



Saint Ignatius' College, Riverview Mathematics Assessment Task 2024

Year 12
Mathematics Advanced
Task 4 Trial HSC Exam
Date: 23 rd August 2024

General Instructions:

- **Reading time : 10 mins**
- **Time Allowed: 3 hours**

- Write using blue or black pen only
- NESAs approved calculators may be used
- Attempt all questions.
- A NESAs Reference Sheet is provided.
- Questions 1 to 10 are all multiple-choice questions worth 1 mark each and are to be answered on the multiple-choice answer sheet provided.
- Questions 11 to 36 are each worth 90 marks and are to be answered on the examination paper.
- Each booklet and the multiple-choice answer sheet must have **your name** and **the initials of your class teacher** on the front cover.
- Marks may not be awarded for missing or carelessly arranged working.

Topics Examined:

All Preliminary and HSC Mathematics topics

SECTION 1

Questions 1 - 10

10 Marks

SECTION 2

Question 11 - 36

90 Marks

Total

100 Marks

Teacher:

- | | |
|----------------|-----|
| • Mr N Mushan | NHM |
| • Ms F Yates | FEY |
| • Mr J Newey | JPN |
| • Mr P Collins | PPC |
| • Ms K Mullan | KXM |

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SECTION 1 (10 marks)

Attempt Questions 1 – 10

1. The domain of $y = \log_e(5 - 2x)$ is:

A. $x < -\frac{5}{2}$

B. $x > -\frac{5}{2}$

C. $x < \frac{5}{2}$

D. $x > \frac{5}{2}$

2. The new equation when $xy=1$ is translated right by 3 units is:

A. $x = \frac{1}{y+3}$

B. $y = \frac{1}{x+3}$

C. $x = \frac{1}{y-3}$

D. $y = \frac{1}{x-3}$

3. At the NSW State Swimming Championships, the time in seconds for competitors from all age groups to finish the 50-metre freestyle is normally distributed with a mean of 27 seconds and a standard deviation of 1.5.

Calculate the percentage of 50-metre freestyle swim competitors who complete the lap in less than 24 seconds.

A. 34%

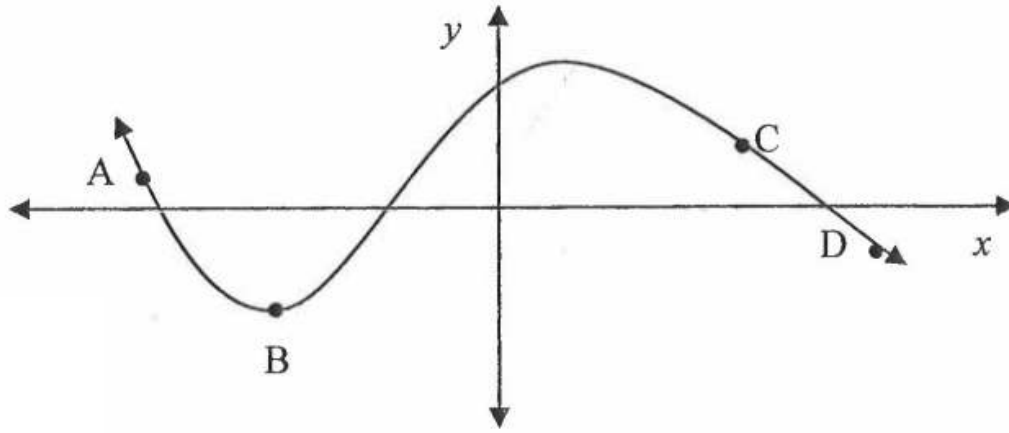
B. 13.5%

C. 2.5%

D. 0.15%

4. Which point on the following diagram relates to the following description.

$$y > 0, \frac{dy}{dx} < 0, \frac{d^2y}{dx^2} < 0$$



- A. A
- B. B
- C. C
- D. D

5. Evaluate $\int_{-1}^1 \frac{1}{e^{2x}} dx$

- A. $1 - e^2$
- B. $e^2 - e^{-2}$
- C. $\frac{e^3 - 1}{3}$
- D. $\frac{e^2 - e^{-2}}{2}$

6. $\int_{-1}^a (2x+1)^3 dx = 10$ where $a > 0$. The value of a is:

- A. 1
- B. 4
- C. 6
- D. 8

7. The curve $y = ax^2 + bx + 4$ has a stationary point at $(3, -5)$. The values of a and b are:

- A. $a = 3; b = -5$
- B. $a = 1; b = -6$
- C. $a = -1; b = 6$
- D. $a = -3; b = 5$

8. The solutions to $\sec(x + \frac{\pi}{4}) = \sqrt{2}$ for $0 \leq x \leq 2\pi$ are:

$x =$

- A. $\frac{\pi}{3}, \frac{5\pi}{3}$
- B. $\frac{\pi}{2}, \frac{5\pi}{2}$
- C. $0, \frac{3\pi}{2}, 2\pi$
- D. $0, \frac{\pi}{4}, \frac{5\pi}{4}, 2\pi$

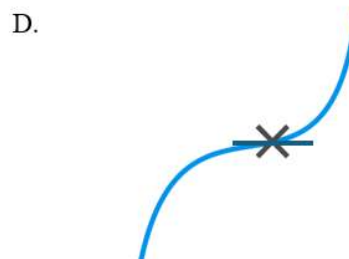
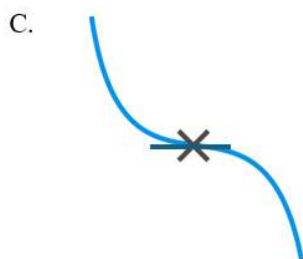
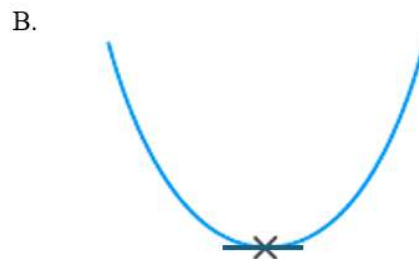
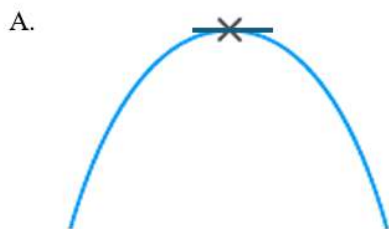
9. If $y = 4^{-x}$, then $\frac{dy}{dx} =$

- A. 4^{-x}
- B. $\frac{1}{x \log_e 4}$
- C. -4^{-x-1}
- D. $\frac{-\log_e 4}{4^x}$

10. If $\frac{d^2y}{dx^2} = 6x - 6$,
 $\frac{dy}{dx} = 0$ when $x = 1$,

and when $x = 2$, $y = -4$

therefore, the nature of the stationary point at $x = 1$ is:



End of Section 1

SECTION 2

Total Marks – 90

Attempt Questions 11-36

[Marks for each part are indicated on the page]

Allow about 2 hours and 45 minutes for this section

Question 11

Marks

A new car purchased for \$27000 depreciates by 15% of its purchase price each year.

(a) What is the value of the car after 1 year?

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(b) What is the value of the car after 3 years? (correct to 2 decimal places)

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(c) When will the value of the car be half of the purchase price?
(correct to 2 decimal places)

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Question 12

Find the 6th term of the geometric sequence if $T_2=3$ and $T_5=\frac{81}{8}$

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Question 13

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{1+\sin x} dx$.

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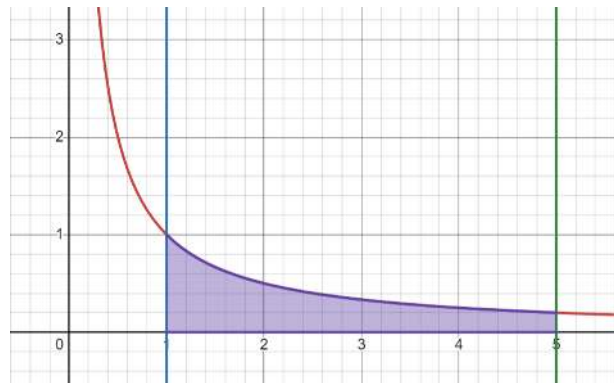
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Question 14

A sketch of $y = \frac{1}{x}$ for the definite integral $\int_1^5 \frac{1}{x} dx$ is shown below.



Use the trapezoidal rule with 5 function values to estimate the area of the shaded region.

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Question 15

Consider the Probability Density Function (PDF) given by

$$f(x) = \begin{cases} \frac{1}{x^2}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $P(X \geq 3)$.

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(b) Write down the formula for $P(A|B)$

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(c) Given $P(X \geq 2) = \frac{1}{2}$, hence, find $P(X \geq 3 | X \geq 2)$

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Question 16

For the curve $y = x^3 - 9x^2 + 24x - 15$

(a) Find the coordinates of the stationary points and determine their nature.

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(b) Find the point of inflection.

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(c) What is the global maximum and minimum of $y = x^3 - 9x^2 + 24x - 15$ for the domain $x \in R [0, 6]$? **2**

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Question 17

Differentiate $\frac{\sin x}{x^2}$.

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Question 18

(a) Evaluate $\int_0^1 \frac{x-1}{x^2-2x+4} dx$. (Write your answer as an exact value)

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(b) Evaluate $\int_{\ln 2}^{2\ln 3} (1-e^x)^2 dx$.

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Question 19

Simplify $\frac{\sin^2 x + \cos^2 x}{\cos^2 x}$.

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Question 20

A particle undergoes straight line motion with velocity $v = \frac{6}{\sqrt{3t+4}}$

where t is time in seconds and distance is in metres.

(a) Find the particle's position x at time t , if initially the particle is at the origin.

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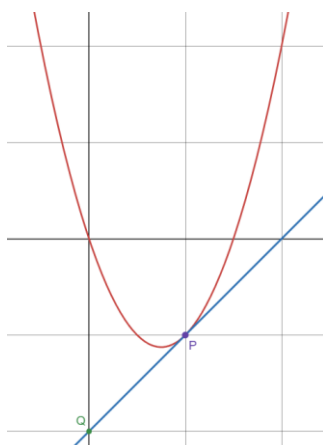
(b) Find the position of the particle 7 seconds later.

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Question 21

The diagram shows the parabola $y = x^2 - 3x$ and a tangent drawn at P.
 The equation of the tangent at P is $y = x - 4$ and it cuts the y-axis at Q.



- (a) Show that the x co-ordinate of P is $x = 2$. **1**

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- (b) Find the **co-ordinates** of Q. **1**

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- (c) A normal is drawn from P and cuts the y-axis at R.
 Find the equation of the normal and the co-ordinates of R. **2**

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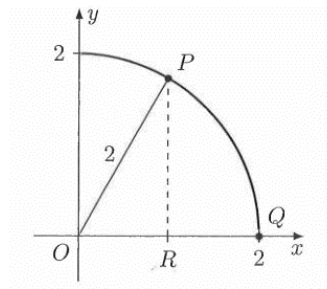
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- (d) Find the area of triangle RPQ. **1**

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Question 22



The diagram shows a point P on a part of $y = \sqrt{4 - x^2}$.

The point P is vertically above R and Q has coordinates (2,0). The point R has coordinates (1,0).

(a) State in radians the size of angle POR.

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(b) Calculate the area of sector OPQ. (Write your answer as an exact value)

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(c) Calculate the area of triangle OPR. (Write your answer as an exact value)

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(d) Hence find $\int_0^1 \sqrt{4 - x^2} dx$ (Write your answer as an exact value)

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Question 23

A continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} 12x^2(1-x), & \text{for } 0 \leq x \leq 1 \\ 0, & \text{for all other values of } x \end{cases}$$

(a) Find the mode of X .

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(b) Find the cumulative distribution function for the given probability density function.

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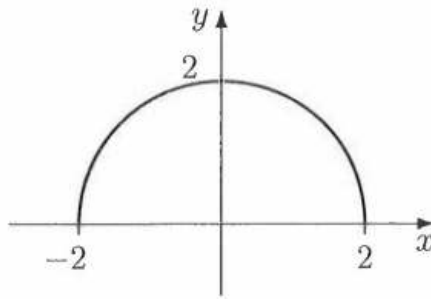
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Question 24

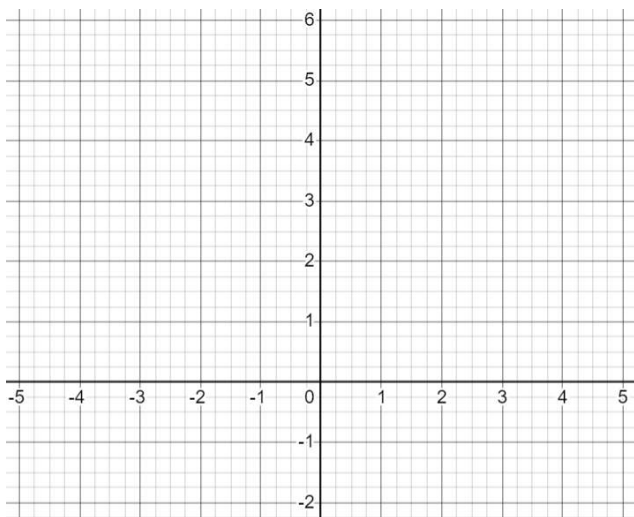
The diagram below shows the graph of $y = f(x)$



Sketch on the axes provided, the following **transformations** of the semi-circle, showing the coordinates of the x and y intercepts.

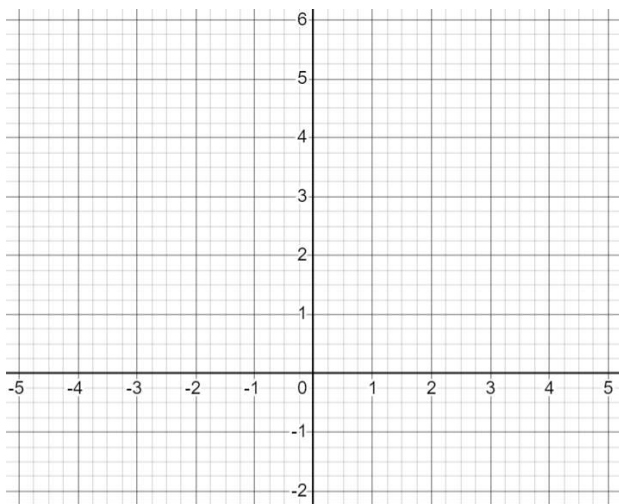
(a) $y = 2f(2x)$

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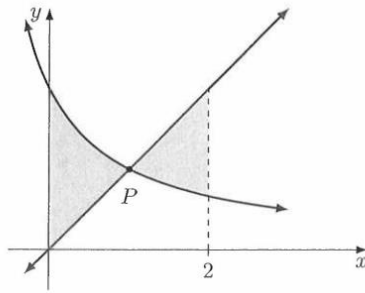


(b) $y = -\frac{1}{2}f\left(\frac{x}{2}\right)$

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Question 25



The shaded region is bounded by the hyperbola $y = \frac{2}{x+1}$, and line $y = x$.

Point P is the intersection of the hyperbola and the line

(a) Show that the x -coordinate of P is 1.

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(b) Find the exact area of the shaded regions.

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Question 26

A mass is bouncing from the end of an elastic string and its height h (metres) above the ground at time t (seconds) is given by $h = 1.6 + 0.4 \cos 2\pi t$

(a) Between what heights is the mass bouncing between?

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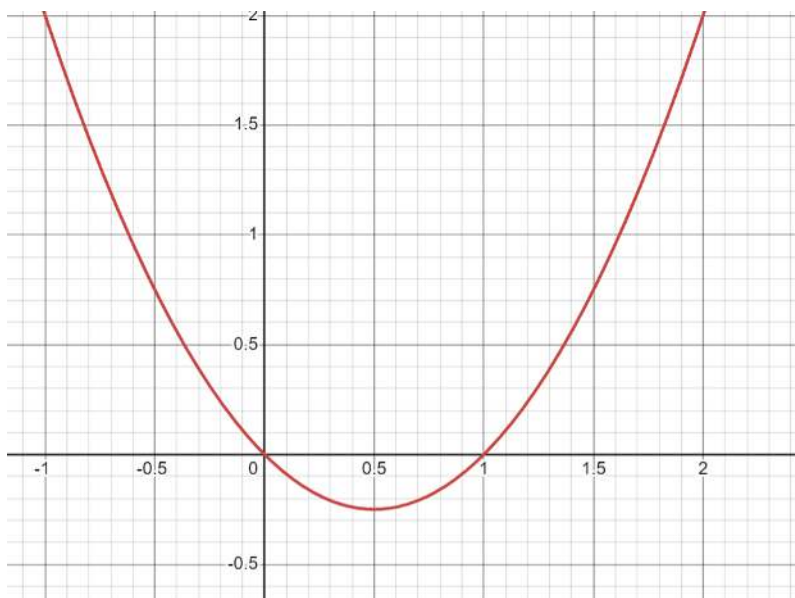
(b) What is the period between when the mass is at its highest point in consecutive bounces?

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Question 27

(a) The graph of $y = f(x)$ is shown below.



On the graph above, sketch the curve, $y = f(2x - 1)$, showing all important features.

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Question 28

A continuous random variable X has a probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{1}{\sqrt{x}}, & 1 \leq x \leq k \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k .

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Question 29

Evaluate $\int_1^2 12x(x^2 + 3)^5 dx$

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Question 30

Danielle completed two class tests this week.

Her results are shown in the table below (class mean and standard deviation shown for each subject).

Subject	Danielle's Mark	Mean	Standard Deviation
Mathematics	72	64	4
Chemistry	78	68	10

(a) In which test did Danielle perform better, relative to her class peers. **2**

(You must show working to support your answer)

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(b) Tina is in Danielle's class and sat the same class tests. Her z-score for Mathematics is 1.5. **1**

What mark did Tina record for Mathematics?

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Question 31

Find the equation of the curve if $\frac{d^2y}{dx^2} = -\frac{4}{(2x-1)^2}$.

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The curve passes through $(5, 2\ln 3)$ and the gradient of the tangent when $x = 1$ is 2.

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Question 32

Find the values of A and B so that $y = A \cos(2x) + B \sin(2x)$ satisfies the equation

$$\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} - 12y = \sin(2x) \text{ for any real values } A \text{ and } B.$$

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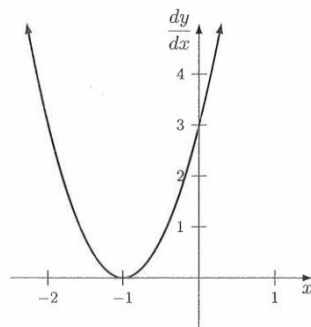
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Question 33

The gradient function $\frac{dy}{dx} = 3(x+1)^2$ of a curve is shown below.



What is the nature of the stationary point at $x = -1$ on $y = f(x)$?

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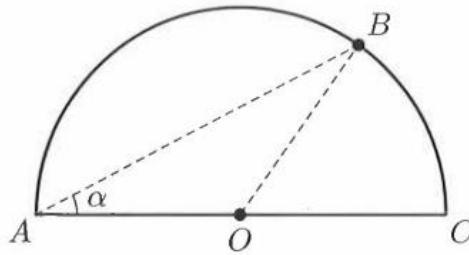
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Question 34

A semi-circular garden with radius 1 km is surrounded by a path. Angela wishes to go from one end of the garden to another (that is move from A to C passing through B) in the maximum possible time to enjoy the ambience of the garden and get some gentle exercise. She decides to walk in a straight line from A to B at a pace of 2 kilometres per hour, and then jog arc BC at a pace of 4 kilometres per hour.



Let $\angle BAC = \alpha$

- (a) Show that $\angle BOC = 2\alpha$, and hence show that Angela runs 2α kilometres. **2**

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- (b) If line $AB = 2 \cos \alpha$, show that the time taken for the total journey is $T = \cos \alpha + \frac{\alpha}{2}$. **1**

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- (e) Find the value for α for which $\frac{dT}{d\alpha}=0$, and determine whether this gives the maximum or minimum value of T.

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- (f) Find how long (to the nearest minute) it takes Angela to complete her journey if she proceeds with the above path (that is move from A to C passing through B).

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Question 35

If $x=6\cos(3t+\frac{3\pi}{4})$ and $\frac{d^2x}{dt^2}=-n^2x$. Find the value of n .

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Question 36

Jean makes a *quarterly* deposit of \$4000 into an account at the beginning of each quarter. The account pays an interest rate of 6% per annum, compounded monthly.

- (a) Show that Jean will have \$16612.16 in the account at the end of the first year. **2**

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- (b) Find the amount in the account at the end of the second year. **2**

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- (c) If Jean wishes to have \$250,000 in the account after 10 years, how much should his quarterly deposit be? **2**

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End of Exam