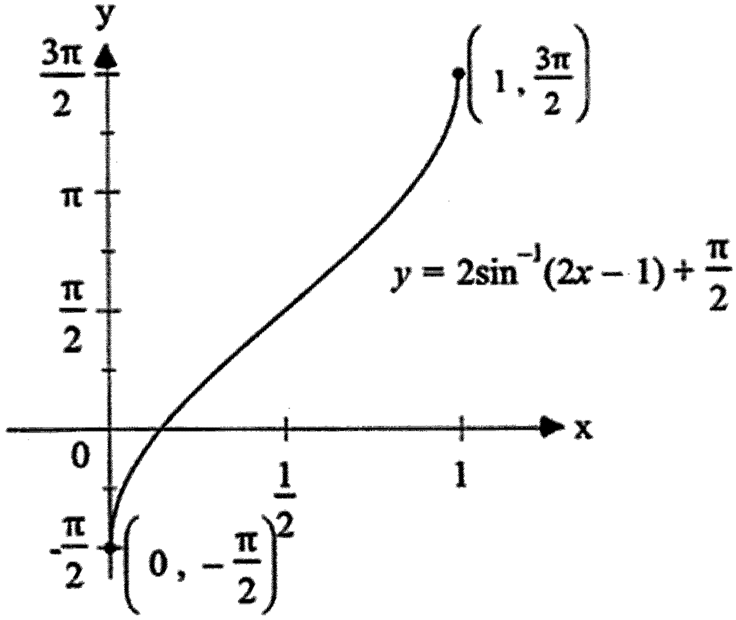
Suggested Solutions

Question 15

b (ii)



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Suggested Solutions

Question 16

a) $\sin 3x - \sin x + \cos 2x = 0$; $0 \le x \le 2\pi$

$\sin(2x + x) - \sin(2x - x) + \cos 2x = 0$

$2\cos 2x \sin x + \cos 2x = 0$ 1

$\cos 2x(2\sin x + 1) = 0$

$\cos 2x = 0$; $0 \le 2x \le 4\pi$

$2x = \pi/2, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ 1

or $\sin x = -\frac{1}{2}$ φ 3, 4

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

1

$x = \pi/4, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\boxed{3}$

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Suggested Solutions

Question 16

b) i) $8\cos^4 x = 3 + 4\cos 2x + \cos 4x$

LHS $= 8\cos^4 x$

$= 2\left(2\cos^2 x\right)^2$

$= 2\left(1 + \cos 2x\right)^2$

$= 2\left(1 + 2\cos 2x + \cos^2 2x\right)$ 1

$= 2\left[1 + 2\cos 2x + \frac{1}{2}\left(1 + \cos 4x\right)\right]$

$= 2 + 4\cos 2x + 1 + \cos 4x$ 1

$= 3 + 4\cos 2x + \cos 4x$

$=$ RHS.

$\boxed{2}$

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Suggested Solutions

Question 16

b (ii)

$$V = \pi \int_0^4 \cos^4 x \, dx$$

$$= \pi \int_0^4 \frac{1}{8} \left(3 + 4\cos 2x + \cos 4x\right) dx$$

$$= \frac{\pi}{8} \left[3x + 2\sin 2x + \frac{1}{4}\sin 4x\right]_0^4$$ 1

$$= \frac{\pi}{8}\left[3\left(\frac{\pi}{4}\right) + 2\sin\left(\frac{\pi}{2}\right) + \frac{1}{4}\sin \pi\right] - 0$$

$$V = \frac{\pi}{8}\left[\frac{3\pi}{4} + 2\right] u^3$$ 1

$\boxed{2}$

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Suggested Solutions

Question 16

C. (i) $^{13}C_5 = 1287$ $\boxed{1}$

(ii) More women than men

$\Rightarrow 5W$: $^{7}C_5 = 21$

$\Rightarrow 4W, 1M$: $^{7}C_4 \times {}^{6}C_1 = 210$

$\Rightarrow 3W, 2M$: $^{7}C_3 \times {}^{6}C_2 = 525$ 1

Total $= 21 + 210 + 525$ 1

$= 756$

$\boxed{2}$

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