



NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS ADVANCED

2024 Year 12 Course Assessment Task 4 (Trial HSC Examination)

Monday, 19 August 2024

General instructions

- Working time – 3 hours.
(plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

SECTION I

- Mark your answers on the answer grid provided (on page 29)

SECTION II

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #:

Class (please ✓)

12MAA.1 – Miss Chaussivert

12MAX.1 – Miss J. Kim

12MAX.2 – Mrs Bhamra

12MAX.3 – Miss Lee

Marker's use only.

QUESTION	1-10	11-14	15-17	18-20	21-23	24-25	26-29	Total
MARKS	$\overline{10}$	$\overline{14}$	$\overline{13}$	$\overline{13}$	$\overline{16}$	$\overline{18}$	$\overline{16}$	$\overline{100}$

Section I

10 marks

Attempt Question 1 to 10

Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided (labelled as page 29).

Questions

Marks

1. What is the solution to the equation $\log_3(2x - 5) = 2$? 1

(A) $x = 1$ (B) $x = \frac{13}{2}$ (C) $x = \frac{11}{2}$ (D) $x = 7$

2. Evaluate $\lim_{x \rightarrow 7} \frac{x^2 + 5x - 84}{x - 7}$ 1

(A) -5 (B) 0 (C) 12 (D) 19

3. For what values of a does the equation $ax^2 + 5x + a = 0$ have no solution? 1

(A) $a > 0$ (C) $-\frac{5}{2} < a < \frac{5}{2}$

(B) $a = \frac{5}{2}$ (D) $a < -\frac{5}{2}$ or $a > \frac{5}{2}$

4. A function f is given by the rule 1

$$f(x) = \begin{cases} (x - 2)^2 + 1, & x \leq 2 \\ 4x - 7, & x > 2 \end{cases}$$

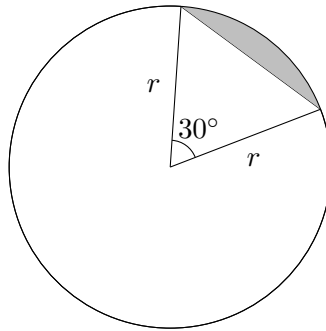
What is the value of $f(f(0))$?

(A) 5 (B) -7 (C) -35 (D) 13

5. The first three terms of an arithmetic series are 5, 9, and 13. 1
What is the 15th term of the series?

(A) 61 (B) 66 (C) 495 (D) 585

6. The diagram shows a circle with radius r centimetres and a sector which subtends an angle of 30° 1



If the area of the shaded segment is 1.7 cm^2 , which of the following is closest to the radius of the circle?

- (A) 0.12 cm (B) 0.34 cm (C) 12.00 cm (D) 144.08 cm
7. Which of the following is an expression for $\int \left(\frac{1}{1 + \sin x} + \frac{1}{1 - \sin x} \right) dx$? 1
- (A) $2 \sec x + c$ (C) $2 \tan x + c$
 (B) $4 \tan x \sec^2 x + c$ (D) $2x + 2 \sec x + c$
8. What is the domain and range of the function $y = \frac{1}{\sqrt{x-9}}$? 1
- (A) $D = \{x : x \geq 9\}$ and $R = \{y : y > 0\}$
 (B) $D = \{x : x > 9\}$ and $R = \{y : y > 0\}$
 (C) $D = \{x : -\infty \leq x \leq \infty\}$ and $R = \{y : -\infty \leq y \leq \infty\}$
 (D) $D = \{x : x \leq -3 \text{ or } x \geq 3\}$ and $R = \{y : y < 0\}$
9. Olivia and Sarah are participating in a fitness program. The probability that Olivia will complete the program is 0.8, and the probability that Sarah will complete the program is 0.7. 1
- What is the probability that only one of them will complete the program?
- (A) 0.14 (B) 0.24 (C) 0.38 (D) 0.56
10. Which of the following is the derivative of $e^{3^x x^6}$. 1
- (A) $3^x x^6 e^{3^x x^6}$ (C) $3^x x^5 (6 + x \log_e 3) e^{3^x x^6}$
 (B) $3^x x^6 e^{(3^x x^6 - 1)}$ (D) $3^x x^5 (3^x x^6 + 6x^5 (\log_e 3) 3^x) e^{3^x x^6}$

Examination continues overleaf...

Section II

90 marks

Attempt Question 11 to 29

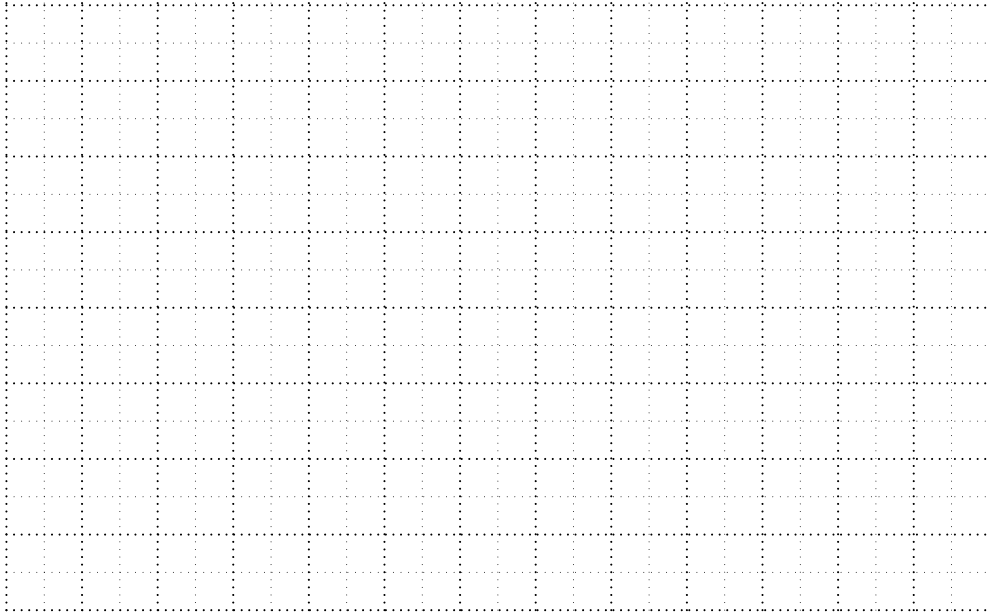
Allow approximately 2 hours and 45 minutes for this section

Write your answers in the space provided.

Question 11 (3 marks)

- (a) Sketch the graph of $y = |x - 1|$ for $x \in [-4, 4]$.

1



- (b) Using the sketch from part (i) or otherwise, solve

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$$|x - 1| = 2x + 4$$

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Question 12 (4 marks)

Given $f(x) = \sqrt{x^2 - 9}$ and $g(x) = x + 5$

- (a) Find positive integers c and d such that $f(g(x)) = \sqrt{(x+c)(x+d)}$ **2**

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- (b) State the domain for which $f(g(x))$ is defined. **2**

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Question 13 (4 marks)

- (a) Show that $\frac{d}{dx}(x \operatorname{cosec} x) = \operatorname{cosec} x - x \cot x \operatorname{cosec} x$ **3**

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- (b) Hence find: **1**

$$\int 2(\operatorname{cosec} x (1 - x \cot x)) dx$$

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Question 14 (3 marks)

Find the equation of the normal to the curve $y = \left(x + \frac{2}{x}\right)^2$ at the point where $x = 2$. **3**
Write the equation of the normal using general form.

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Question 15 (3 marks)

A and B are two events such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A|B) = 0.5$
(a) Show that $P(A \cup B) = 0.4$. **2**

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(b) Find the value of $P(\overline{A} \cap \overline{B})$ **1**

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Question 16 (4 marks)

The discrete random variable X has the following distribution:

x	0	1	2	3	4
$P(X = x)$	0.1	$5a^2$	0.2	0.2	$2.5 + 11a$

- (a) Show that $a = -\frac{1}{5}$. **3**

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- (b) Find the mean of the distribution. **1**

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Question 17 (6 marks)

There are 4 red and 3 black discs in a bag. Ms Kim and Ms Lee are playing a game in which they take turns drawing a disc from the bag and then replacing it.

To win the game, Ms Kim must draw a red disc and for Ms Lee to win, she must draw a black disc. They continue taking turns until there is a winner. Ms Kim draws a disc first.

(a) Find the probability that Ms Kim wins on her first draw. **1**

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(b) Find the probability that Ms Kim wins in three or less of her turns. **2**

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(c) Find the probability that Ms Kim wins the game. **3**

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Question 18 (2 marks)

(a) Prove the following identity:

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$$\frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta} = 2 \sec \theta.$$

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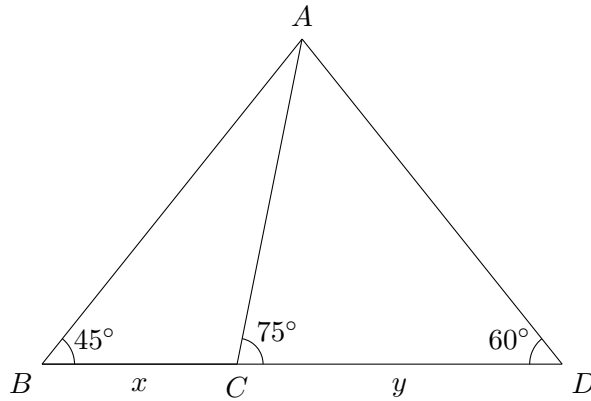
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Question 19 (7 marks)

Refer to the following diagram:



(a) Show that $\frac{x}{y} = \frac{\sqrt{3}}{2}$.

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(b) Find the smallest positive value of α (where α and θ are in degrees) such that

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$$\sin(\alpha - \theta) = \sin \theta$$

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- (c) Hence, show that the ratio of the area of $\triangle ABC$ to the area of $\triangle ADC$ is $\sqrt{3} : 2$. **3**

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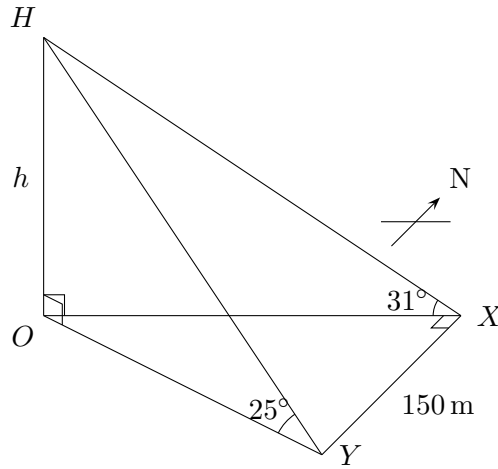
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Question 20 (4 marks)

From a point X due east of a tower, the angle of elevation of the top of the tower H is 31° . From another point Y due south of X , the angle of elevation is 25° . $XY = 150$ m.



- (a) Show that $h = \frac{150 \tan 25^\circ \tan 31^\circ}{\sqrt{\tan^2 31^\circ - \tan^2 25^\circ}}$. **3**

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- (b) Hence, find the height of the tower. **1**

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Question 21 (4 marks)

A particle is moving such that its displacement in metres after t seconds is given by the equation

$$x = -1.5 + 3 \sin 2t$$

- (a) State the maximum displacement of the particle. **1**

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- (b) Find the first four times when the particle is at the origin. **3**

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Question 22 (9 marks)

A particle travels in a straight line. Its velocity \dot{x} at time t is given by

$$\dot{x} = (3t^2 - 12t + 9) \text{ metres per second}$$

- (a) Find the expression for the particle’s displacement x in terms of t , if the particle was initially at $x = 2$. **2**

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- (b) Find an expression for the particle’s acceleration in terms of t . **1**

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- (c) At what times is the particle at rest? **1**

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- (d) Draw a velocity-time graph that represents the motion of this particle, showing all important features. **2**

(e) Find the total distance travelled between $t = 0$ to $t = 3$.

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Question 23 (3 marks)

Given $\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$ and using only translations, reflections and dilations, carefully state all of the transformations required to transform the function $y = \sin x$ to obtain the function $y = \sin^2 x$.

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Question 24 (13 marks)

Consider the function:

$$f(x) = \log_e (x^2 + 1)$$

- (a) Find the stationary point(s) of $f(x)$ and determine their nature. **3**

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- (b) Find the coordinates of the point(s) of inflection of $f(x)$. **3**

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- (c) Hence, sketch the curve of $f(x)$, showing all key features.

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- (d) Use the Trapezoidal Rule with four subintervals to find an approximation to:

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$$\int_1^3 \log_e(x^2 + 1) dx$$

Give your answer correct to 3 decimal places.

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- (e) State whether the approximation found in part (d) is greater or less than the exact value of $\int_1^3 \log_e(x^2 + 1) dx$. Use your sketch in part (c) to provide a brief justification for your answer.

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Examination continues overleaf...

Question 25 (5 marks)

An open-topped fish tank of volume 90 m^3 is to be made in the shape of a rectangular prism of length $2x$ metres, width x metres, and height h metres. Materials cost \$15 per square metre for the base of the tank and \$20 per square metre for the sides of the tank.

Show that the total cost $\$C$ of making the fish tank is given by

$$C = 30x^2 + \frac{5\,400}{x}$$

and hence find the dimensions of the fish tank with the least total cost.

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Question 26 (4 marks)

Mrs Bhamra and Ms Chaussivert's classes each sat twenty class tests. Ms Bhamra's class results on the tests are displayed in the box-and-whisker plot shown in part (a).

(a) Ms Chaussivert's 5-number summary for her class' test results is **1**

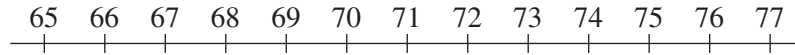
67, 69, 71, 73 75

Draw a box-and-whisker plot to display Ms Chaussivert's class results below that of Mrs Bhamra's class results.

Ms Bhamra



Ms Chaussivert



(b) Mrs Bhamra's class claims that they scored better than Ms Chaussivert's class. Are they correct? Justify your answer by referring to the summary statistics and the skewness of the distributions. **3**

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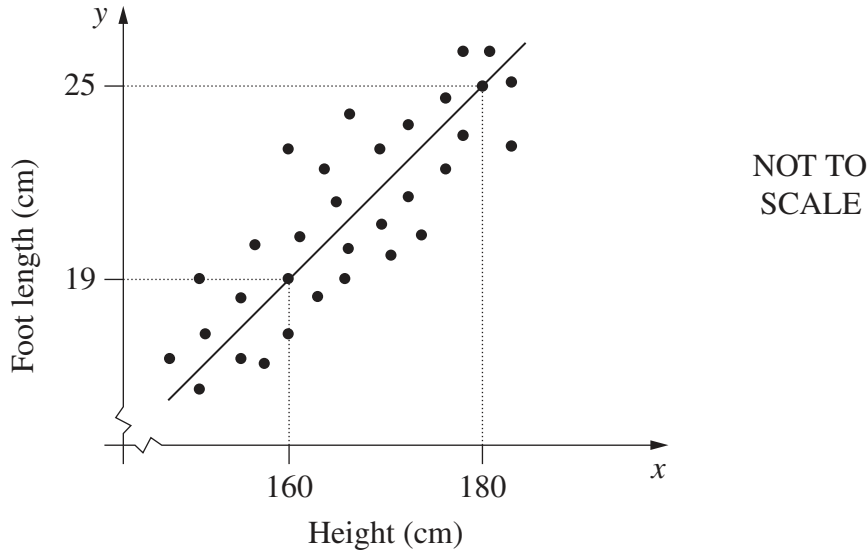
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Examination continues overleaf...

Question 27 (4 marks)

Each member of a group of males had his height and foot length measured and recorded. The results were graphed and a line of fit drawn.



- (a) Why does the value of the y intercept have no meaning in this situation? **1**

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- (b) George is 10 cm taller than his brother Harry. Use the line of fit to estimate the difference in their foot lengths. **1**

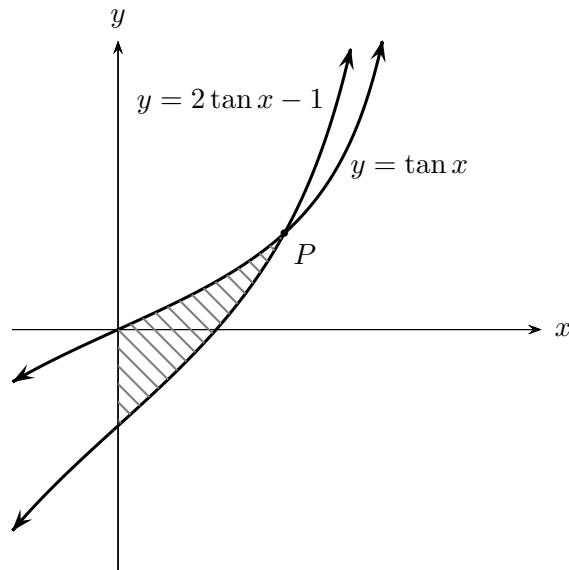
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- (c) Sam calculated a correlation coefficient of -1.2 for the data. Give TWO reasons why Sam must be incorrect. **2**

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Question 28 (4 marks)

The diagram below shows a sketch of parts of the graphs of $y = \tan x$ and $y = 2 \tan x - 1$. The graphs intersect at the point P . 4



Show that the area shaded is equal to $\left(\frac{\pi}{4} - \frac{1}{2} \log_e 2\right)$ square units.

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Examination continues overleaf...

Question 29 (4 marks)

The level of a local dam is being monitored. After t days, the rate at which the volume of water in the dam is changing is given by 4

$$R = \frac{1\,000}{1+t} - 500$$

where R is the rate of change of the volume of water in the dam, measured in megalitres per day.

The volume of water in the dam at the end of the fifth day of monitoring was half the volume at the end of the fourth day.

Find an expression for the volume of water V in the dam, t days after the monitoring began.

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