Student Number

Hunters Hill High School

2024 **TRIAL EXAMINATION**

Mathematics Extension 1

General Instructions	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • For questions in Section II, show relevant mathematical reasoning and/ or calculations
Total marks: 70	**Section I – 10 marks** (pages 3–9) • Attempt Questions 1–10 • Allow about 15 minutes for this section **Section II – 60 marks** (pages 10–16) • Attempt Questions 11–14 • Allow about 1 hour and 45 minutes for this section

Section I

10 Marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

---

1. For which values of x is $|x - 1| \geq 4$?

(A) $-3 \leq x \leq 5$

(B) $x \leq -3, \quad x \geq 5$

(C) $x \geq 5$

(D) $-3 < x \leq 5$

2. Given the substitution $t = \tan\frac{\theta}{2}$, the expression $\frac{1 - \cos\theta}{\sin\theta}$ can be written as

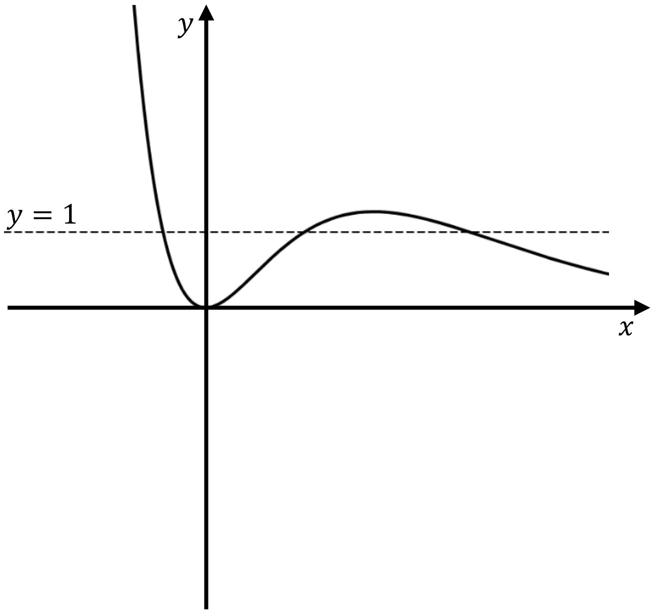
(A) t

(B) 0

(C) $\frac{1}{t}$

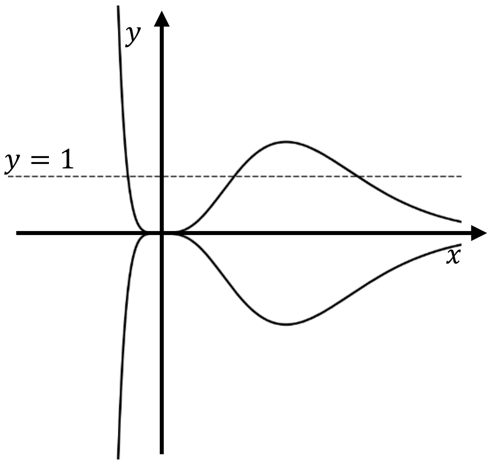
(D) $2t$

3. This graph shows $y = f(x)$.

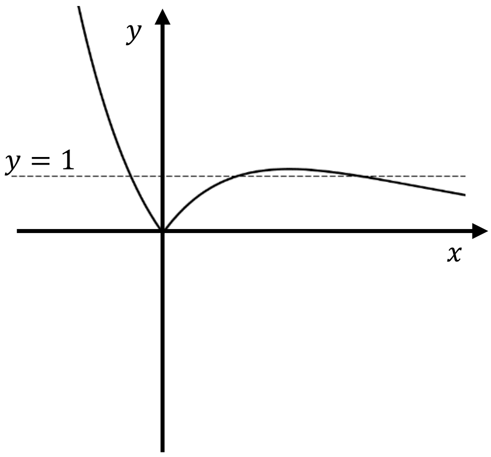


Which of the following graphs best represents $y^2 = f(x)$?

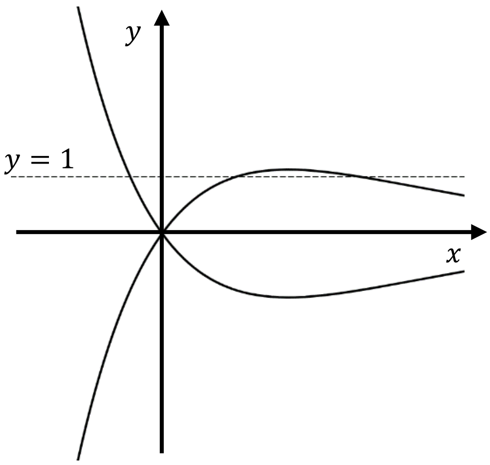
(A)



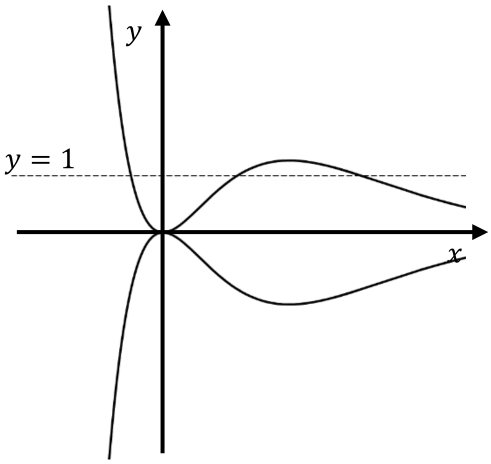
(B)



(C)



(D)



4. In how many ways can 7 people, chosen from a group of 12 people, be arranged around a table?

(A) $\dfrac{11!}{4!\,7}$

(B) $\dfrac{11!}{4!}$

(C) $\dfrac{12!}{5!\,7}$

(D) $\dfrac{12!}{5!}$

5. The polynomial $P(x) = 3x^3 + \alpha x^2 + \beta x + \gamma$ has roots 3 and -2, with one of them being a double root.

What is a possible value of β?

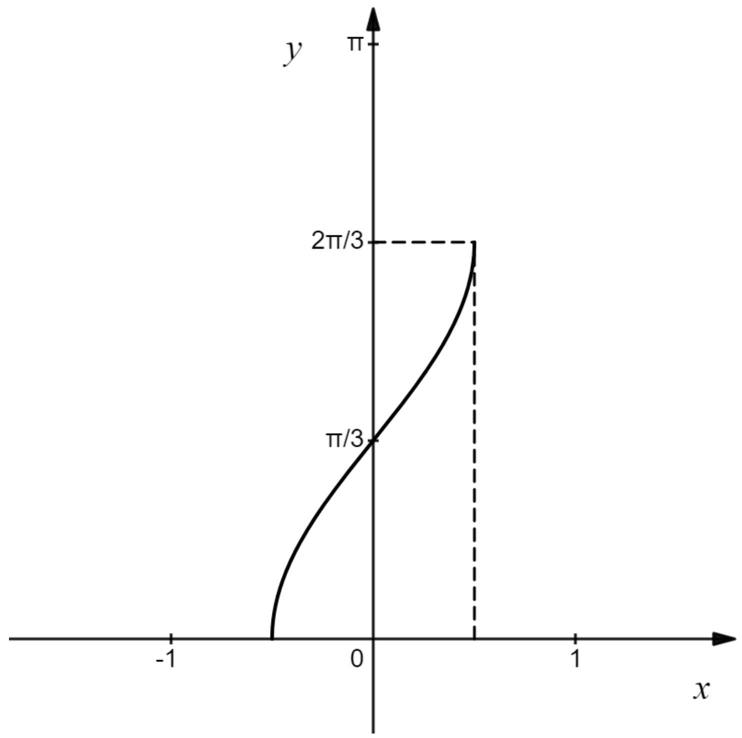
(A) -3

(B) -9

(C) 3

(D) 9

6. Which function best describes the following graph?



(A) $y = \frac{3}{2}\cos^{-1}\left(-\frac{x}{2}\right)$

(B) $y = \frac{2}{3}\cos^{-1}\left(-\frac{x}{2}\right)$

(C) $y = \frac{3}{2}\cos^{-1}(-2x)$

(D) $y = \frac{2}{3}\cos^{-1}(-2x)$

7. The integral of $\cos^2 2x$ is

(A) $\frac{1}{2}(1+\cos 4x)+c$

(B) $\frac{1}{2}(1-\cos 4x)+c$

(C) $\frac{1}{2}\left(x+\frac{1}{4}\sin 4x\right)+c$

(D) $\frac{1}{2}\left(x-\frac{1}{4}\sin 4x\right)+c$

8. Which of the following vector pairs creates an acute angle of approximately 25°?

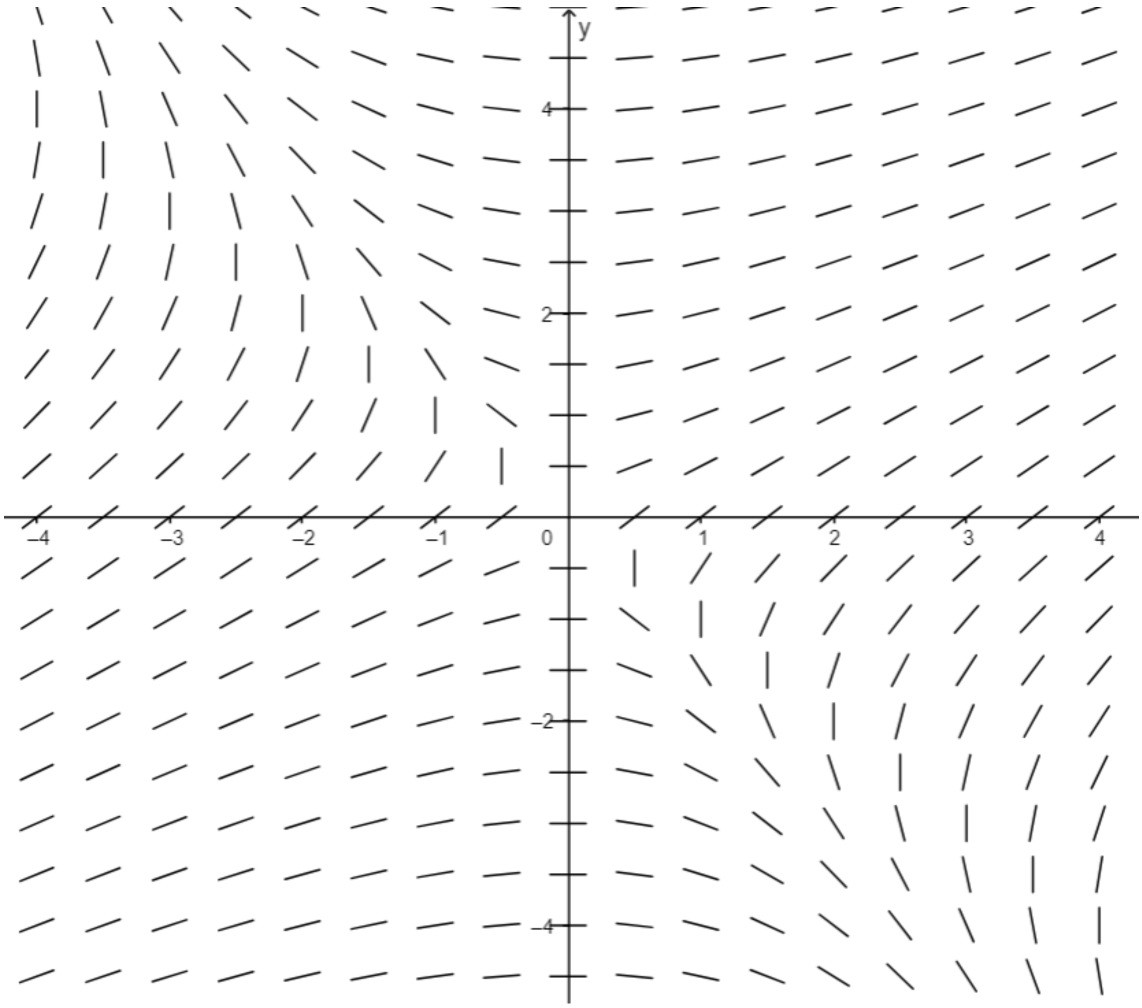
(A) $\underset{\sim}{u} = -2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$

(B) $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{v} = -\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j}$

(C) $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{v} = 0\underset{\sim}{i} + 3\underset{\sim}{j}$

(D) $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{v} = \underset{\sim}{i} + 7\underset{\sim}{j}$

9.



The differential equation that best represents the slope field above is

(A) $\dfrac{dy}{dx} = \dfrac{x+y}{x}$

(B) $\dfrac{dy}{dx} = \dfrac{x}{x+y}$

(C) $\dfrac{dy}{dx} = \dfrac{x}{y-2}$

(D) $\dfrac{dy}{dx} = \dfrac{y}{x-2}$

10. Let X be a discrete random variable with binomial distribution, such that $X \sim Bin(2.4, 1.68)$.

The values of n (the number of independent trials) and p (the probability of success in each trial) are

(A) $n = 8,\quad p = 0.3$

(B) $n = 6,\quad p = 0.3$

(C) $n = 8,\quad p = 0.4$

(D) $n = 6,\quad p = 0.4$

End of Section I

Section II

60 Marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Begin each question in a new writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (14 marks) Begin a new Writing Booklet.

(a) Convert the following parametric equations to Cartesian form, with y as the subject. **2**

$$x = 3 - t^2$$
$$y = 2t + 1$$

(b) Solve $\dfrac{2}{x-1} \geq x$. **3**

(c) Integrate $\displaystyle\int \frac{dx}{5 + 4x^2}$ **2**

(d) Define vectors $\underset{\sim}{a} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, $\underset{\sim}{b} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$ and $c = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$.

i. Find $2\underset{\sim}{a} - \underset{\sim}{c}$. **1**

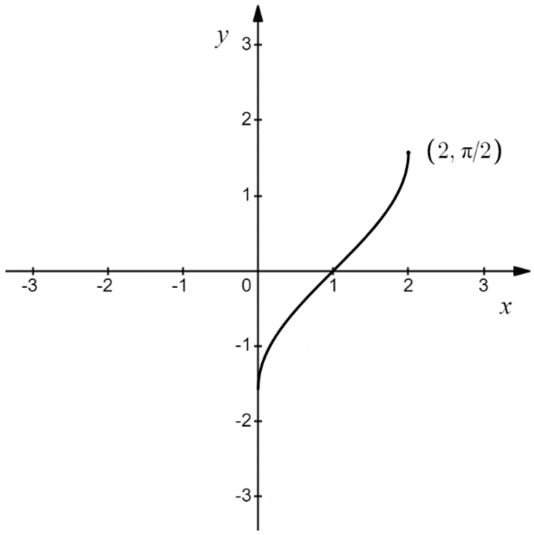
ii. Find $\left|\underset{\sim}{a}\right|$. **1**

iii. Find $\underset{\sim}{\hat{b}}$, the unit vector in the direction of $\underset{\sim}{b}$. **1**

iv. Find $proj_{\underset{\sim}{a}}\left(\underset{\sim}{c}\right)$. **2**

Question 11 continues on next page

(e) The graph of function $f(x) = \sin^{-1}(x-1)$ is shown below.



i. State the range of the inverse function, $f^{-1}(x)$. **1**

ii. How many points of intersection exist between $f(x)$ and $f^{-1}(x)$? Justify your answer. **1**

End of Question 11

Question 12 (16 marks) Begin a new Writing Booklet.

(a) If $\cos\theta = -\dfrac{5}{8}$ and $\tan\theta < 0$, **2**

determine the exact value of $\sin\theta$ in its simplest form.

(b) A polynomial, $P(x)$, is given by $P(x) = x^4 - 11x^3 + 24x^2 + 4x - 32$.

i. Show that $P(x)$ has a zero, $x = 2$, of multiplicity two. **2**

ii. By polynomial division, deduce the remaining zeros and hence write $P(x)$ as a product of its linear factors. **3**

(c) A school survey about student experiences in Year 12 consists of six questions. Each question has two possible answers, True or False, and students are required to answer every question. **2**

How many students must complete the survey to ensure that at least three sets of responses are identical?

(d) Solve $\sin\left(x + \dfrac{\pi}{3}\right)\cos\left(x - \dfrac{\pi}{3}\right) = \dfrac{1+\sqrt{3}}{4}, 0 \le x \le 2\pi$. **4**

(e) Prove by mathematical induction that $3^{2n+1} + 2^{n+2}$ is divisible by 7 for integers $n \ge 1$. **3**

End of Question 12

Question 13 (15 marks) Begin a new Writing Booklet.

(a) Using the substitution $u = \sin^2 x$, evaluate $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin 2x}{\sin^2 x - 1}\,dx$. **3**

(b) A spherical, latex balloon is filled with helium to an initial radius of 20 cm, allowing it to float. Over time, helium escapes through pores in the surface of the balloon, such that its radius decreases at a rate of 1.6 cm h^{-1}.

i. At what rate is the volume (in $\text{cm}^3\ \text{h}^{-1}$) of the balloon decreasing when its radius reaches 10 cm?
Answer correct to 1 decimal place. **2**

ii. The balloon is considered to be 'deflated' once its volume reaches 800 cm^3.
How many hours will it take the balloon to reach this state? **2**

(c) In ceramic kilns, the thermocouple is the device that reports the temperature to the user. **4**

In an old kiln, the thermocouple is faulty and fails somewhere around the 900°C mark, meaning the user is unable to determine whether the kiln is correctly reaching higher temperatures.

Six hours after the kiln begins to cool, the temperature is recorded at 750°C. A second measurement of 500°C was recorded after another three hours.

The rate of change of the temperature in the kiln is proportional to the difference between its temperature and the room temperature of 80°C. This can be modelled using Newton's Law of Cooling,

$$\frac{dT}{dt} = -k(T - A),$$

where T is the temperature in degrees Celsius at time t hours after the kiln has begun to cool.

Calculate the initial temperature of the kiln.

Question 13 continues on next page

(d) Consider the differential equation $\frac{dy}{dx} = xy - 2x$.

i. Find the gradient of the tangent to the solution curve that passes through the point $(2, 3)$, at that point. **1**

ii. Find the equation of the solution curve to the DE that passes through the point $(2, 3)$. **3**

End of Question 13

Question 14 (15 marks) Begin a new Writing Booklet.

(a) The latest product in a range of torches is considered faulty if its bulb produces light of less than 2000 lumens. **3**

The manufacturer has concluded that the chance of producing a faulty torch is 4.3%.

A random sample of 40 torches is selected post-manufacturing and the strength of its light is tested.

Assuming the sample proportion is normally distributed, use the table of values provided to find the probability that the percentage of torches with faulty bulbs lies between 5% and 10%.

(b) Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x}\right)^6$. **3**

Question 14 continues on next page

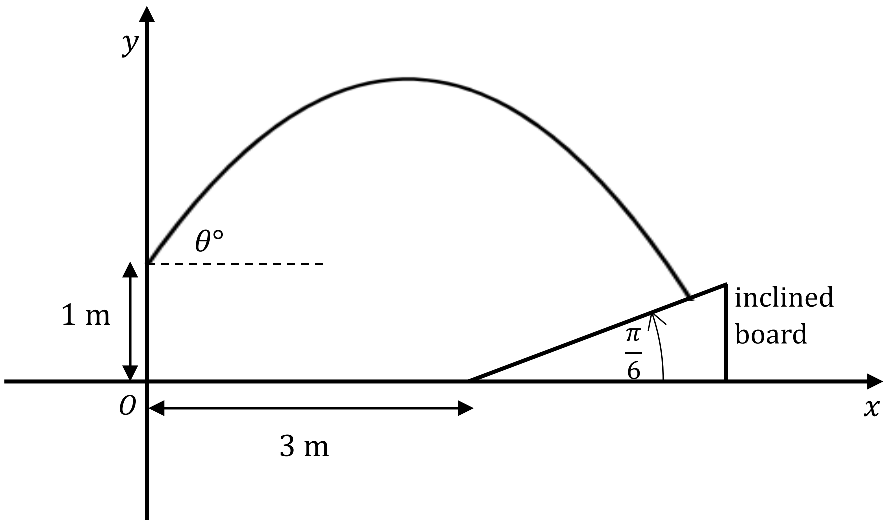
(c) In a game of cornhole, players take turns throwing a small, fabric bean bag at a hole in an inclined board.

Let the origin O of a Cartesian coordinate system be at the point where a player stands with $\underset{\sim}{i}$ representing the unit vector in the positive x direction and $\underset{\sim}{j}$ representing the unit vector in the positive y direction. Distances are measured in metres and time is measured in seconds.

In this instance, the board is inclined at an angle of 30° and begins 3 m from the player, as shown in the diagram below. A given bag leaves a player's hand 1 m above the origin at an angle $\theta°$ above the horizontal.

The position vector of the bean bag t seconds after leaving the player's hand is given by

$$\underset{\sim}{r}(t) = (3t)\underset{\sim}{i} + (1 + 5t - 4.9t^2)\underset{\sim}{j}, \qquad t \geq 0.$$



i. Find the speed, in metres per second, of the bean bag when it leaves the player's hand. **2**

ii. Find the angle $\theta°$. **1**

iii. Find the maximum height above O reached by the bag. Answer in metres, correct to 1 decimal place. **2**

iv. Find the length of time the bag spends in the air. **3**

v. Given the above equation results in the bag passing through the hole, find the distance of the hole from the bottom of the board, along its length. **1**

End of Examination

Section 1

Thursday, 25 July 2024 11:04 AM

HHHS
2024 Trial Exam
Mathematics Extension 1

Solutions and Marking Guidelines

1. $|x-1| \geq 4$

$\therefore x-1 \leq -4 \quad x-1 \geq 4$

$x \leq -3 \quad x \geq 5$

$\Rightarrow B$

2. $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{1 - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}}$

$= \dfrac{1+t^2-(1-t^2)}{2t}$

$= \dfrac{2t^2}{2t}$

$= t \quad \Rightarrow A$

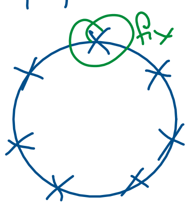
3. $y^2 = f(x) \Rightarrow y = \pm\sqrt{f(x)}$

if $f(x) > 1$, $\sqrt{f(x)} < f(x)$

if $0 < f(x) < 1$, $\sqrt{f(x)} > f(x)$

$\Rightarrow C$

4. 7 people from 12.



ways $= {}^{12}C_7 \times 6!$

$= \dfrac{12!\, 6!}{7!\, 5!}$

$= \dfrac{12!}{5!\, 7}$

$\Rightarrow C$

5. $P(x) = 3x^3 + \alpha x^2 + \beta x + \gamma$

roots $3, -2, -2$ ① or $3, 3, -2$ ②

① $\dfrac{\beta}{3} = 3(-2) + (-2)^2 + (-2).3$

$\beta = -24$ ✗

or ② $\dfrac{\beta}{3} = 3^2 + 3(-2) + (-2)3$

$\beta = -9$

$\Rightarrow B$

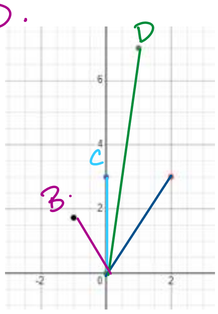
6. domain halved $\Rightarrow 2x$
range is $\frac{2}{3}$ of normal.
reflection $\Rightarrow -x$

$\Longrightarrow$ D

7. $\cos 2nx = 2\cos^2 nx - 1$

$\int \cos^2 2x\,dx = \frac{1}{2}\int (1+\cos 4x)\,dx$
$= \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right)$
$\Rightarrow$ C

8.



A: pair are perpendicular.

C: 2, 3, x — 33.7°

B > C

$\Rightarrow$ D

9. constant $g' = 0$ along $x = 0$
means x is a factor of numerator
so B or C.

test $(1,-1)$, m should be undefined
B: $\frac{1}{1-1}$ is undefined

C: $\frac{1}{-1-2} = -\frac{1}{3}$

$\Rightarrow$ B

10. $X \sim Bin(2.4, 1.68)$

$\therefore np = 2.4 \quad npq = np(1-p) = 1.68$

$1 - p = \frac{1.68}{2.4}$

$p = 0.3$ so A or B.

A: $np = 8 \times 0.3 = 2.4$ $\Rightarrow$ A.

Question 11

Thursday, 25 July 2024 11:04 AM

a)

$x = 3 - t^2 \quad -i$

$y = 2t + 1 \quad -ii$

from ii $t = \frac{1}{2}(y-1)$

subst into i

$x = 3 - \left[\frac{1}{2}(y-1)\right]^2$

$(y-1)^2 = 4(3-x)$

1 – makes y the subject
– reaches half function + or –

2 – substitutes correctly into other.

b)

$\frac{2}{x-1} \geq x$

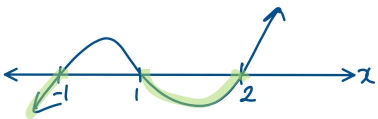
$2(x-1) \geq x(x-1)^2$

$x(x-1)^2 - 2(x-1) \leq 0$

$(x-1)\left[x(x-1) - 2\right] \leq 0$

$(x-1)(x^2 - x - 2) \leq 0$

$(x-1)(x-2)(x+1) \leq 0$



$\therefore x \leq -1, \quad 1 < x \leq 2$

1 – multiplies by square

2 – correctly factored

3 – correct solution

c) $\int \frac{dx}{5+4x^2} = \frac{1}{2}\int \frac{2dx}{5+(2x)^2}$

$= \frac{1}{2\sqrt{5}} \tan^{-1}\frac{2x}{\sqrt{5}} + C$

1 – chooses $\tan^{-1}$ or includes $\frac{1}{2}, 2$ in integral

2 – correct ans

d) i $\underset{\sim}{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \quad \underset{\sim}{b} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}, \quad \underset{\sim}{c} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$2\underset{\sim}{a} - \underset{\sim}{c} = 2\begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ -8 \end{pmatrix}$

1 – correct ans

ii $|\underset{\sim}{a}| = \sqrt{4^2 + (-1)^2}$

$= \sqrt{17}$

1 – correct ans

iii $\hat{\underset{\sim}{b}} = \dfrac{\underset{\sim}{b}}{|\underset{\sim}{b}|}$

$= \dfrac{1}{\sqrt{(-2)^2+(-3)^2}} \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

$= \dfrac{1}{\sqrt{13}} \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

1 – correct ans

iv $\text{proj}_{\underset{\sim}{a}} \underset{\sim}{c} = \dfrac{\underset{\sim}{c} \cdot \underset{\sim}{a}}{|\underset{\sim}{a}|^2} \underset{\sim}{a}$

$= \dfrac{2(4)+(6)(-1)}{(\sqrt{17})^2} \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$= \dfrac{2}{17} \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

1 – substitutes into formula or finds $\underset{\sim}{c} \cdot \underset{\sim}{a}$.

2 – correct ans

e) i for $f(x)$, domain is $[0, 2]$

$\therefore$ range of $f^{-1}(x)$ is $[0, 2]$

1 – correct ans

ii There are no points of intersection between $f(x)$ and $f^{-1}(x)$.
as all points on $f(x)$ lie below $y=x$

1 – correct ans + justification

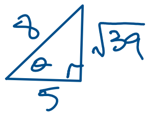
Question 12

Thursday, 25 July 2024 11:04 AM

a) $\cos\theta = -\frac{5}{8}$

$\left.\begin{array}{l}\cos\theta < 0 \\ \tan\theta < 0\end{array}\right\} \Rightarrow \theta$ in 2^{nd} quadrant

and $\sin\theta > 0$



$\therefore \sin\theta = \frac{\sqrt{39}}{8}$

1 – justifies correct quadrant or finds $\sin\theta$!

2 – both.

b) i $P(x) = x^4 - 11x^3 + 24x^2 + 4x - 32$

$P'(x) = 4x^3 - 33x^2 + 48x + 4$

$P(2) = 2^4 - 11(2)^3 + 24(2)^2 + 4(2) - 32$
$= 2(8) - 11(8) + 12(8) + 8 - 4(8)$
$= 15(8) - 15(8)$
$= 0$

$P'(2) = 4(2)^3 - 33(2)^2 + 48(2) + 4$
$= 4(8 - 33 + 24 + 1)$
$= 0$

as $P(2) = P'(2) = 0$, $x = 2$ is a root of multiplicity 2

1 – differentiates or shows $P(2) = 0$

2 – shows $P(2) = P'(2) = 0$

ii $(x-2)(x-2)$ is a factor of $P(x)$
$\Rightarrow x^2 - 4x + 4$ is also a factor

```
                x^2 - 7x - 8
x^2-4x+4 ) x^4 - 11x^3 + 24x^2 + 4x - 32
           x^4 -  4x^3 +  4x^2
           ------------------
                 -7x^3 + 20x^2
                 -7x^3 + 28x^2 - 28x
                 -------------------
                         -8x^2 + 32x
                         -8x^2 + 32x - 32
                         ----------------
                                        0
```

$\therefore P(x) = (x-2)^2(x^2 - 7x - 8)$
$= (x-2)^2(x-8)(x+1)$

1 – notes $x^2 - 4x + 4$ is a factor

2 – performs a correct division step
- successive division by $x-2$ for correct answer

3 – correct factors

c) with two outcomes per question, there are $2^6 = 64$ arrangements

by pigeonhole principle, minimum number of students for three identical responses is

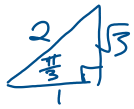
$64 \times 2 + 1 = 129$

1 – states 2^6 arrangements

2 – correct ans, stating PHP.

d) $\sin\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right) = \frac{1+\sqrt{3}}{4}$

$\frac{1}{2}\left[\sin 2x + \sin \frac{2\pi}{3}\right] = \frac{1+\sqrt{3}}{4}$ — 1 – uses product to sum



$\sin 2x + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

$\sin 2x = \frac{1}{2}$ — 2 – value for $\sin 2x$

$0 \leq x \leq 2\pi$

$0 \leq 2x \leq 4\pi$.

$\sin > 0 \Rightarrow$ angle in 1st + 2nd quads

$2x = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}, 3\pi - \frac{\pi}{6}$ — 3 – correct x in $[0, 2\pi]$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ — 4 – correct solutions

e). Prove $3^{2n+1} + 2^{n+2}$ divisible by 7, $n \geq 1$

Prove true for $n=1$ — 1 – proves base case

$3^{2(1)+1} + 2^{1+2} = 3^3 + 2^3$

$= 27 + 8$

$= 35$ which is divisible by 7

$\therefore$ true for $n=1$

Assume true for $n=k$

$\therefore 3^{2k+1} + 2^{k+2} = 7M$ (M is an integer) – induction hypothesis

Prove true for $n=k+1$

$3^{2(k+1)+1} + 2^{(k+1)+2} = 3^2 . 3^{2k+1} + 2.2^{k+2}$

$= 9(7M - 2^{k+2}) + 2.2^{k+2}$ – by hypothesis — 2 – use hypothesis in proof.

$= 7(9M) - 9.2^{k+2} + 2^{k+2}$

$= 7(9M - 2^{k+2})$ which is divisible by 7 — 3 – correct proof

$\therefore$ by principle of induction, expression divisible by 7 for $n \geq 1$.

Question 13

Thursday, 25 July 2024 11:04 AM

a) $I = \int_0^{\frac{\pi}{3}} \frac{\sin 2x}{\sin^2 x - 1}\,dx$

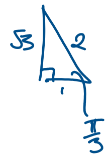
let $u = \sin^2 x$

$du = 2\sin x.\cos x\,dx$

$= \sin 2x\,dx$

for $x = 0$, $u = 0$

$x = \frac{\pi}{3}$, $u = \frac{3}{4}$



1 – differentiates $\sin^2 x$

$\therefore I = \int_0^{\frac{3}{4}} \frac{du}{u-1}$

$= \left[\ln|u-1|\right]_0^{\frac{3}{4}}$

$= \ln\left|\frac{3}{4}-1\right| - \ln|0-1|$

$= \ln\frac{1}{4}$

$= -2\ln 2$ $\quad [-1.38\ (2dp)]$

2 – correct subst

3 – correct ans

b) i $\quad V = \frac{4}{3}\pi r^3 \quad \frac{dr}{dt} = -1.6$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

$= 4\pi r^2 \times (-1.6)$

at $r = 10$

$\frac{dV}{dr} = 4\pi(10)^2 \times (-1.6)$

$= -2010.6193\ cm^3 h^{-1}$

1 – finds $\frac{dV}{dr}$

2 – correctly ans using chain rule

ii $\quad \frac{dr}{dt} = -1.6$

$\int dr = \int -1.6\,dt$

$r = -1.6t + c$

given $r_0 = 20cm$,

$20 = -1.6(0) + c$

$c = 20$

$r = 20 - 1.6t$

1 – find $r = f(t)$

Given $V = 800$

$\frac{4}{3}\pi r^3 = 800$

$r^3 = \frac{600}{\pi}$

$r = \sqrt[3]{\frac{600}{\pi}}$

$$t = \frac{20 - \sqrt[3]{\frac{600}{4}}}{1.6}$$

$$= 8.90074 \text{ h.}$$

∴ balloon deflated after 9 hours

2 - ans

c) $\frac{dT}{dt} = -k(T - A)$

$$\int \frac{dT}{T-A} = \int -k\,dt$$

$$\ln|T-A| = -kt + c$$

$$T - A = e^{-kt+c}$$

$$T = A + Be^{-kt} \quad (B = e^c)$$

1 - exp for $T = f(t)$ from separation of variables

as $t \to \infty$, $T \to 80$

$$80 = A + Be^{-\infty}$$

$$\therefore A = 80$$

2 - finds value of A

and $T = 80 + Be^{-kt}$

at $t = 6$, $T = 750$

$$750 = 80 + Be^{-k.6}$$

$$\frac{670}{B} = e^{-6k}$$

$$B = 670e^{6k} \quad -i$$

3 - substitute a condition correctly

at $t = 9$, $T = 500$

$$500 = 80 + Be^{-k.9}$$

$$B = 420e^{9k} \quad -ii$$

equating i and ii

$$670e^{6k} = 420e^{9k}$$

$$\therefore e^{3k} = \frac{67}{42}$$

$$k = \frac{1}{3}\ln\frac{67}{42}$$

$$= 0.1556743$$

$$B = 420e^{9(0.1556743)}$$

$$= 1705.0057$$

$$\therefore T = 80 + 1705\, e^{-0.05068314t}$$

at $t=0$,

$T = 1795°C$ — 4 - correct ans

d) i

$\frac{dy}{dx} = xy - 2x$

$= 2(3) - 2(2)$

$= 2$ — 1 - correct ans

ii

$\frac{dy}{dx} = x(y-2)$

$\int \frac{dy}{y-2} = \int x\,dx$ — 1 - seperates variables

$\ln|y-2| = \frac{x^2}{2} + c$ — 2 - correct integration

at $x=2$, $y=3$

$\ln|3-2| = \frac{2^2}{2} + c$

$c = -2$

$\ln|y-2| = \frac{x^2}{2} - 2$ — 3 - correct ans

Question 14

Thursday, 25 July 2024 11:04 AM

a) $E(\hat{p}) = p$
$= 0.043$

$Var(\hat{p}) = \frac{p(1-p)}{n}$
$= \frac{0.043(1-0.043)}{40}$
$= 0.00102878$

$\sigma(\hat{p}) = 0.0320745$

$\therefore \hat{p} \sim N(0.043, 0.0320745)$

5%: $z \doteq \frac{0.05 - 0.043}{0.0320745}$
$\doteq 0.2182418$

10%: $z \doteq \frac{0.1 - 0.043}{0.0320745}$
$\doteq 1.7771$

$P(5\% \leq \hat{p} \leq 10\%) \doteq P(0.22 \leq Z \leq 1.78)$
$= 0.9625 - 0.5871$
$= 0.3754$

1- finds $Var(\hat{p})$

2- finds one z-score

3- finds probability

b) $\left(2x^2 - \frac{3}{x}\right)^6 = \sum_{n=0}^{6} \binom{6}{n} (2x^2)^{6-n} \left(-\frac{3}{x}\right)^n$

term independent of x when
$(x^2)^{6-n} \cdot (x^{-1})^n = x^0$

equate powers
$2(6-n) - n = 0$

$12 - 3n = 0$
$n = 4$

$\therefore$ term is $\binom{6}{4} 2^{6-4} (-3)^4 = 4860$

1- writes in sigma notation

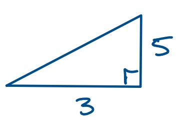
2- exp. to find n

3- answer

c)

i $\underset{\sim}{v}(t) = 3\underset{\sim}{i} + (5 - 9.8t)\underset{\sim}{j}$.

$\underset{\sim}{v}(0) = 3\underset{\sim}{i} + 5\underset{\sim}{j}$.



1- finds a vector for t=0

$v^2 = 5^2 + 3^2$

$v = \sqrt{34}\ ms^{-1}$

$= 5.830952$ — 2 - ans

ii $\theta = \tan^{-1}\frac{5}{3}$

$= 1.030377$ rad

$= 59°2''$ — 1 - ans

iii at max height $\underset{\sim}{v_j} = 0$ — 1 - lets $\underset{\sim}{v_j} = 0$

$5 - 9.8t = 0$

$t = \frac{5}{9.8}$

$= 0.5102041$

height $= 1 + 5\left(\frac{5}{9.8}\right) - 4.9\left(\frac{5}{9.8}\right)^2$

$= 2.2755102$ m — 2 - ans

iv Solve for t when $y = -\sqrt{3} + \frac{x}{\sqrt{3}}$ — 1 - equation of ramp

$1 + 5t - 4.9t^2 = -\sqrt{3} + \frac{3t}{\sqrt{3}}$ as $\underset{\sim}{s_i} = 3t = x$ — 2 - equating.

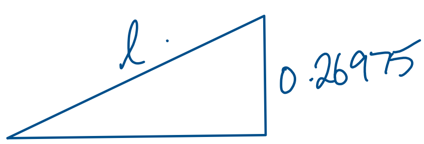
$= -\sqrt{3} + \sqrt{3}\,t$

$4.9t^2 + (\sqrt{3} - 5)t - (\sqrt{3} + 1) = 0$

$t = \frac{-(\sqrt{3}-5) + \sqrt{(\sqrt{3}-5)^2 + 4(4.9)(\sqrt{3}+1)}}{2(4.9)}$ taking pos. — 3 - ans

$= 1.15$ s.

v



$3(1.15) - 3$

$= 0.45$

$l = 0.52466$ m — 1 - ans