



SYDNEY BOYS HIGH SCHOOL

NESA Number:

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Name:

Maths Class: Circle

12U 12M

12Z2 12Z4

11A 11B 11M1 11M2

2024

YEAR 12
TASK 4
TRIAL HSC

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- All answers, unless otherwise stated, should be left in simplified, exact form
- Marks may **NOT** be awarded for messy or badly arranged work
- For questions in Section II, show ALL relevant mathematical reasoning and/or calculations
- Answers that rely totally on calculator technology may not necessarily receive full marks.

Total Marks: 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 38)

- Attempt all Questions in Section II
- Allow about 2 hours and 45 minutes for this section

Examiner: WB

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

1 Which of the following gives the domain of $y = \frac{1}{\sqrt{4-5x}}$?

A. $x > \frac{4}{5}$

B. $x < \frac{4}{5}$

C. $x \geq \frac{4}{5}$

D. $x \leq \frac{4}{5}$

2 The average mass of 6 bags of lollies is 0.4 kg.

What could be the mass of the heaviest bag?

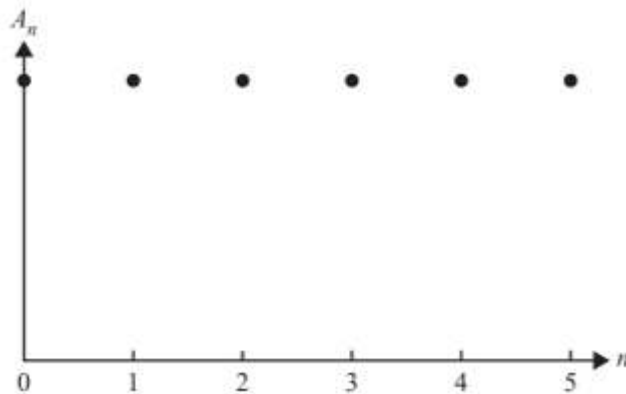
A. 0.3 kg

B. 0.45 kg

C. 2.5 kg

D. 3 kg

3 The graph below represents the value A_n , in dollars, of an annuity investment for five time periods.



Which of the following recurrence relations could match this graphical representation?

A. $A_0 = 200\,000$, $A_{n+1} = 1.015A_n - 2500$

B. $A_0 = 200\,000$, $A_{n+1} = 1.025A_n - 5000$

C. $A_0 = 200\,000$, $A_{n+1} = 1.035A_n - 5500$

D. $A_0 = 200\,000$, $A_{n+1} = 1.04A_n - 6000$

8 Let $f(x) = ax^m$ and $g(x) = bx^n$, where $a, b, m,$ and n are positive integers.

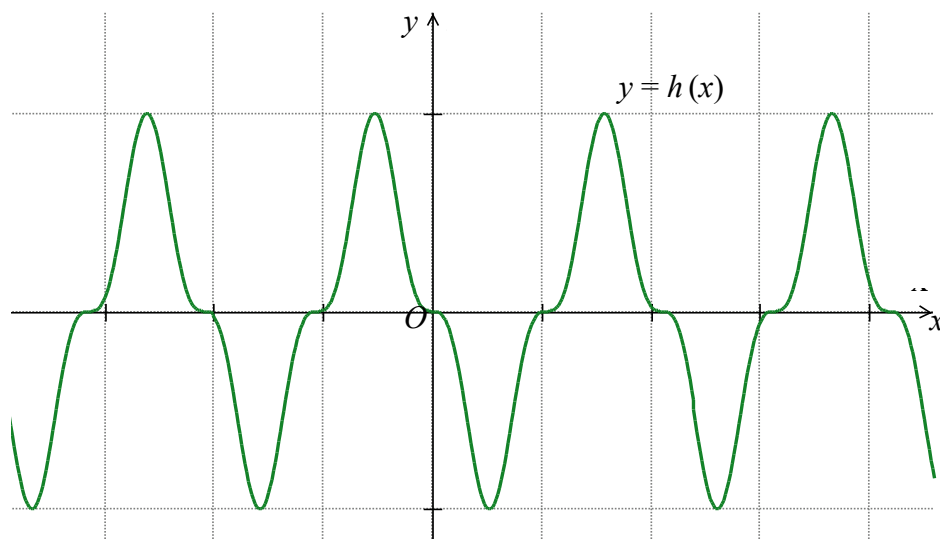
If $f'(x)$ is an antiderivative of $g(x)$, then which one of the following MUST be true?

- A. $\frac{m}{n}$ is an integer.
- B. $\frac{n}{m}$ is an integer.
- C. $\frac{a}{b}$ is an integer.
- D. $\frac{b}{a}$ is an integer.

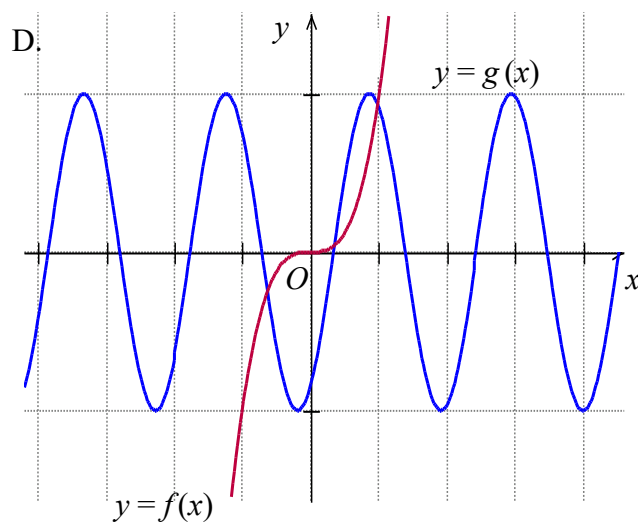
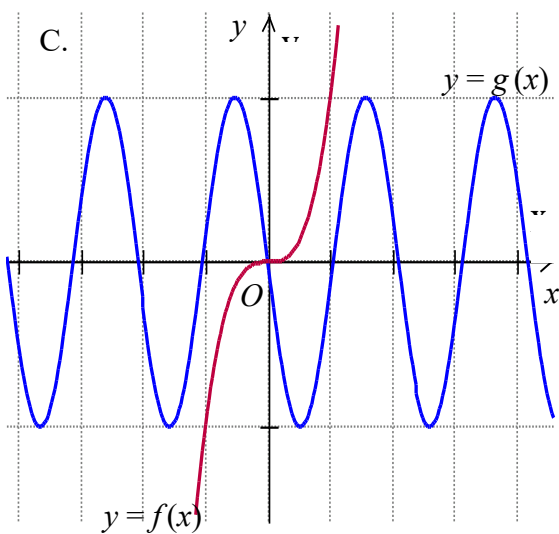
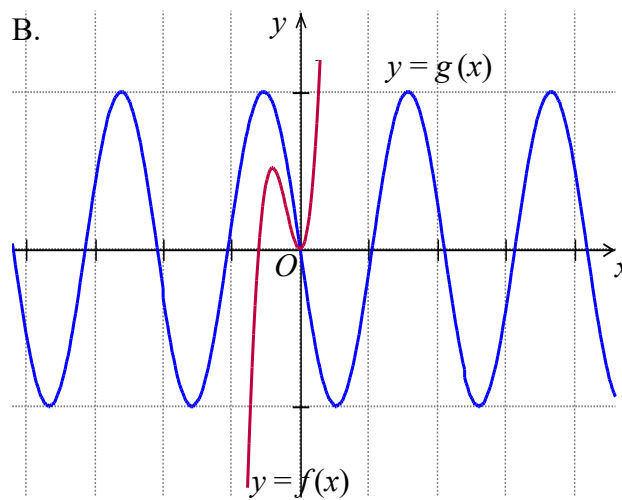
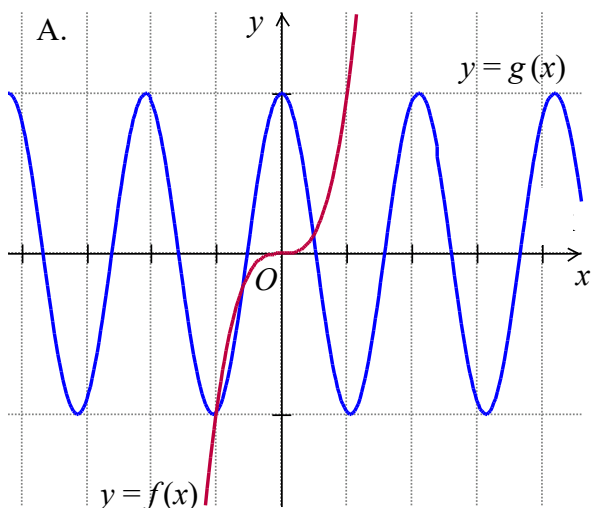
9 What is the maximum value of $\cos(2x + 30^\circ)(1 - \cos(2x + 30^\circ))$?

- A. $\frac{1}{4}$
- B. $\frac{1}{2}$
- C. $\frac{3}{4}$
- D. 1

10 The diagram shows the graph of the composite function $h(x) = f(g(x))$.



Which of the following is the best option to represent the graphs of the two functions $f(x)$ and $g(x)$?



End of Section I



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YEAR 12 Mathematics Advanced

HSC Task #4 (THSC)

Part A

Section II

Part A 20 marks Attempt Questions 11–17

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

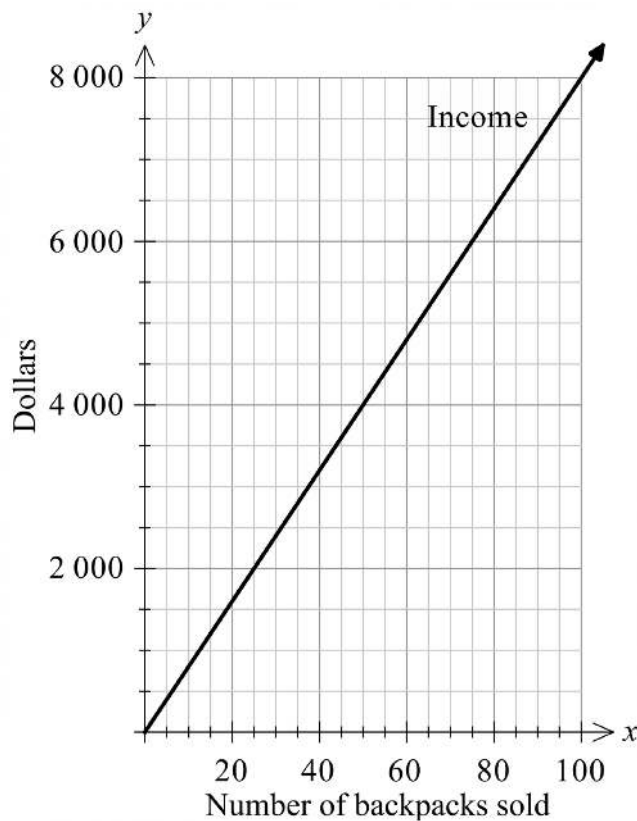
Question 11 (4 marks)

Jaime owns a company producing and selling backpacks. The income function is $y = 80x$, where x is the number of backpacks sold. The cost of producing these backpacks includes a set-up cost of \$4500 and additional costs of \$30 per backpack.

- (a) Write down the cost function in the form $y = mx + c$. 1

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- (b) Draw the cost function on the number plane below. 1



- (c) Hence, or otherwise, determine Jaime's break-even point?
(i.e. how many backpacks does he need to sell to cover his costs?) 2

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Question 12 (2 marks)

Find $\int 2x^3 + 6x^2 - 5x \, dx$

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Question 13 (2 marks)

Find $\int \frac{2}{4x + 1} \, dx$

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Question 14 (3 marks)

Evaluate $\int_0^1 4e^{2x} \, dx$

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Question 15 (2 marks)

Find $\int \sin 2x + \frac{1}{\sqrt{x}} \, dx$

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Question 16 (2 marks)

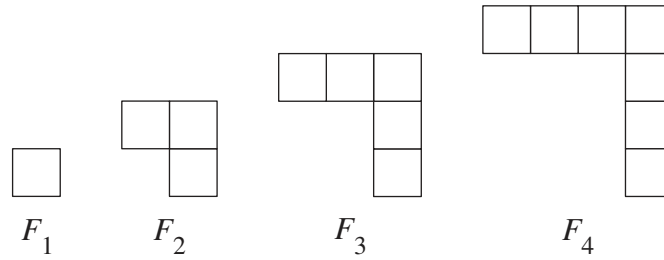
Find $\int \tan x \, dx$

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Question 17 (5 marks)

The diagram below shows the first four figures that form an arithmetic progression.



(a) Write down the constant difference. **1**

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(b) Show that $F_{88} = 175$. **2**

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(c) Show that the number of squares required to construct the first n figures is n^2 . **2**

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End of Part A



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YEAR 12 Mathematics Advanced

HSC Task #4 (THSC)

Part B

Section II

Part B **18 marks**
Attempt Questions 18 – 24

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 18 (3 marks)

Find the sum of the geometric progression $6 - 12 + 24 - \dots + 1536$. **3**

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Question 19 (2 marks)

The magnitude, I , of a current varies inversely with the resistance, R , in a circuit. **2**

If $R = 10$ when $I = 24$, find the equation of I in terms of R .

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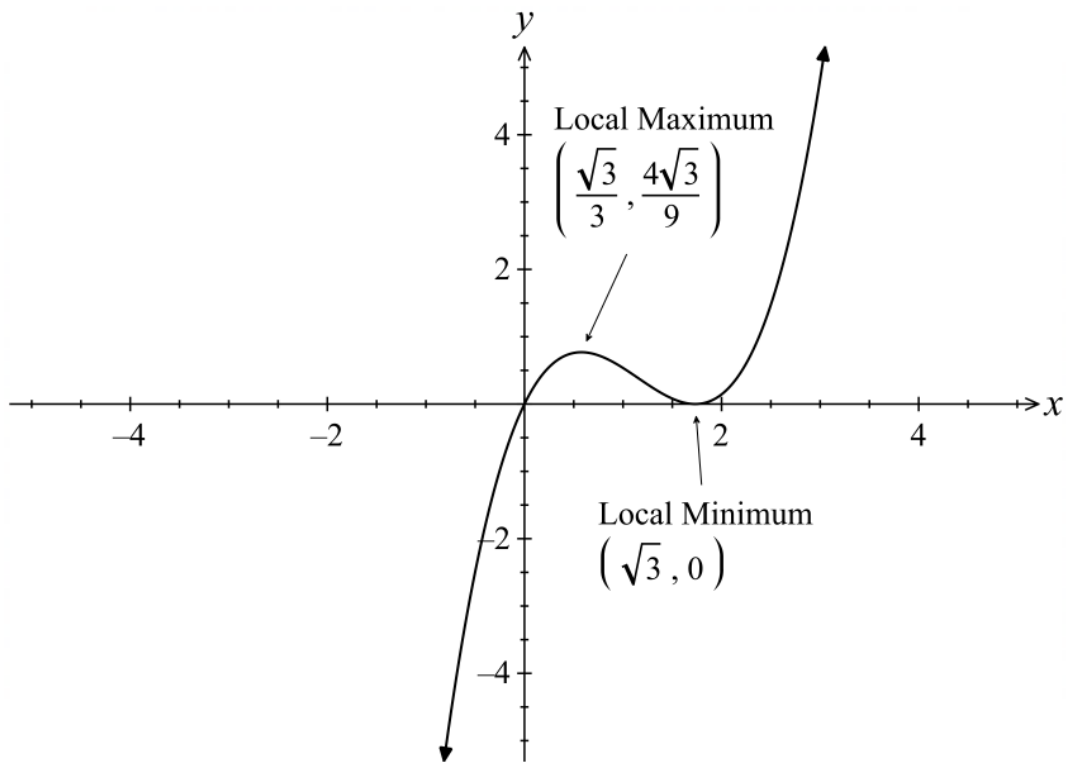
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Question 20 (2 marks)

The graph below is that of $f(x) = x^3 - (2\sqrt{3})x^2 + 3x$.

2



The functions $g(x)$ and $h(x)$ are defined by the following equations:

$$g(x) = f(-x)$$

$$h(x) = g(x + a)$$

The graph of $h(x)$ is tangential to the x -axis at the point $(5, 0)$.

Find the exact value of a .

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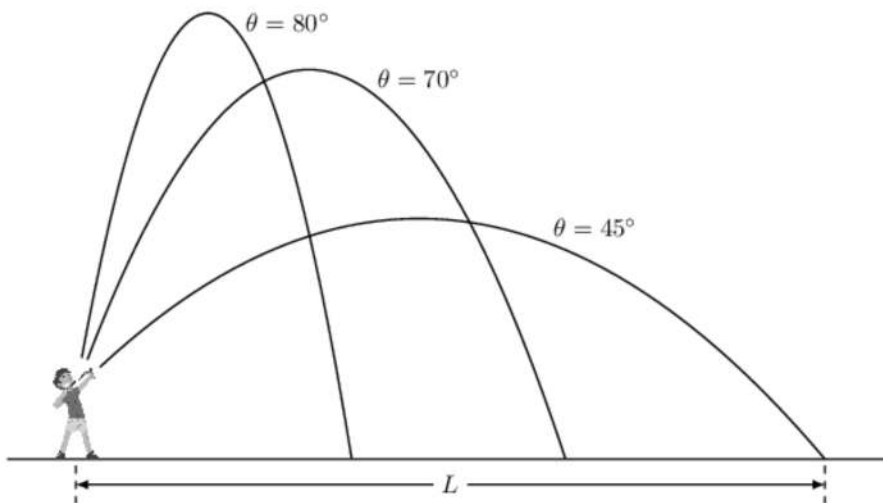
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Question 21 (3 marks)

Kai launches stones from a slingshot into the air. Each time he launches a stone, he records the angle, θ , in degrees at which the stone is launched, and the horizontal distance, L , in metres, which the stone travels from his shooting position to where it first lands.

The diagram below illustrates how the stones travel in the air when launched at different angles.



Kai analyses his results and concludes:

$$\frac{dL}{d\theta} = -0.08\theta + 3.3, \quad 0^\circ \leq \theta \leq 90^\circ.$$

- (a) Determine whether the graph of L versus θ is increasing or decreasing at $\theta = 50^\circ$ **1**

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Kai observes that when the angle is 30° , the sling stone will travel a horizontal distance of 90 m.

- (b) Find an expression for L in terms of θ . **2**

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Question 22 (2 marks)

Let $f(x)$ be a function such that $\int_0^3 f(x) dx = 8$.

(a) Find the value of $\int_0^3 2f(x) dx$. **1**

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(b) If $\int_c^d f(x-2) dx = 8$, find the values of c and d . **1**

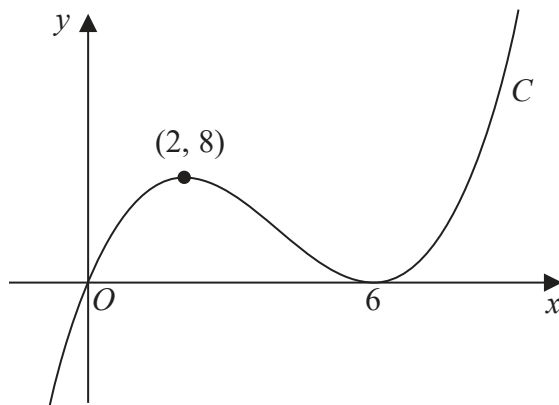
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Question 23 (2 marks)

Differentiate $5x^2 \tan\left(\frac{1}{x}\right)$. **2**

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Question 24 (4 marks)



The figure above shows a sketch of a curve C with equation $y = f(x)$, where $f(x)$ is a cubic expression in x .

The curve

- passes through the origin
- has a maximum turning point at $(2, 8)$
- has a minimum turning point at $(6, 0)$

(a) Write down the values of x for which $f'(x) < 0$. **1**

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The line with equation $y = k$, where k is a constant, intersects C at only one point.

(b) Find the set of values of k . **1**

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(c) Find the equation of C . **2**

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End of Part B



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YEAR 12 Mathematics Advanced

HSC Task #4 (THSC)

Part C

Section II

Part C 18 marks Attempt Questions 25 – 29

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 25 (3 marks)

The table below shows the future value interest factors for an annuity of \$1.
The contributions are made at the beginning of each year.

3

<i>Period</i>	<i>Interest rate per period</i>				
	1%	2%	3%	4%	5%
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019

Johnny deposits \$5000 into a savings account at the beginning of each year for 8 years.

For the first 6 years, the interest rate is 3% p.a., compounding annually.

During the 7th and 8th years, the interest rate is 2.5% p.a., compounding annually.

What is the total amount that has been added to Johnny's account at the end of 8 years?

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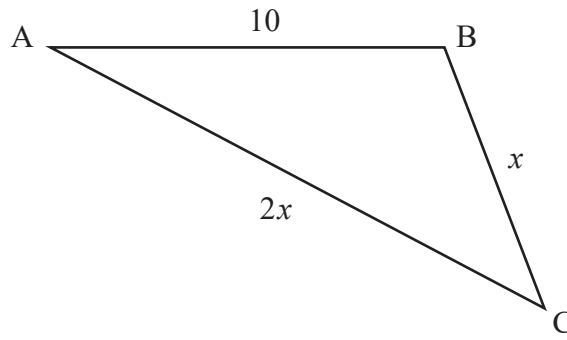
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Question 26 (4 marks)

The diagram below shows triangle ABC , with $AB = 10$, $BC = x$, $AC = 2x$, and $\cos \hat{C} = \frac{3}{4}$.

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Find the area of the triangle.

Leave your answer in the form $\frac{p\sqrt{q}}{2}$, where p and q are integers.

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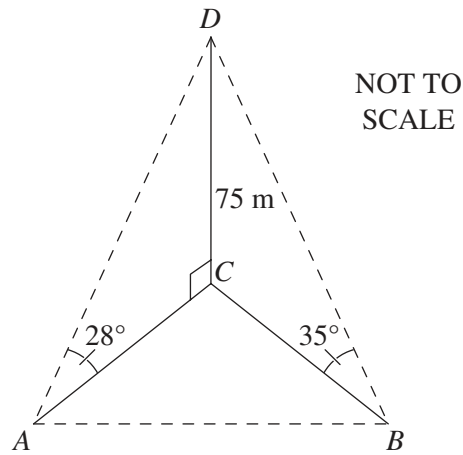
Question 27 (3 marks)

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The diagram shows a 75 m vertical tower, represented by line DC .

Points A and B are in the same horizontal plane as the base of the tower, point C .

The angle of elevation from point A to point D is 28° , and the angle of elevation from point B to point D is 35° .



The bearing of point C from point A is 050° T, and the bearing of point C from point B is 300° T.

Find the distance between points A and B , correct to the nearest metre.

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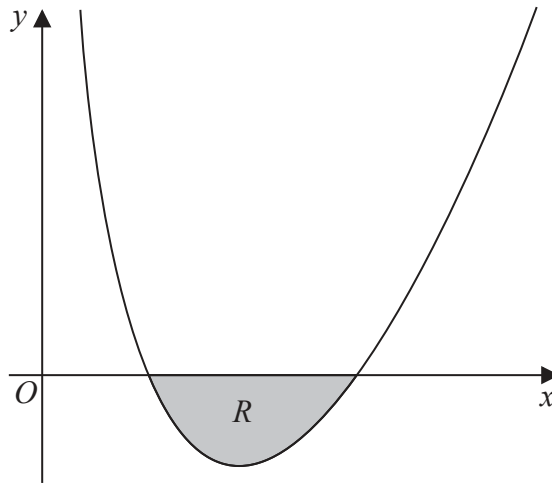
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Question 28 (4 marks)

The diagram below shows a sketch of part of a curve with equation

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$$y = \frac{(x-2)(x-4)}{4\sqrt{x}}, x > 0$$



The region R , shown in the diagram above, is bounded by the curve and the x -axis.

Find the exact area of R , writing your answer in the form $\frac{a\sqrt{2}+b}{5}$, where a and b are integers.

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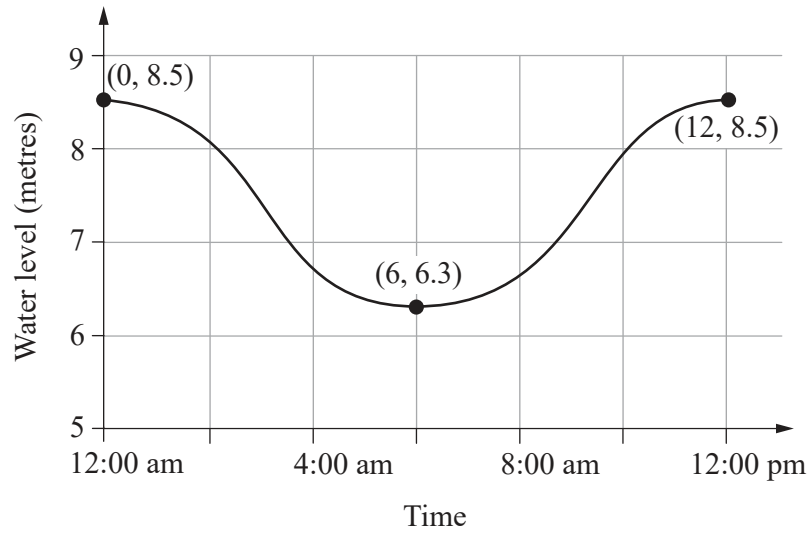
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Question 29 (4 marks)

The graph below shows the water level under a bridge over a 12-hour period.



- (a) Let $W(t)$ be the function that models the water level at a time t after 12.00 am.

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Find the exact value of n such that $W(t) = 1.1 \cos(nt) + 7.4$.

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Question 29 continues on page 24



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YEAR 12 Mathematics Advanced

HSC Task #4 (THSC)

Part D

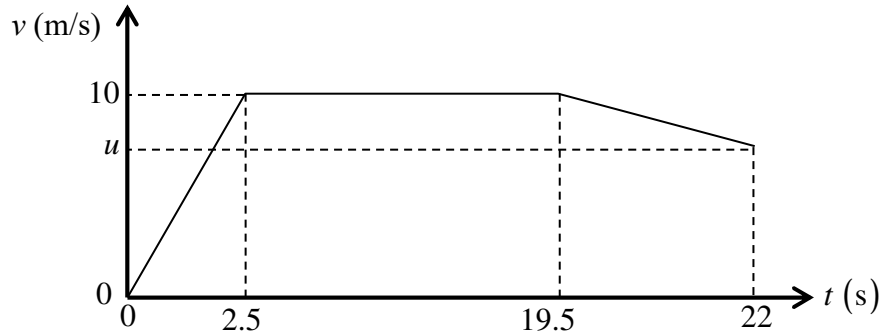
Section II

Part D 17 marks Attempt Questions 30 – 34

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 30 (5 marks)



The diagram above shows the speed-time graph of a sprinter running a 200 m race in a straight horizontal track.

The sprinter starts from rest and achieves, with a uniform acceleration, his top speed of 10 m/s in 2.5 s. He maintains this speed for 17 s when he experiences a cramp.

The sprinter then slows down, with a uniform acceleration. He crosses the finishing line with a speed of u m/s, 22 s after the start of the race.

By only using the graph above,

- (a) calculate the distance covered by the sprinter until he experienced a cramp.

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Question 30 continues on page 28

Question 30 (continued)

(b) show that $u = 4$ m/s

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(c) calculate the acceleration of the sprinter at the last part of the race i.e. $19.5 \leq t \leq 22$.

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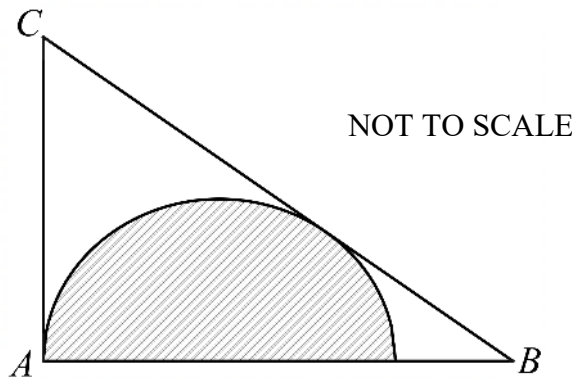
End of Question 30

Question 31 (3 marks)

In the diagram below, a semi-circle is inscribed in ΔABC , where $AC = 3$ cm, $AB = 4$ cm, and $BC = 5$ cm. Side BC is tangential to the semi-circle.

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Assuming that a radius of a circle is perpendicular to a tangent at its point of contact, find the exact area of the semi-circle.



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Question 32 (2 marks)

At a certain location, a biologist measures the width of a river to be 12 m.
He also records the depth of the river at 2 m interval widths as shown.

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Width (m)	0	2	4	6	8	10	12
Depth (m)	0.52	2.15	3.70	4.27	3.32	1.28	0.59

Use the trapezoidal rule to calculate an approximation to the cross-sectional area of the river.

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Question 33 (4 marks)

The function f is defined by

$$f(x) = \frac{e^{3x}}{4x^2 + k},$$

where k is a positive constant.

- (a) Find a function $g(x)$ such that $f'(x) = (12x^2 - 8x + 3k)g(x)$. **2**

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- (b) Given that the curve with equation $y = f(x)$ has at least one stationary point, find the range of possible values of k . **2**

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Question 34 (3 marks)

In a game, there are three different spinning wheels labelled A , B , and C .

Each of the three wheels is divided into small sectors, each of equal size.

In each sector a player can get either “Win Holiday “or “Win car”.

The information about the spinning wheels is as shown in the table.

Daniel is to play the game and he is to spin a wheel at random.

	Spinning Wheels		
	A	B	C
Win Holiday	6	2	8
Win Car	3	6	2
Total	9	8	10

(a) What is the probability that Daniel selects wheel A and Win Car?

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(b) Given that Daniel wins a car, what is the probability that he selected wheel C ?

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End of Part D



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YEAR 12 Mathematics Advanced

HSC Task #4 (THSC)

Part E

Section II

Part E 17 marks Attempt Questions 35 – 36

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 35 (5 marks)

(a) Prove that $(1 - \sin x)(\sec x + \tan x) \equiv \cos x$.

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(b) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x (1 - \sin x)(\sec x + \tan x) dx$.

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Question 36 (7 marks)

Georgie has borrowed \$25 000 from a bank with interest accrued and repayments made monthly. The following table lists the progress for the first 6 months of the loan.

Month Number	Opening Balance	Monthly interest	Monthly Repayment	End Balance
1	\$25,000.00	\$112.50	\$400.00	\$24,712.50
2	\$24,712.50	\$111.21	\$400.00	\$24,423.71
3	\$24,423.71	\$109.91	\$400.00	\$24,133.61
4	\$24,133.61	\$108.60	\$400.00	\$23,842.21
5	\$23,842.21	\$107.29	\$400.00	\$23,549.50
6	\$23,549.50	\$105.97	\$400.00	\$23,255.48
7		A		B

(a) What is the per annum rate of interest? 1

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(b) Determine the values of *A* and *B*. 2

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Question 35 continues on page 36

