North Sydney Girls High School

Teacher: ____________________

Student Number

2024 **HSC TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading Time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 44)

- Attempt Questions 11 – 32
- Allow about 2 hours and 45 minutes for this section

Question	1–10	11–19	20-27	28-32	Total
Mark	/10	/30	/33	/27	/100

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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1 A box contains m yellow balls and 20 black balls. If a ball is randomly drawn from the box, then the probability of drawing a yellow ball is $\frac{1}{5}$. What is the value of m?

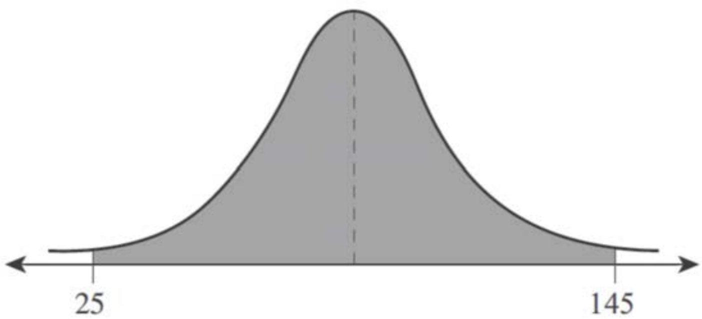
A. 4

B. 5

C. 15

D. 25

2 The graph shows a normal distribution. Approximately 99.7% of the area under the curve is bounded by $x = 25$ and $x = 145$.



Which of the following gives the correct mean and variance of the distribution?

A. mean = 60 and variance = 20

B. mean = 60 and variance = 400

C. mean = 85 and variance = 20

D. mean = 85 and variance = 400

3 What is the domain of the function $f(x)=\sqrt{25-x^2}+\sqrt{x}$?

A. $[0,5]$

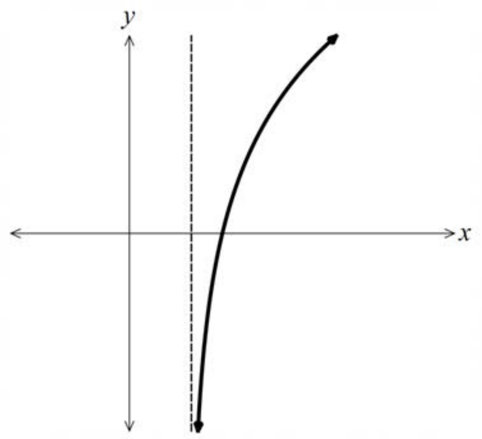
B. $(0,5)$

C. $(-\infty,0)\cup(5,\infty)$

D. $(-\infty,\infty)$

4 Which of the following is a possible sketch of $y=a\log_e(x-b)$ where $a<0$ and $b<0$?

A



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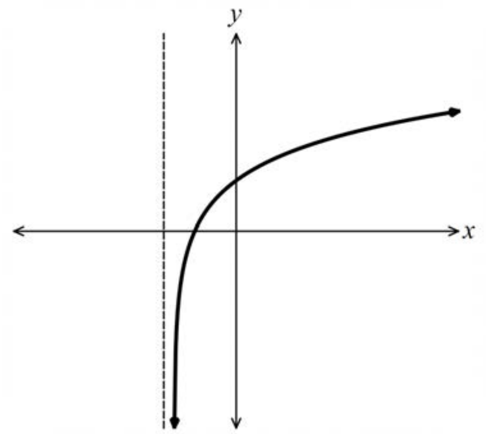
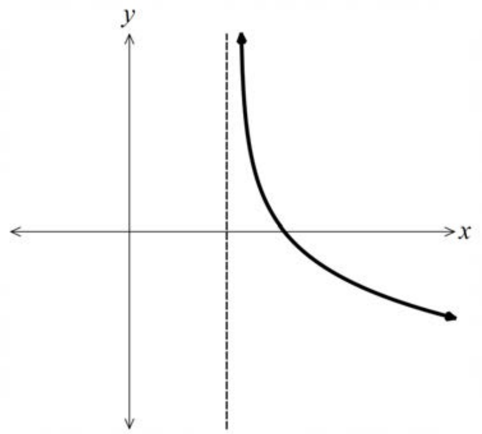


C



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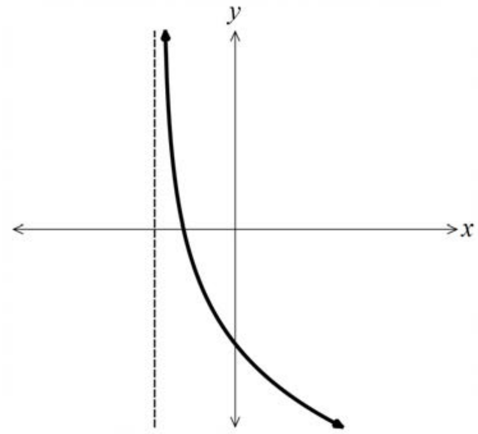


5 The cost $\$c$ of making a box of cookies is inversely proportional to the square of the number of boxes n made.

Which of the following is a possible equation of the model?

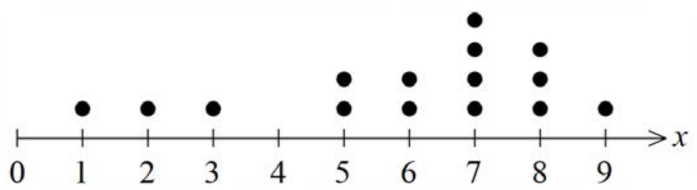
A. $cn^2 = 10$

B. $cn^2 = -10$

C. $c^2 n = 10$

D. $c^2 n = -10$

6 Consider the dot plot shown.



Which row of the table correctly describes the data?

	Statistical measure	*Shape*
A.	Mode = 7	Positively Skewed
B.	Median = 7	Negatively Skewed
C.	Mean = 7	Positively Skewed
D.	Range = 4	Negatively Skewed

7 Which of the following is the derivative of $\log_3 3x$?

A. $\dfrac{1}{x}$

B. $\dfrac{1}{x\ln 3}$

C. $\dfrac{1}{3x}$

D. $\dfrac{1}{3x\ln 3}$

8 Consider the function $h(x) = 2\sin 2x + 1$.

Which of the following is a correct statement about $h(x)$?

A. $h(x)$ is increasing where $0 \le x \le \dfrac{\pi}{2}$.

B. Maximum value is 3 and the minimum value is -3.

C. $h(x) = -h(-x)$ for all real x.

D. $h(x)$ has the same period as $g(x) = \cot x$.

9 Let $f'(x) = g'(x) - 1$, where $f(0) = 2$ and $g(0) = 1$.

Which of the following is an expression for $f(x)$?

A. $f(x) = g(x) - x$

B. $f(x) = g'(x) - 1$

C. $f(x) = g(x) - x + 1$

D. $f(x) = g(x)$

10 Given $f(x)$ is a quadratic function with a vertex at $(3,-4)$, which of the following must be true?

A. $f(x)=0$ has one or more integer roots.

B. $f(x)-3=0$ has one or more rational roots.

C. $f(x)+4=0$ has one or more real roots.

D. $f(x)+5=0$ has no real roots.

End of Section I

Student Number

Mathematics Advanced
Section II Answer Booklet 1

1

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–19 (30 marks)
Booklet 2 – Attempt Questions 20–27 (33 marks)
Booklet 3 – Attempt Questions 28–32 (27 marks)

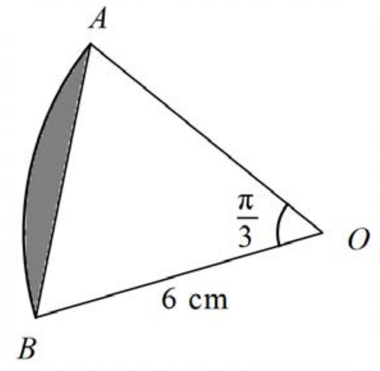
Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

The diagram shows a sector of circle with centre O and radius 6 cm, where $\angle AOB = \frac{\pi}{3}$. **2**



Find the exact value of the area of the shaded segment.

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Question 12 (2 marks)

Differentiate $\frac{x^3}{\ln x}$ with respect to x. **2**

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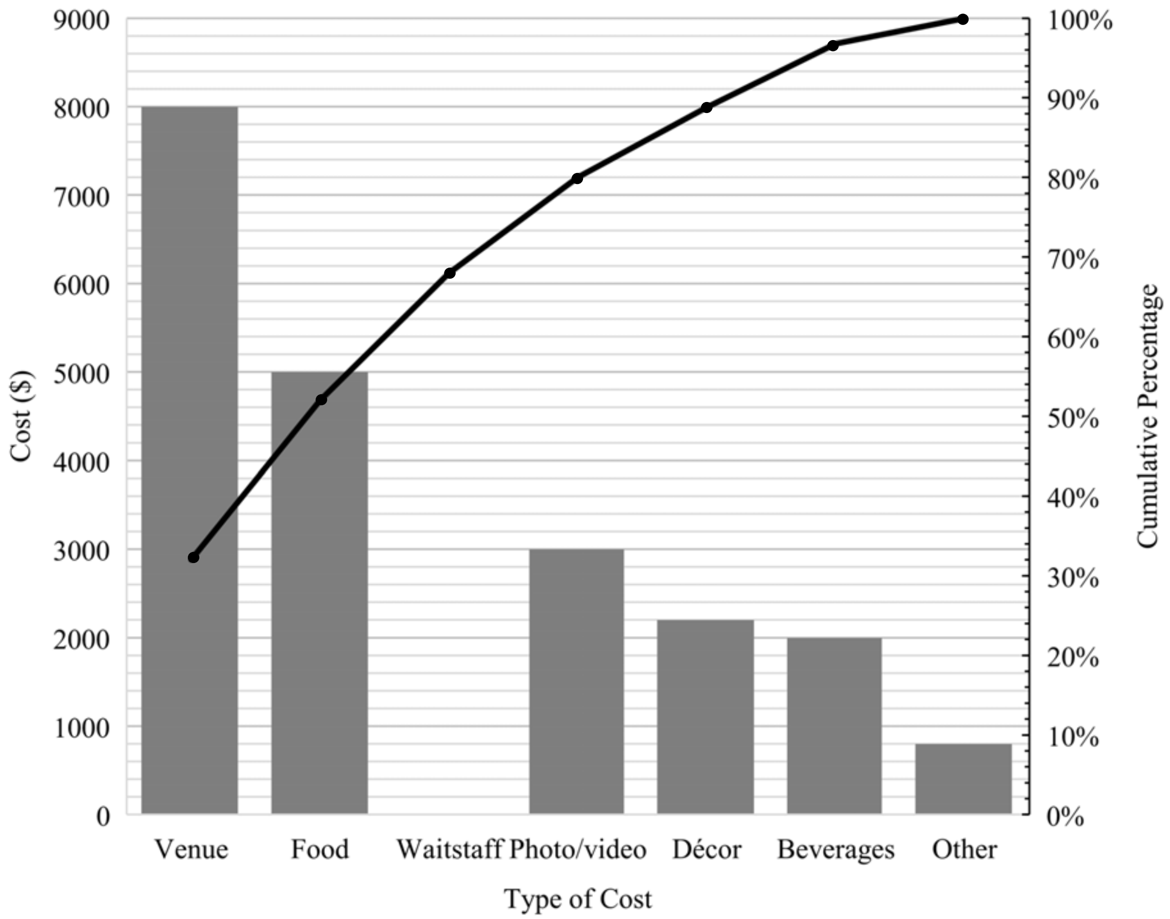
Question 13 (3 marks)

The following Pareto chart shows the distribution of costs for an end-of-year event.



(a) Use the graph to estimate the percentage of the total cost spent on 'Waitstaff'? **1**

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(b) Find the total cost for this event. **1**

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(c) Find the cost spent on 'Waitstaff'. **1**

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Question 14 (3 marks)

The sum of the first n terms of an arithmetic series is given by $S_n = \frac{1}{2}n(3n+2)$.

(a) Find the first three terms of the series. **2**

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(b) Find an expression for the nth term of the series. **1**

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Question 15 (3 marks)

Find the equation of the normal to the curve $y = e^{\cos(\pi x)}$ at the point where $x = \frac{1}{2}$. **3**

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Question 16 (3 marks)

Show that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$. **3**

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Question 17 (6 marks)

In a high school, there are eight students who study both Maths and Music.

Marks for these students from the most recent assessments are shown below.

Student	A	B	C	D	E	F	G	H	Mean	σ
Music	61	52	70	65	48	50	56	55		
Maths	80	70	85	90	75	75	78	73	78.25	6.6

(a) Find the mean and standard deviation of the Music assessment. Give your answer correct to nearest 3 decimal places. **2**

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(b) Student A claims that she is performing better in Music than in Maths. Is this claim correct? Justify your answer with appropriate calculations. **2**

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Question 17 continues on page 17

Question 17 (continued)

(c) Find the correlation coefficient Maths and Music exam scores. **1**

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(d) One student concludes that ‘people who are good at Maths are good at Music’. **1**
Does the data support this statement? Justify your answer.

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End of Question 17

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Question 18 (3 marks)

Solve $\tan\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ for $-\pi \le x \le \pi$ **3**

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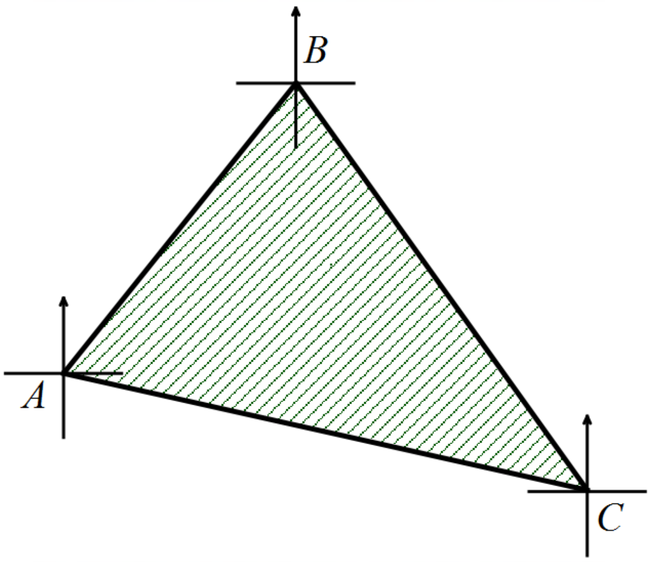
Question 19 (5 marks)

Bill runs around a triangular shaped park, starting and ending at gate *A*. He runs 12 km from *A* to *B* on a bearing of $N29^{\circ}E$, then runs 20 km from *B* to *C* on a bearing of $S31^{\circ}E$. Lastly, he runs back to *A*.



(a) Find the exact area of the park. **2**

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(b) Find the total distance Bill ran. **3**

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Proceed to Booklet 2 for Questions 20-27

Question 20 (4 marks)

A random variable is normally distributed with mean 0 and standard deviation 1.
The table gives the probability that this random variable is less than z for different values of z.

z	0	0.5	1	1.5	2	2.5
$P(Z < z)$	0.500	0.692	0.841	0.933	0.977	0.994

In a particular city, the weekly rents for apartments are normally distributed with a mean of \$750 and a standard deviation of \$100.

(a) Find the probability that the weekly rent of an apartment chosen at random is less than \$600. **2**

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(b) Sam's budget for the weekly rent is \$600. She has refined her searches and produced a list such that she is only looking at properties which are less than the mean weekly rent. Find the probability that an apartment that Sam chooses at random from this list is within her budget. **2**

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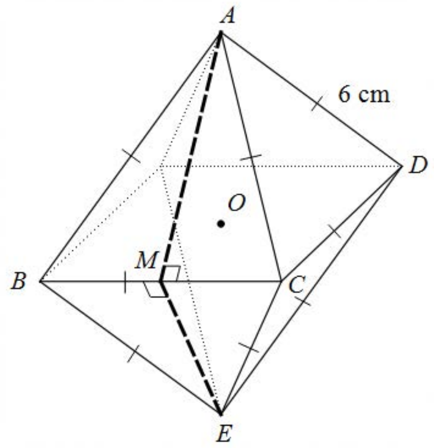
Question 21 (3 marks)

Two identical right pyramids with square bases and equilateral triangular faces are joined to form a regular octahedron. **3**

The following diagram shows a regular octahedron with edge length 6 cm and centre at *O*. *AM* and *ME* are perpendicular heights of ΔABC and ΔBCE respectively and point *M* is the midpoint of *BC*.



Find the size of $\angle AME$, to the nearest degree.

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Question 22 (6 marks)

A continuous random variable X has the probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{kx}{x^2+1}, & \text{for } 1 \le x \le 5 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Show that $k = \dfrac{2}{\ln 13}$. **3**

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Question 22 continues on page 25

(b) Find the median m. **3**

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End of Question 22

Please turn over

Question 23 (4 marks)

The motion of a particle is given by $x = 10t + 4e^{-3.5t} - 4$, where x is the displacement from the starting position in metres and t is time after the initial position in seconds.

(a) Find the initial velocity. **2**

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(b) Sketch the velocity-time graph. **2**

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Question 24 (2 marks)

Let $f(x)$ be an even function. If $\int_0^6 f(x)\,dx = 12$ and $\int_4^6 f(x)\,dx = -5$ evaluate $\int_{-4}^4 2f(x)\,dx$. **2**

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Question 25 (6 marks)

At a market, a stallholder is hosting free-to-play mini games.
If the player wins against the stallholder, they will receive \$25 as a reward. If they lose, they have to pay \$5 to the stallholder. If the game ends in a draw, no money is exchanged.

The random variable X represents the amount of money a player wins per mini game, where the probability of winning a game is independent of each other.

The distribution of X is given in the table below.

x	-5	0	25
$P(X=x)$	$0.29-2k^2$	0.87	$k-0.28$

(a) Show that $2k^2-k+0.12=0$. **1**

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(b) Find the value of k, giving reasons for your answer. **2**

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Question 25 continues on page 29

(c) Find $E(X)$. **1**

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A player decides to play 3 mini games.

(d) How much is the player expected to win or lose? **1**

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(e) What is the probability that the player will lose exactly two of the three games? **1**

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End of Question 25

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Question 26 (4 marks)

A spring is extended to its maximum length and released.

The length of the spring l cm, t seconds after release is modelled by the function $l = a + b\cos 2t$.

(a) Given that the minimum length of the spring is 3 cm and the maximum length is 11 cm, find the values of a and b. **2**

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(b) Sketch a graph of the model in the domain $0 \le t \le 2\pi$. **2**

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Question 27 (4 marks)

Consider the function $f\left(\frac{x}{3}-1\right)=\frac{1}{x-2}-6$.

(a) List a set of transformations that, when applied in order, would transform $f(x)$ to $f\left(\frac{x}{3}-1\right)$. **2**

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(b) Find the equation of $y=f(x)$. **2**

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Proceed to Booklet 3 for Questions 28-32

Student Number

Mathematics Advanced
Section II Answer Booklet 3

3

Booklet 3 – Attempt Questions 28–32 (27 marks)

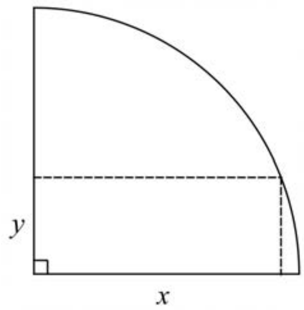
Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 28 (7 marks)

A new logo for MathsCompany is to be made by inscribing a rectangle inside a quadrant of radius 3 cm as shown below.



The length and width of the rectangle are x and y cm respectively and $x^2 + y^2 = 9$.

(a) Show that the area of the rectangle is given by **1**

$$A = x\sqrt{9 - x^2}$$

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(b) Show that **2**

$$\frac{dA}{dx} = \sqrt{9 - x^2} - \frac{x^2}{\sqrt{9 - x^2}}$$

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Question 28 continues on page 35

(c) Hence find the maximum area of the rectangle. **4**

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End of Question 28

Please turn over

Question 29 (7 marks)

The first three terms of a geometric series are $6\cos\theta$, $2(\sin\theta - 2)$ and $3\tan\theta$.

(a) Show that $2\sin^2\theta - 17\sin\theta + 8 = 0$. **2**

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(b) Find the exact value of θ in radians, given that it is obtuse. **2**

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Question 29 continues on page 37

(c) Find the limiting sum of the series, leaving your answer in the form $k\left(\sqrt{3}+1\right)$. **3**

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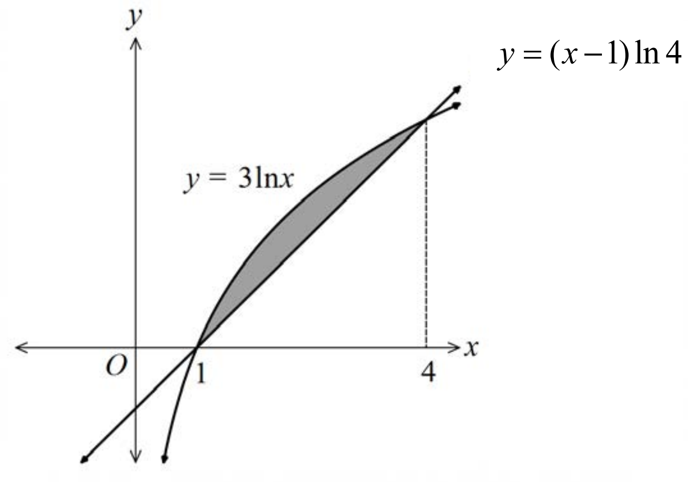
End of Question 29

Please turn over

Question 30 (4 marks)

The shaded region is enclosed by the curve $y = 3\ln x$ and line $y = (x-1)\ln 4$, as shown in the diagram.



(a) Use the Trapezoidal rule with 3 sub-intervals to find an approximate area of the shaded region. Give your answer correct to three significant figures. **3**

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Question 30 continues on page 39

(b) Explain whether the approximation is an overestimate or underestimate of the true area of the shaded region. **1**

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End of Question 30

Please turn over

Question 31 (4 marks)

Let A and B be two events. Suppose that $P(A) = 0.8$, $P(B \mid A) = 0.45$ and $P(B \mid \overline{A}) = 0.6$.

(a) Find $P(B)$. **3**

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(b) Is event A independent from event B? Justify your answer with relevant calculations. **1**

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Question 32 (5 marks)

(a) Find $\frac{d}{dx}\left(xe^{mx}\right)$. **1**

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(b) Hence find $\int xe^{mx}dx$. **2**

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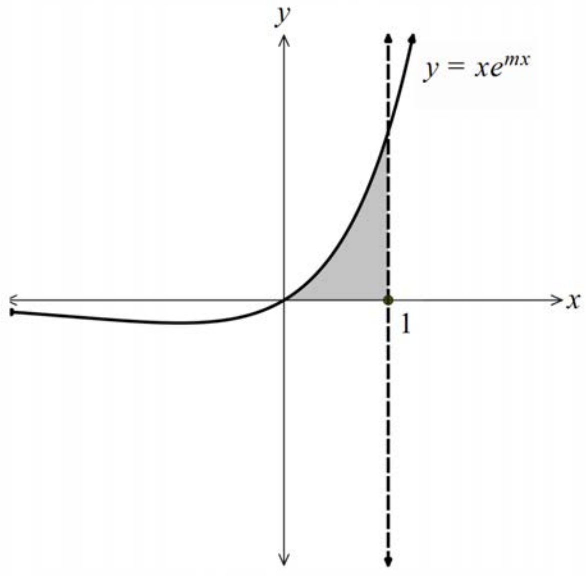
Question 32 continues on page 43

(c) The area of the region bounded by the curve $y = xe^{mx}$, the x-axis and $x = 1$ is shown below. Given that the area of this area is $\frac{1}{m}$, find m. **2**



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Teacher: ________________________

North Sydney Girls High School

Student Number

2024 **HSC TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading Time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 44)

- Attempt Questions 11 – 32
- Allow about 2 hours and 45 minutes for this section

Question	1–10	11–19	20-27	28-32	Total
Mark	/10	/30	/33	/27	/100

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

---

1 A box contains m yellow balls and 20 black balls. If a ball is randomly drawn from the box, then the probability of drawing a yellow ball is $\frac{1}{5}$. What is the value of m?

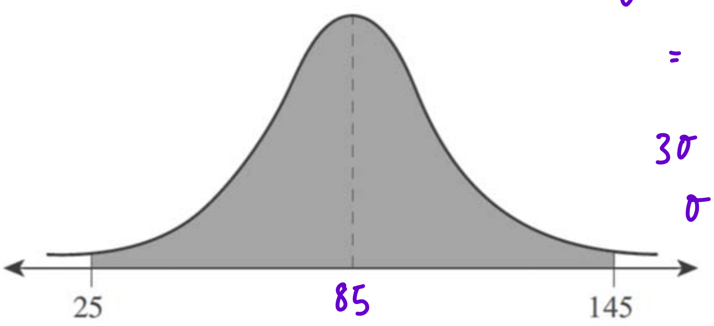
A. 4

B. 5

C. 15

D. 25

2 The graph shows a normal distribution. Approximately 99.7% of the area under the curve is bounded by $x = 25$ and $x = 145$.



Which of the following gives the correct mean and variance of the distribution?

A. mean = 60 and variance = 20

B. mean = 60 and variance = 400

C. mean = 85 and variance = 20

D. mean = 85 and variance = 400

3 What is the domain of the function $f(x)=\sqrt{25-x^2}+\sqrt{x}$?

$[-5,5]\cap[0,\infty) = [0,5]$

A. $[0,5]$

B. $(0,5)$

C. $(-\infty,0)\cup(5,\infty)$

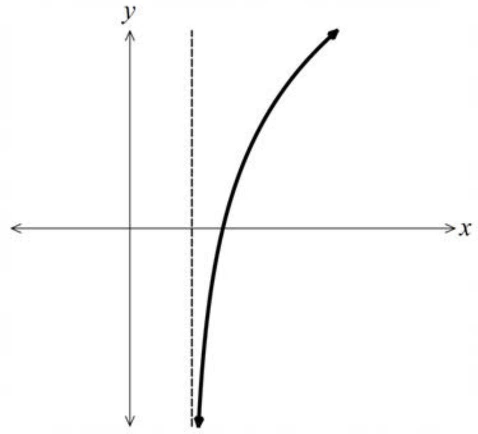
D. $(-\infty,\infty)$

4 Which of the following is a possible sketch of $y=a\log_e(x-b)$ where $a<0$ and $b<0$?

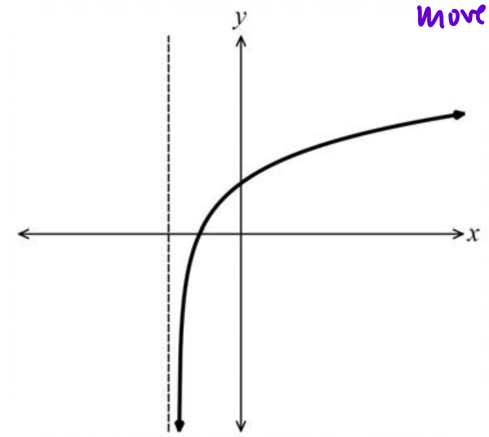
reflect about x-axis

$a\log_e(x+\ )$ move left

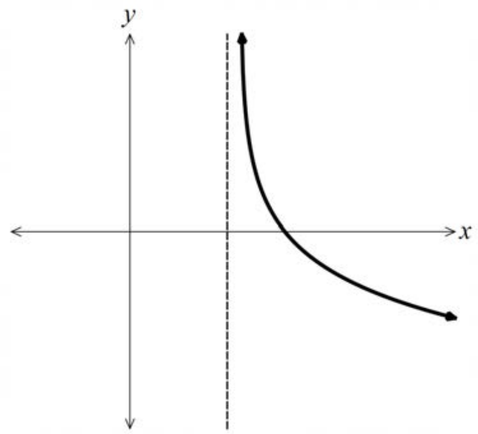
A.



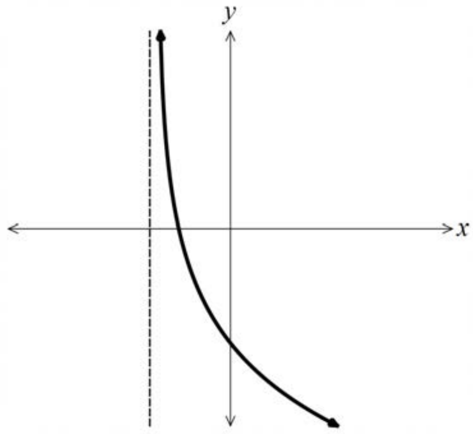
B.



C.



D.



$c \propto \frac{1}{n^2}$

5 The cost $\$c$ of making a box of cookies is inversely proportional to the square of the number of boxes n made.

Which of the following is a possible equation of the model?

A. $cn^2 = 10$

B. $cn^2 = -10$

C. $c^2 n = 10$

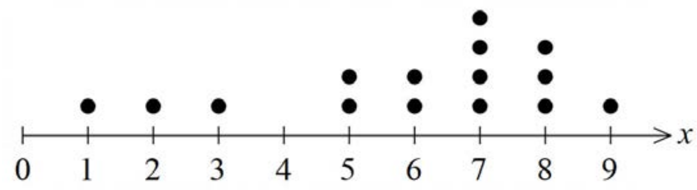
D. $c^2 n = -10$

$c = \frac{k}{n^2}$

$cn^2 = k$

since c and n^2 are positive

6 Consider the dot plot shown.



mean = 5.9$\dot{3}$
mode = 7
median = 7
range = 8
negatively skewed

Which row of the table correctly describes the data?

	Statistical measure	*Shape*
A.	Mode = 7	Positively Skewed
B.	Median = 7	Negatively Skewed
C.	Mean = 7	Positively Skewed
D.	Range = 4	Negatively Skewed

7 Which of the following is the derivative of $\log_3 3x$?

A. $\dfrac{1}{x}$

B. $\dfrac{1}{x\ln 3}$

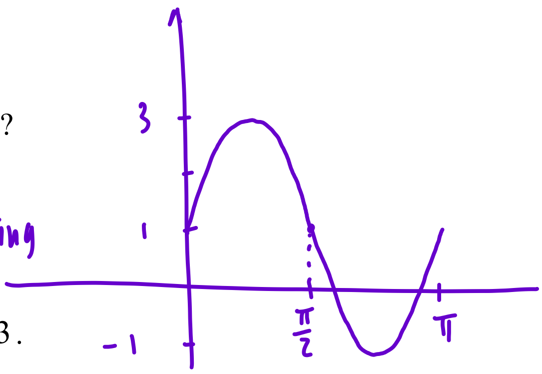
C. $\dfrac{1}{3x}$

D. $\dfrac{1}{3x\ln 3}$

$\frac{d}{dx}(\log_3 3x) = \frac{3}{\ln 3 \times 3x} = \frac{1}{x\ln 3}$

8 Consider the function $h(x) = 2\sin 2x + 1$.

Which of the following is a correct statement about $h(x)$?



A. $h(x)$ is increasing where $0 \le x \le \dfrac{\pi}{2}$. not only increasing

B. Maximum value is 3 and the minimum value is -3. ↳ −1

C. $h(x) = -h(-x)$ for all real x. not an odd function

D. $h(x)$ has the same period as $g(x) = \cot x$. period = π

9 Let $f'(x) = g'(x) - 1$, where $f(0) = 2$ and $g(0) = 1$.

Which of the following is an expression for $f(x)$?

A. $f(x) = g(x) - x$

B. $f(x) = g'(x) - 1$

C. $f(x) = g(x) - x + 1$

D. $f(x) = g(x)$

$f(x) = \int g'(x) - 1 \, dx = g(x) - x + c$

when $x = 0$

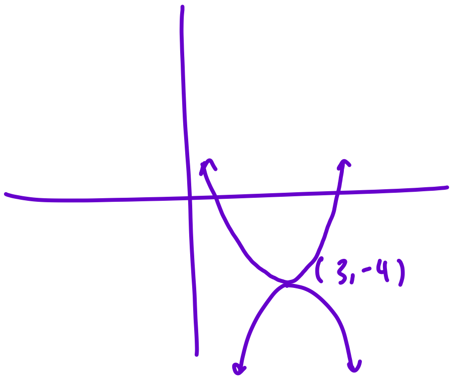
$f(0) = g(0) - 0 + c$

$2 = 1 - 0 + c$

$c = 1$

$\therefore f(x) = g(x) - x + 1$

10 Given $f(x)$ is a quadratic function with a vertex at $(3,-4)$, which of the following must be true?



A. $f(x)=0$ has one or more integer roots.

B. $f(x)-3=0$ has one or more rational roots.

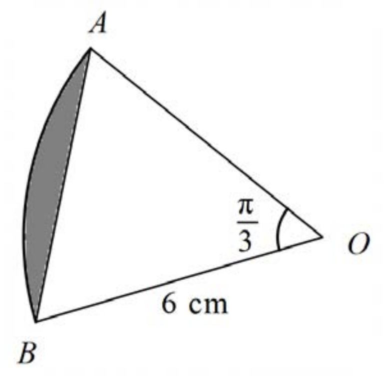
C. $f(x)+4=0$ has one or more real roots.

D. $f(x)+5=0$ has no real roots.

End of Section I

Question 11 (2 marks)

The diagram shows a sector of circle with centre O and radius 6 cm, where $\angle AOB = \dfrac{\pi}{3}$. **2**



Find the exact value of the area of the shaded segment.

$A = A_{sector} - A_{\triangle AOB}$

$= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} - \frac{1}{2} \times 6 \times 6 \sin\frac{\pi}{3}$ ✓ one formula correctly simplified

$= 6\pi - 18 \times \frac{\sqrt{3}}{2}$

$= 6\pi - 9\sqrt{3}$ cm^2 ✓ correct answer, exact value

Question 12 (2 marks)

Differentiate $\dfrac{x^3}{\ln x}$ with respect to x. **2**

$u = x^3 \quad v = \ln x$

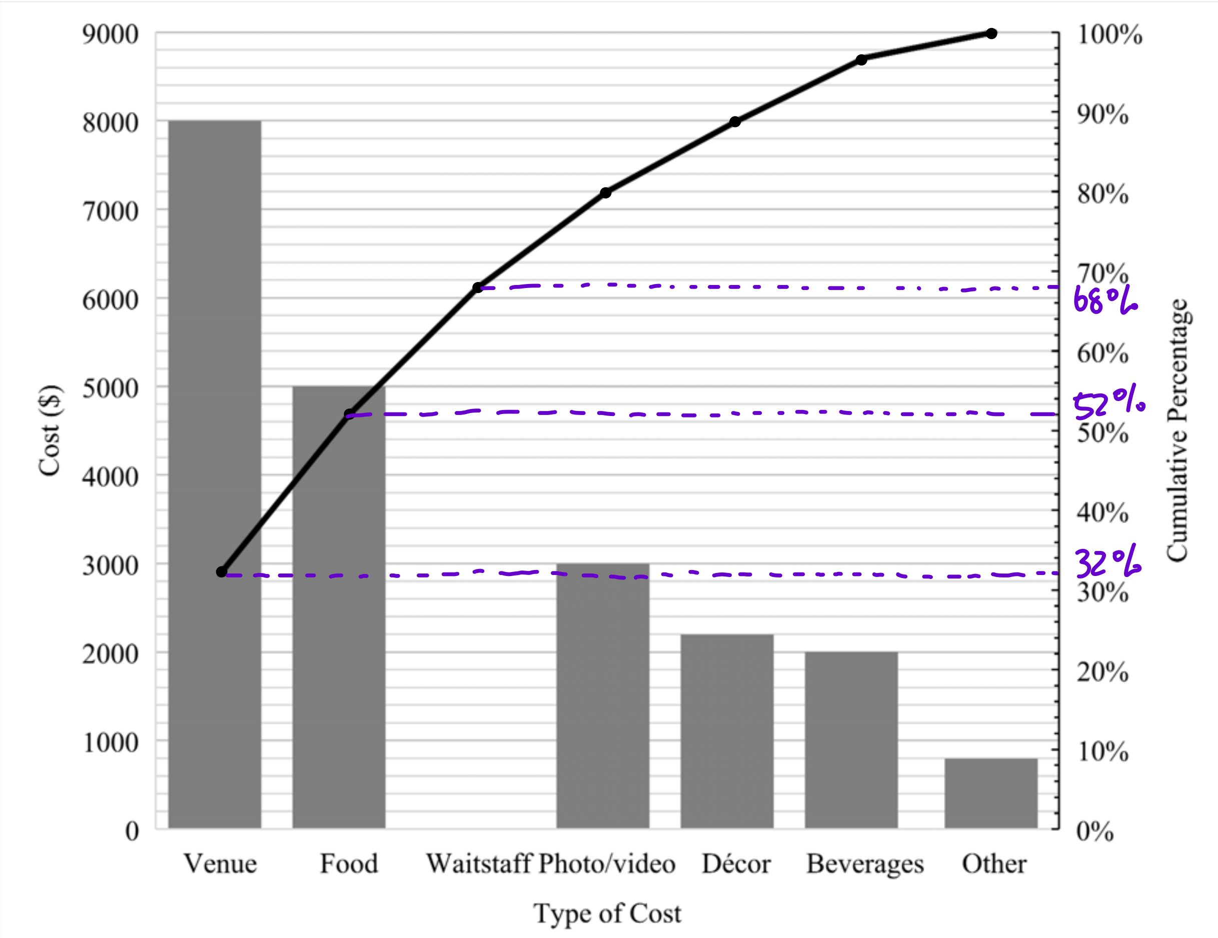
$u' = 3x^2 \quad v' = \frac{1}{x}$ ✓

$\frac{d}{dx}\left(\frac{x^3}{\ln x}\right) = \frac{\ln x \times 3x^2 - x^3 \cdot \frac{1}{x}}{(\ln x)^2}$ ✓

$= \frac{3x^2 \ln x - x^2}{(\ln x)^2}$

Question 13 (3 marks)

The following Pareto chart shows the distribution of costs for an end-of-year event.



(a) Use the graph to estimate the percentage of the total cost spent on 'Waitstaff'? **1**

68% − 52% = 16%

(±1% for each estimation)

(b) Find the total cost for this event. **1**

32% of total cost = 8000

total cost = 8000 ÷ 0.32

= $25000

(c) Find the cost spent on 'Waitstaff'. **1**

16% × 25000 = $4000

Question 14 (3 marks)

The sum of the first n terms of an arithmetic series is given by $S_n = \frac{1}{2}n(3n+2)$.

(a) Find the first three terms of the series. **2**

$T_1 = S_1$

$= \frac{1}{2}(1)(3(1)+2)$

$= \frac{5}{2}$ ✓ (or of equivalent merit)

$T_2 = S_2 - S_1$

$= \frac{1}{2}(2)(3(2)+2) - \frac{5}{2}$

$= \frac{11}{2}$ ✓$\frac{1}{2}$

$T_3 = S_3 - S_2$

$= \frac{1}{2}(3)(3(3)+2) - 8$

$= \frac{17}{2}$ ✓$\frac{1}{2}$

(b) Find an expression for the nth term of the series. **1**

$a = \frac{5}{2}$ $d = 3$

$T_n = \frac{5}{2} + (n-1)3$

$= \frac{5}{2} + 3n - 3$

$= 3n - \frac{1}{2}$

OR $T_n = S_n - S_{n-1}$

$= \frac{1}{2}n(3n+2) - \frac{1}{2}(n-1)(3(n-1)+2)$

$= \frac{1}{2}[3n^2 + 2n - (n-1)(3n-1)]$

$= \frac{1}{2}[3n^2 + 2n - (3n^2 - 4n + 1)]$

$= \frac{1}{2}(6n-1)$

Question 15 (3 marks)

Find the equation of the normal to the curve $y = e^{\cos(\pi x)}$ at the point where $x = \frac{1}{2}$. **3**

$y' = -\sin \pi x \times \pi \times e^{\cos \pi x}$ ✓

$x = \frac{1}{2}$ $y' = -\pi \sin \frac{\pi}{2} e^{\cos \frac{\pi}{2}}$

$= -\pi (1) e^{0}$

$= -\pi$

$= m_T$

$m_N = -\frac{1}{m_T}$

$= \frac{1}{\pi}$ ✓

$x = \frac{1}{2}$ $y = e^{\cos \frac{\pi}{2}}$

$= e^{0}$

$= 1$

$m_N = \frac{1}{\pi}$, $(\frac{1}{2}, 1)$

$y - 1 = \frac{1}{\pi}(x - \frac{1}{2})$

$\pi y - \pi = x - \frac{1}{2}$

$2x - 2\pi y + (2\pi - 1) = 0$ ✓

(OR $y = \frac{1}{\pi}x - \frac{1}{2\pi} + 1$)

Question 16 (3 marks)

Show that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$. **3**

$$LHS = \frac{\sec\theta(1-\cos^4\theta)}{1+\cos^2\theta}$$

$$= \frac{\sec\theta(1-\cos^2\theta)(1+\cos^2\theta)}{1+\cos^2\theta} \checkmark$$

$$= \sec\theta \times \sin^2\theta \checkmark$$

$$= \frac{1}{\cos\theta} \times \sin^2\theta$$

$$= \frac{\sin\theta}{\cos\theta} \times \sin\theta \checkmark$$

$$= \tan\theta\sin\theta$$

$$= RHS$$

Question 17 (6 marks)

In a high school, there are eight students who study both Maths and Music.

Marks for these students from the most recent assessments are shown below.

Student	A	B	C	D	E	F	G	H	Mean	σ
Music	61	52	70	65	48	50	56	55		
Maths	80	70	85	90	75	75	78	73	78.25	6.6

(a) Find the mean and standard deviation of the Music assessment. Give your answer correct to nearest 3 decimal places. **2**

mean = 57.125 ✓

σ = 7.149 ✓

(b) Student A claims that she is performing better in Music than in Maths. Is this claim correct? Justify your answer with appropriate calculations. **2**

$z_{music} = \frac{61 - 57.125}{7.149}$

$= 0.542\ldots$ ✓ finds at least one z-score

$z_{maths} = \frac{80 - 78.25}{6.6}$

$= 0.2\dot{6}\dot{5}$

Since $z_{music} > z_{maths}$, she is correct. ✓

Question 17 continues on page 17

Question 17 (continued)

(c) Find the correlation coefficient Maths and Music exam scores. **1**

r = 0.831 (3d.p.)

(d) One student concludes that 'people who are good at Maths are good at Music'. **1**
Does the data support this statement? Justify your answer.

there is a strong positive correlation between Maths and Music scores, which supports this statement

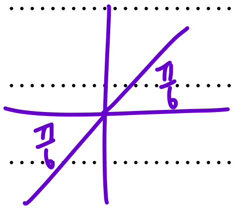
End of Question 17

Please turn over

Question 18 (3 marks)

Solve $\tan\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ for $-\pi \le x \le \pi$ **3**

$-\pi + \frac{\pi}{4} \le x + \frac{\pi}{4} \le \pi + \frac{\pi}{4}$



related angle $= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$ ✓

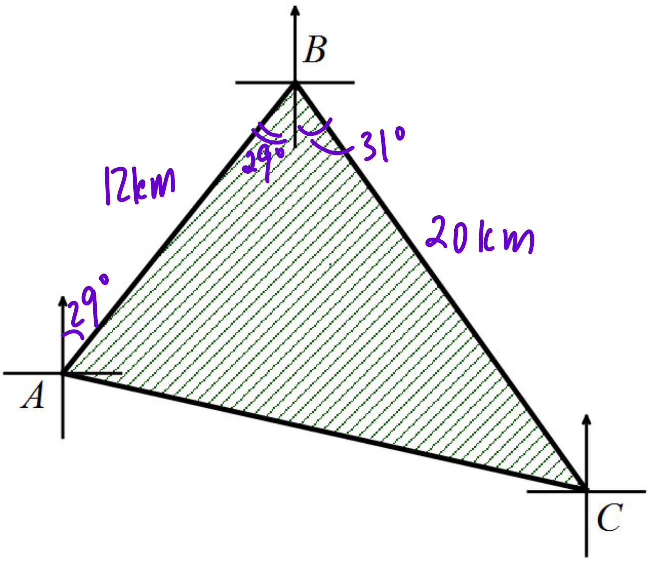
$x + \frac{\pi}{4} = \frac{\pi}{6}, \ \pi + \frac{\pi}{6}, \ -\pi + \frac{\pi}{6}$ ✓

$x = \frac{\pi}{6} - \frac{\pi}{4}, \ \frac{7\pi}{6} - \frac{\pi}{4}$

$= -\frac{\pi}{12}, \ \frac{11\pi}{12}$ ✓

Question 19 (5 marks)

Bill runs around a triangular shaped park, starting and ending at gate A. He runs 12 km from A to B on a bearing of $N29^\circ E$, then runs 20 km from B to C on a bearing of $S31^\circ E$. Lastly, he runs back to A.



(a) Find the exact area of the park. **2**

$\angle ABC = 29^\circ + 31^\circ$

$= 60^\circ$ ✓

$A = \frac{1}{2} \times 12 \times 20 \times \sin 60^\circ$

$= 60\sqrt{3}$ km^2 ✓ ('exact' answer assessed in Q11)

(b) Find the total distance Bill ran. **3**

$AC^2 = 12^2 + 20^2 - 2 \times 12 \times 20 \cos 60^\circ$ ✓

$= 304$

$AC = \sqrt{304}$ ✓

$= 4\sqrt{19}$

total distance $= 12 + 20 + 4\sqrt{19}$

$= 32 + 4\sqrt{19}$ km ✓

$\left(= 49.4355957\ldots \right.$

$\left. \approx 49.44 \text{ km} \right)$

Proceed to Booklet 2 for Questions 20-27

Question 20 (4 marks)

A random variable is normally distributed with mean 0 and standard deviation 1.

The table gives the probability that this random variable is less than z for different values of z.

z	0	0.5	1	1.5	2	2.5
$P(Z < z)$	0.500	0.692	0.841	0.933	0.977	0.994

In a particular city, the weekly rents for apartments are normally distributed with a mean of \$750 and a standard deviation of \$100.

(a) Find the probability that the weekly rent of an apartment chosen at random is less than \$600. **2**

$P(X < 600) = P(Z < -1.5)$ ✓

$= 1 - P(Z < 1.5)$

$= 1 - 0.933$ ✓

$= 0.067$

$= 6.7\%$

(b) Sam's budget for the weekly rent is \$600. She has refined her searches and produced a list such that she is only looking at properties which are less than the mean weekly rent. Find the probability that an apartment that Sam chooses at random from this list is within her budget. **2**

$P(X < 600 \mid X < 750) = \dfrac{P(X < 600 \cap X < 750)}{P(X < 750)}$ ✓

$= \dfrac{P(X < 600)}{P(X < 750)}$

$= \dfrac{0.067}{0.5}$ ✓

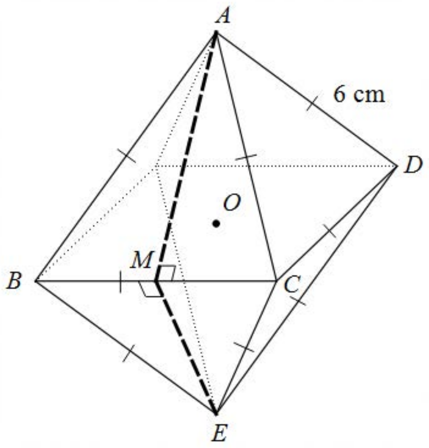
$= 0.134$

$= 13.4\%$

Question 21 (3 marks)

Two identical right pyramids with square bases and equilateral triangular faces are joined to form a regular octahedron. **3**

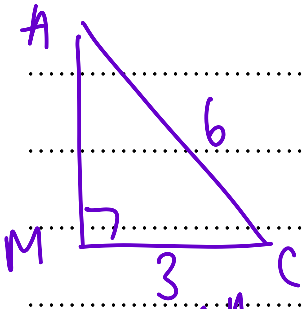
The following diagram shows a regular octahedron with edge length 6 cm and centre at O. AM and ME are perpendicular heights of ΔABC and ΔBCE respectively and point M is the midpoint of BC.



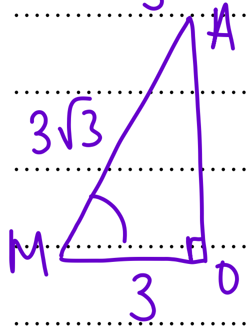
Find the size of $\angle AME$, to the nearest degree.

$\angle AME = 2 \times \angle AMO$ ($\Delta AMO \equiv \Delta EMO$, SSS)

OK using cosine rule
- AM ✓
- AE ✓
- angle ✓



$AM = \sqrt{6^2 - 3^2}$
$= \sqrt{27}$
$= 3\sqrt{3}$ ✓



$\cos^{-1} \angle AMO = \frac{3}{3\sqrt{3}}$

$MO = \frac{1}{2} \times 6$
$= 3$

$\angle AMO = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (acute) ✓

$\angle AME = 2\angle AMO$
$= 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
$= 109.47122...$
$\approx 109°$ (nearest degree) ✓

Question 22 (6 marks)

A continuous random variable X has the probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{kx}{x^2+1}, & \text{for } 1 \le x \le 5 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Show that $k = \dfrac{2}{\ln 13}$. **3**

$\frac{k}{2}\int_1^5 \frac{2x}{x^2+1}\,dx = 1$ ✓

$\frac{k}{2}\left[\ln|x^2+1|\right]_1^5 = 1$ ✓

$k(\ln 26 - \ln 2) = 2$

$k \ln 13 = 2$

$k = \frac{2}{\ln 13}$ ✓

Question 22 continues on page 25

Question 22 (continued)

(b) Find the median m. **3**

$$\frac{2}{\ln 13}\int_1^m \frac{x}{x^2+1}\,dx = 0.5 \quad \checkmark\tfrac{1}{2}$$

$$\frac{1}{\ln 13}\Big[\ln|x^2+1|\Big]_1^m = 0.5$$

$$\frac{1}{\ln 13}\left(\ln(m^2+1) - \ln 2\right) = 0.5 \quad \checkmark\tfrac{1}{2} \quad (m^2+1>0)$$

$$\ln\left(\frac{m^2+1}{2}\right) = \frac{1}{2}\ln 13$$

$$\ln\left(\frac{m^2+1}{2}\right) = \ln 13^{\frac{1}{2}} \quad \checkmark$$

$$\frac{m^2+1}{2} = \sqrt{13}$$

$$m^2+1 = 2\sqrt{13}$$

$$m^2 = 2\sqrt{13}-1$$

$$m = \sqrt{2\sqrt{13}-1} \quad \checkmark \quad (1 \leq m \leq 5)$$

End of Question 22

$(m \approx 2.492208\ldots)$

Please turn over

Question 23 (4 marks)

The motion of a particle is given by $x = 10t + 4e^{-3.5t} - 4$, where x is the displacement from the starting position in metres and t is time after the initial position in seconds.

(a) Find the initial velocity. **2**

$\frac{dx}{dt} = 10 + 4 \times -3.5e^{-3.5t}$ ✓

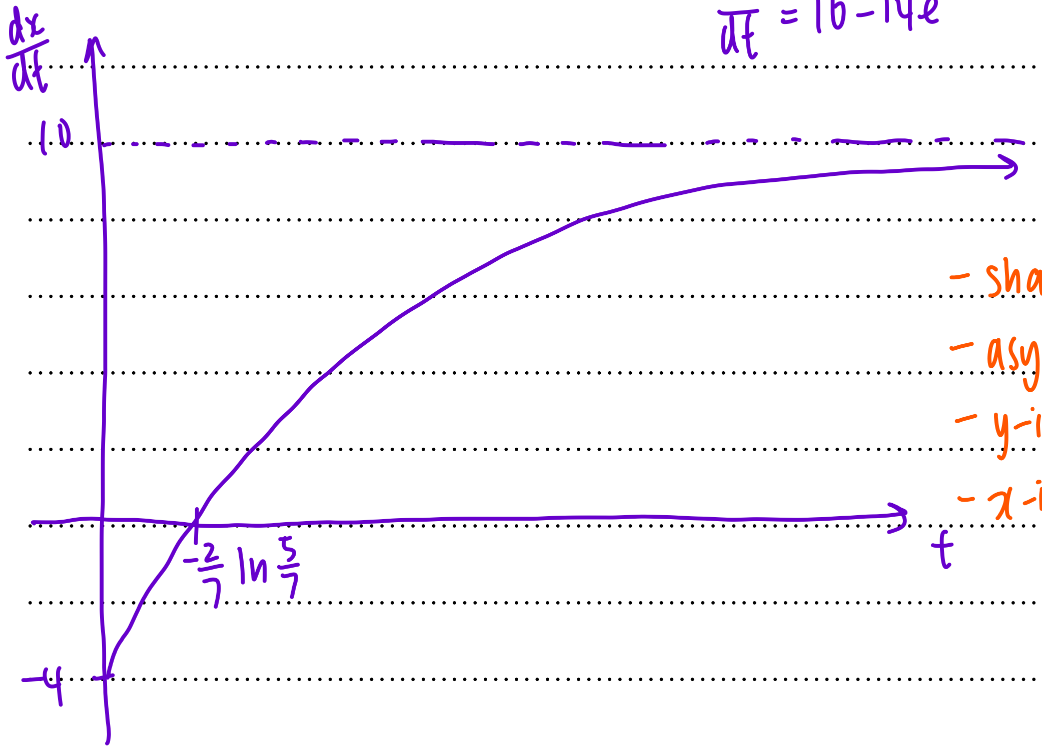
$= 10 - 14e^{-3.5t}$

$t=0$ $\quad \frac{dx}{dt} = 10 - 14e^{0}$

$= -4$ m/s ✓

(b) Sketch the velocity-time graph. **2**

$\frac{dx}{dt} = 10 - 14e^{-3.5t}$



($\frac{1}{2}$ each)

- shape
- asymptote
- y-intercept
- x-intercept

$\frac{dx}{dt} = 0$

$10 - 14e^{-3.5t} = 0$

$e^{-3.5t} = \frac{5}{7}$

$-3.5t = \ln\frac{5}{7}$ $\quad \rightarrow t = \frac{\ln\frac{5}{7}}{-3.5}$

$= -\frac{2\ln\frac{5}{7}}{7}$

Question 24 (2 marks)

Let $f(x)$ be an even function. If $\int_0^6 f(x)\,dx = 12$ and $\int_4^6 f(x)\,dx = -5$ evaluate $\int_{-4}^4 2f(x)\,dx$. **2**

$$\int_0^4 f(x)\,dx = \int_0^6 f(x)\,dx - \int_4^6 f(x)\,dx$$

$$= 12 - (-5)$$

$$= 17$$ ✓

$$\int_{-4}^4 2f(x)\,dx = 2\int_{-4}^4 f(x)\,dx$$

OR

$$= 2 \times 2\int_0^4 f(x)\,dx \quad \text{(even function)}$$

$$= 2 \times 2 \times 17$$

$$= 68$$ ✓ correct answer

Question 25 (6 marks)

At a market, a stallholder is hosting free-to-play mini games.

If the player wins against the stallholder, they will receive \$25 as a reward. If they lose, they have to pay \$5 to the stallholder. If the game ends in a draw, no money is exchanged.

The random variable X represents the amount of money a player wins per mini game, where the probability of winning a game is independent of each other.

The distribution of X is given in the table below.

x	-5	0	25
$P(X=x)$	$0.29-2k^2$	0.87	$k-0.28$

(a) Show that $2k^2-k+0.12=0$. **1**

$\Sigma P(X=x) = 1$

$0.29-2k^2+0.87+k-0.28 = 1$

$0.88-2k^2+k = 1$

$\therefore 2k^2-k+0.12=0$

(b) Find the value of k, giving reasons for your answer. **2**

$k = \dfrac{1 \pm \sqrt{1-4\times2\times0.12}}{2\times2}$

$= \dfrac{1 \pm \sqrt{0.04}}{4}$

$= 0.3$ or 0.2 ✓

but $P(X=25) = k-0.28$, where $0 \leq k-0.28 \leq 1$

$0.28 \leq k \leq 1.28$

$\therefore k=0.3$ ✓ must include reason

Question 25 continues on page 29

Question 25 (continued)

(c) Find $E(X)$. **1**

$E(X) = -5(0.29-2(0.3)^2) + 0.87(0) + 25(0.3-0.28)$

$= -0.05$

A player decides to play 3 mini games.

(d) How much is the player expected to win or lose? **1**

$E(X) \times 3 = -\$0.15$

$\therefore$ the player is expected to lose 15 cents

(e) What is the probability that the player will lose exactly two of the three games? **1**

outcomes: $LL\bar{L}$, $L\bar{L}L$, $\bar{L}LL$

$P(L) = 0.29 - 2(0.3)^2$

$= 0.11$

$P(\text{lose exactly twice}) = (0.11)^2 \times (1-0.11) \times 3$

$= 0.032307$

$= 3.2307\%$

End of Question 25

Please turn over

Question 26 (4 marks)

A spring is extended to its maximum length and released.

The length of the spring l cm, t seconds after release is modelled by the function $l = a + b\cos 2t$.

(a) Given that the minimum length of the spring is 3 cm and the maximum length is 11 cm, find the values of a and b. **2**

$a = \frac{11+3}{2}$ (centre)

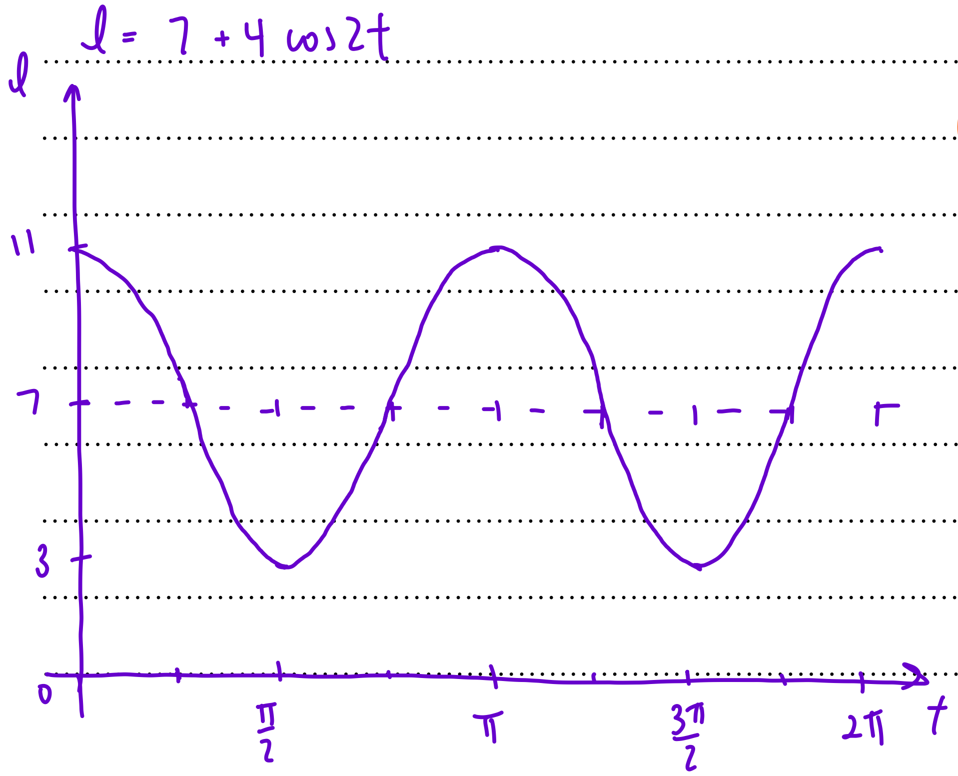
$= 7$ ✓

$b = \frac{11-3}{2}$ (amplitude)

$= 4$ ✓

(b) Sketch a graph of the model in the domain $0 \le t \le 2\pi$. **2**

$l = 7 + 4\cos 2t$



(1 mark)
- shape
- scale (x- and y- axes)

Question 27 (4 marks)

Consider the function $f\left(\frac{x}{3}-1\right)=\frac{1}{x-2}-6$.

(a) List a set of transformations that, when applied in order, would transform $f(x)$ to $f\left(\frac{x}{3}-1\right)$. **2**

method 1

① translate right by 1 unit ✓

② dilate horizontally by a factor of 3 ✓

method 2

$f\left(\frac{1}{3}(x-3)\right)$

① dilate horizontally by a factor of 3

② translate right by 3 units

(b) Find the equation of $y=f(x)$. **2**

method 1

$f(x-1)=f\left(\frac{3x}{3}-1\right)$ dilate horizontally by a factor of $\frac{1}{3}$

$=\frac{1}{3x-2}-6$ ✓

$f(x)=f((x+1)-1)$ translate left by 1 unit

$=\frac{1}{3(x+1)-2}-6$

$=\frac{1}{3x+1}-6$ ✓

method 2

$f\left(\frac{x}{3}\right)=f\left(\frac{x+3}{3}-1\right)$ translate left by 3 units

$=\frac{1}{x-2+3}-6$

$=\frac{1}{x+1}-6$

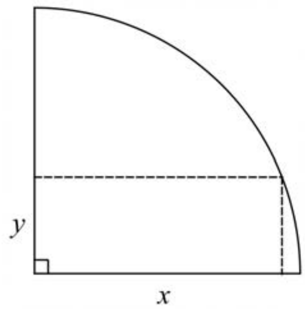
Proceed to Booklet 3 for Questions 28-32

$f(x)=f\left(\frac{3x}{3}-1\right)$ dilate horizontally by a factor of $\frac{1}{3}$

$=\frac{1}{3x+1}-6$

Question 28 (7 marks)

A new logo for MathsCompany is to be made by inscribing a rectangle inside a quadrant of radius 3 cm as shown below.



The length and width of the rectangle are x and y cm respectively and $x^2 + y^2 = 9$.

(a) Show that the area of the rectangle is given by **1**

$$A = x\sqrt{9 - x^2}$$

$x^2 + y^2 = 9$

$y^2 = 9 - x^2$

$y = \sqrt{9 - x^2}$ (y is length)

$A_{rectangle} = x \times y$

$= x\sqrt{9 - x^2}$

(b) Show that **2**

$$\frac{dA}{dx} = \sqrt{9 - x^2} - \frac{x^2}{\sqrt{9 - x^2}}$$

$u = x$ $\quad v = (9 - x^2)^{\frac{1}{2}}$

$u' = 1$ $\quad v' = \frac{1}{2}(9 - x^2)^{-\frac{1}{2}} \cdot -2x$

$= -\frac{x}{\sqrt{9 - x^2}}$ ✓

$\frac{dA}{dx} = vu' + uv'$

$= \sqrt{9 - x^2} \times 1 + x \times -\frac{x}{\sqrt{9 - x^2}}$ ✓

$= \sqrt{9 - x^2} - \frac{x^2}{\sqrt{9 - x^2}}$

Question 28 continues on page 35

Question 28 (continued)

(c) Hence find the maximum area of the rectangle. **4**

for max/min A, $\frac{dA}{dx} = 0$

$$\sqrt{9-x^2} - \frac{x^2}{\sqrt{9-x^2}} = 0$$

$\times\sqrt{9-x^2}$ $\times\sqrt{9-x^2}$

$$9-x^2 - x^2 = 0$$ ✓

$$9-2x^2 = 0$$

$$2x^2 = 9$$

$$x^2 = \frac{9}{2}$$

$$x = \frac{3}{\sqrt{2}} \quad (x \text{ is length})$$

$$= \frac{3\sqrt{2}}{2}$$ ✓

test nature

x	2.1	$\frac{3\sqrt{2}}{2}$	2.2
$\frac{dA}{dx}$	0.084	0	-0.333
	/	—	\

✓

$\therefore$ max A when $x = \frac{3\sqrt{2}}{2}$ cm

$$\max A = x\sqrt{9-x^2}$$

$$= \frac{3}{\sqrt{2}}\sqrt{9-\frac{9}{2}}$$

$$= \frac{9}{2} \text{ cm}^2$$ ✓

End of Question 28

Please turn over

Question 29 (7 marks)

The first three terms of a geometric series are $6\cos\theta$, $2(\sin\theta - 2)$ and $3\tan\theta$.

(a) Show that $2\sin^2\theta - 17\sin\theta + 8 = 0$. **2**

$\frac{T_3}{T_2} = \frac{T_2}{T_1}$ for geometric series

$\frac{3\tan\theta}{2(\sin\theta - 2)} = \frac{2(\sin\theta - 2)}{6\cos\theta}$

$18\tan\theta\cos\theta = 4(\sin\theta - 2)^2$

$9\frac{\sin\theta}{\cos\theta}\cos\theta = 2(\sin^2\theta - 4\sin\theta + 4)$

$9\sin\theta = 2\sin^2\theta - 8\sin\theta + 8$

$2\sin^2\theta - 17\sin\theta + 8 = 0$

(b) Find the exact value of θ in radians, given that it is obtuse. $\frac{\pi}{2} < \theta < \pi$ **2**

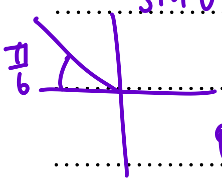
let $m = \sin\theta$

$2m^2 - 17m + 8 = 0$

$(2m - 1)(m - 8) = 0$

$m = \frac{1}{2}$, $m = 8$

$\sin\theta = \frac{1}{2}$ no solution



$\theta = \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

Question 29 continues on page 37

Question 29 (continued)

(c) Find the limiting sum of the series, leaving your answer in the form $k\left(\sqrt{3}+1\right)$. **3**

$a = 6\cos\frac{5\pi}{6}$

$= 6 \times -\frac{\sqrt{3}}{2}$

$= -3\sqrt{3}$ ✓

$r = \frac{2(\sin\frac{5\pi}{6} - 2)}{-3\sqrt{3}}$

$= \frac{2(\frac{1}{2}-2)}{-3\sqrt{3}}$

$= \frac{-3}{-3\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$ ✓

since $|r|<1$, limiting sum exists.

$S_\infty = \frac{-3\sqrt{3}}{1-\frac{1}{\sqrt{3}}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{-9}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{-9(\sqrt{3}+1)}{2}$

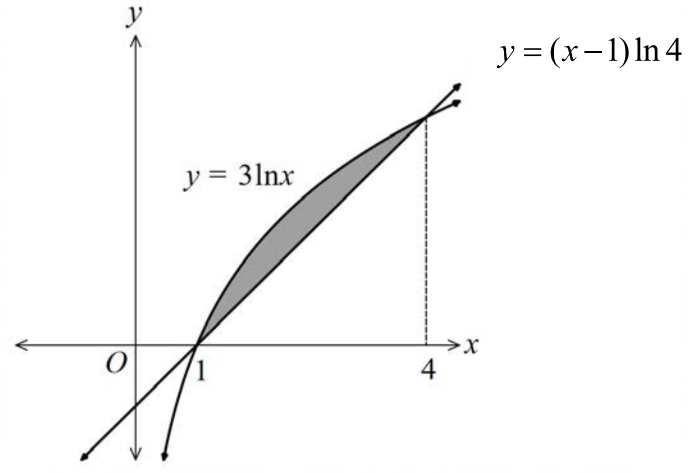
$= -\frac{9}{2}(\sqrt{3}+1)$ ✓

End of Question 29

Please turn over

Question 30 (4 marks)

The shaded region is enclosed by the curve $y = 3\ln x$ and line $y = (x-1)\ln 4$, as shown in the diagram.



(a) Use the Trapezoidal rule with 3 sub-intervals to find an approximate area of the shaded region. Give your answer correct to three significant figures. **3**

method 1

$A = \int_1^4 3\ln x - (x-1)\ln 4 \, dx$ ✓

x	1	2	3	4
$3\ln x - (x-1)\ln 4$	0	$3\ln 2 - \ln 4$	$3\ln 3 - 2\ln 4$	0

$A \approx \frac{4-1}{2\times 3}\left[0 + 0 + 2\left(3\ln 2 - \ln 4 + 3\ln 3 - 2\ln 4\right)\right]$ ✓

$\approx \frac{1}{2}\left(2\left(3\ln 2 - 3\ln 4 + 3\ln 3\right)\right)$

$\approx 1.216395\ldots$

≈ 1.22 units^2 (3 s.f.) ✓

method 2

use Trapezoidal rule to estimate area under $y = \ln x$, then subtract the area of the triangle.

Question 30 continues on page 39

Question 30 (continued)

(b) Explain whether the approximation is an overestimate or underestimate of the true area of the shaded region. **1**

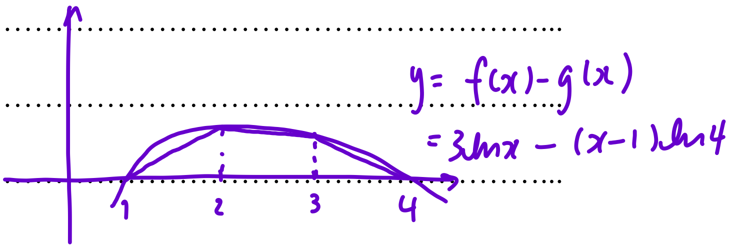
method 1-1 let $f(x) = 3\ln x$ and $g(x) = (x-1)\ln 4$ ✓½

$f(x)$ is concave down ✓½ and $g(x)$ has 0 concavity for $1 \le x \le 4$.

$\therefore$ $f(x) - g(x)$ is concave down for $1 \le x \le 4$

$\therefore$ the approximation of $\int_1^4 f(x) - g(x)\, dx$ using Trapezoidal rule is an underestimation.

method 1-2 using a diagram



method 2 $y = 3\ln x$ is concave down

(given they used exact value of the triangle)

$\therefore$ underestimation

End of Question 30

Please turn over

Question 31 (4 marks)

Let A and B be two events. Suppose that $P(A) = 0.8$, $P(B \mid A) = 0.45$ and $P(B \mid \overline{A}) = 0.6$.

(a) Find $P(B)$. **3**

$P(B|A) = \frac{P(B \cap A)}{P(A)}$

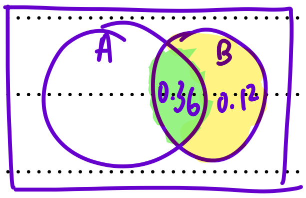
$0.45 = \frac{P(B \cap A)}{0.8}$

$\therefore P(B \cap A) = 0.36$ ✓

$P(B|\bar{A}) = \frac{P(B \cap \bar{A})}{P(\bar{A})}$

$0.6 = \frac{P(B \cap \bar{A})}{1 - 0.8}$

$P(B \cap \bar{A}) = 0.12$ ✓



$P(B) = P(A \cap B) + P(\bar{A} \cap B)$

$= 0.36 + 0.12$

$= 0.48$ ✓

(b) Is event A independent from event B? Justify your answer with relevant calculations. **1**

$P(A) \times P(B) = 0.8 \times 0.48$

$= 0.384$

$\neq P(A \cap B)$

$\therefore$ A is NOT independent from B

Question 32 (5 marks)

(a) Find $\frac{d}{dx}\left(xe^{mx}\right)$. **1**

$\frac{d}{dx}\left(xe^{mx}\right) = e^{mx}\cdot 1 + x \cdot me^{mx}$

$= e^{mx} + mxe^{mx}$

(b) Hence find $\int xe^{mx}\,dx$. **2**

$\frac{d}{dx}\left(xe^{mx}\right) = e^{mx} + mxe^{mx}$

$\int \frac{d}{dx}\left(xe^{mx}\right) dx = \int e^{mx} + mxe^{mx}\, dx$

$xe^{mx} + C = \frac{1}{m}e^{mx} + m\int xe^{mx}\, dx$ ✓ or work of equivalent merit

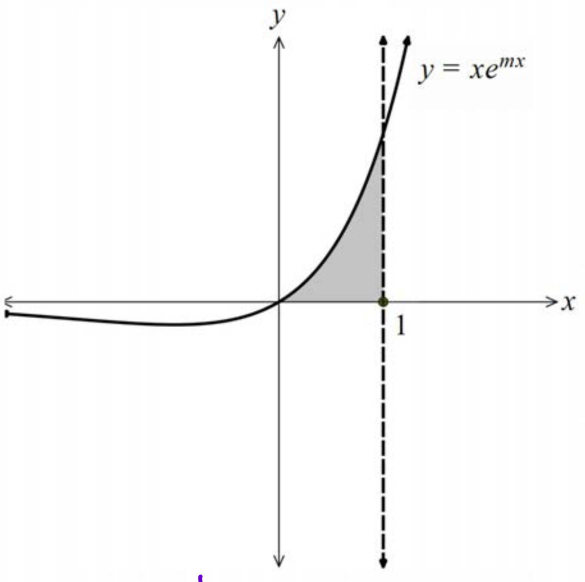
$m\int xe^{mx}\, dx = xe^{mx} - \frac{1}{m}e^{mx} + C$

$\int xe^{mx}\,dx = \frac{1}{m}e^{mx}\left(x - \frac{1}{m}\right) + C$ ✓

Question 32 continues on page 43

Question 32 (continued)

(c) The area of the region bounded by the curve $y = xe^{mx}$, the x-axis and $x = 1$ is shown below. Given that the area of this area is $\frac{1}{m}$, find m. **2**



$A = \int_0^1 xe^{mx}\,dx = \frac{1}{m}$

$\frac{1}{m}\left[e^{mx}\left(x - \frac{1}{m}\right)\right]_0^1 = \frac{1}{m}$

$e^m\left(1 - \frac{1}{m}\right) - e^0\left(0 - \frac{1}{m}\right) = 1$ ✓

$e^m\left(1 - \frac{1}{m}\right) + \frac{1}{m} - 1 = 0$

$e^m\left(1 - \frac{1}{m}\right) - \left(1 - \frac{1}{m}\right) = 0$

$\left(1 - \frac{1}{m}\right)\left(e^m - 1\right) = 0$

$1 - \frac{1}{m} = 0 \qquad e^m = 1$

$\frac{1}{m} = 1 \qquad m = 0$

$m = 1 \qquad$ but $m \neq 0$ as $A = \frac{1}{m}$

$\therefore m = 1$ ✓

End of paper