



Student Number

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# Sydney Girls High School

## 2024

TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

### Mathematics Advanced

- General Instructions**
- Reading time – 10 minutes
  - Working time – 3 hours
  - Write using black pen
  - Calculators approved by NESA may be used
  - A reference sheet is provided at the back of this paper
  - For questions in Section II, show relevant mathematical reasoning and/ or calculations

**Total Marks:****100****Section I – 10 marks (pages 3–6)**

- Attempt Questions 1–10
- Allow about 15 minutes for this section

**Section II – 90 marks (pages 9–34)**

- Attempt Questions 11–36
- Allow about 2 hours and 45 minutes for this section

**THIS IS A TRIAL PAPER ONLY**

It does not necessarily reflect the format or the content of the 2020 HSC Examination Paper in this subject.

Question	M.C	11-20	21-24	25-29	30-34	35-37	Total
Marks	/10	/31	/14	/16	/14	/15	/100

**Section I**

**10 marks**

**Attempt Questions 1-10**

**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for questions 1-10

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**1** What is  $\int x^2(2x^3 + 1)^5 dx$ .

A.  $\frac{x^3}{18}(2x^3 + 1)^6 + C$

B.  $\frac{1}{6}(2x^3 + 1)^6 + C$

C.  $\frac{x^2}{36}(2x^3 + 1)^6 + C$

D.  $\frac{1}{36}(2x^3 + 1)^6 + C$

**2** What is the domain of the function  $\ln(x + 2) + \sqrt{9 - x}$ ?

A.  $(-2, 9)$

B.  $[-2, 9]$

C.  $(-2, 9]$

D.  $[-2, 9)$

3 Find  $\int (e + \sin 2x) dx$ .

A.  $\frac{e^2}{2} - \frac{1}{2} \cos 2x + c$

B.  $ex - \frac{1}{2} \cos 2x + c$

C.  $\frac{e^2}{2} - 2 \cos 2x + c$

D.  $ex - 2 \cos 2x + c$

4 Which of the following is an expression for  $\frac{d}{dx} \log_7 4x$ ?

A.  $\frac{7}{4x \ln 7}$

B.  $\frac{1}{x \ln 7}$

C.  $\frac{4}{x \ln 7}$

D.  $\frac{\ln 7}{\ln 4x}$

5 Consider the series  $\log_a 54 + \log_a 18 + \log_a 6 + \dots$

Which of the following statements best describe the series?

A. A geometric series with common ratio of  $-3$ .

B. A geometric series with common ratio of  $-\log_a 3$ .

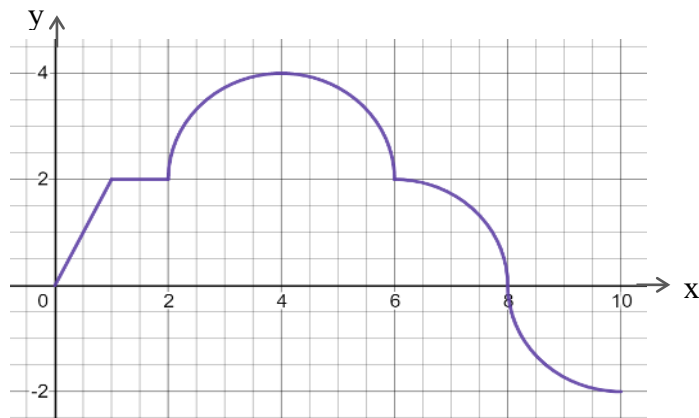
C. An arithmetic series with common difference  $-3$

D. An arithmetic series with common difference of  $-\log_a 3$ .

- 6 Bella recently did a Maths test and an English test. The class scores on each test were normally distributed. The mean on the English test was 60%, and the standard deviation was 10%. Bella's mark was 72%.  
The mean on the Maths test was 73%, and the standard deviation was 5%.  
What was Bella's mark in the Maths test, if her z-scores in English and Maths were equal?

- A. 72%
- B. 78%
- C. 79%
- D. 83%

- 7 The diagram shows the graph of  $y = f(x)$ , which is made of two-line segments, a semi-circle and two quarters of a circle.



Which of the following is the value of  $\int_0^{10} f(x)dx$ ?

- A.  $2\pi + 11$
- B.  $4\pi + 11$
- C.  $4\pi + 12$
- D.  $2\pi + 12$

8 Let  $h(x)=f(g(x))$ , with  $f(4)=1$ ,  $f'(4)=6$ ,  $g(2)=4$  and  $g'(2)=4$ . What is the equation of the tangent to the graph of  $y=h(x)$  at  $x=2$ ?

- A.  $y = x - 6$
- B.  $y = 24x - 2$
- C.  $y = 6x - 11$
- D.  $y = 24x - 47$

9 Simplify  $\cot(270^\circ + x^\circ)\cos(-x^\circ) + \cos(90^\circ + x^\circ)$

- A.  $-2 \sin x$
- B.  $\sin x + \cos x$
- C.  $0$
- D.  $2 \sin x$

10 A biased four-sided die is rolled.

The following table gives the probability of each score.

Score	1	2	3	4
Probability	$\frac{7}{16}$	$\frac{5}{16}$	$3^k$	$3^{2k+1}$

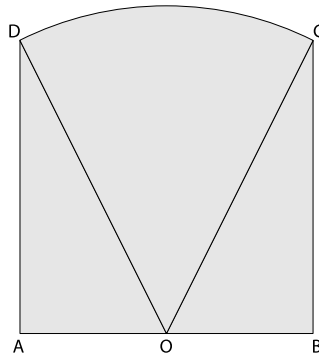
What is the probability of rolling 3?

- A.  $\frac{1}{4}$
- B.  $\frac{1}{6}$
- C.  $\frac{1}{8}$
- D.  $\frac{1}{5}$

**Question 11 (5 marks)**

A shape consists of three sides of a square together with a sector,  $ODC$ , where  $O$  is the midpoint of  $AB$ , as shown below (not to scale).

$$DA = AB = BC = 2 \text{ cm}$$



- a) Find the exact length of  $DO$ . (1)

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- b) If  $\angle DOC = \theta$ , show that  $\cos \theta = \frac{3}{5}$ . (2)

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- c) Hence find the perimeter of the shape  $ABCD$  correct to 2 decimal places. (2)

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**Question 12 (3 marks)**

The following table shows the probability distribution of a discrete random variable with an expected value of two.

$x$	0	1	2	3	4
$P(X = x)$	0.2	0.1	$a$	$b$	0.2

- a) Calculate the values of  $a$  and  $b$ . (2)

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- b) Hence calculate the standard deviation, correct to two decimal places. (1)

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**Question 13 (2 marks)**

Show that the curve  $y = 3x^2 - 5 \ln x$  is concave up for all values of  $x > 0$ . (2)

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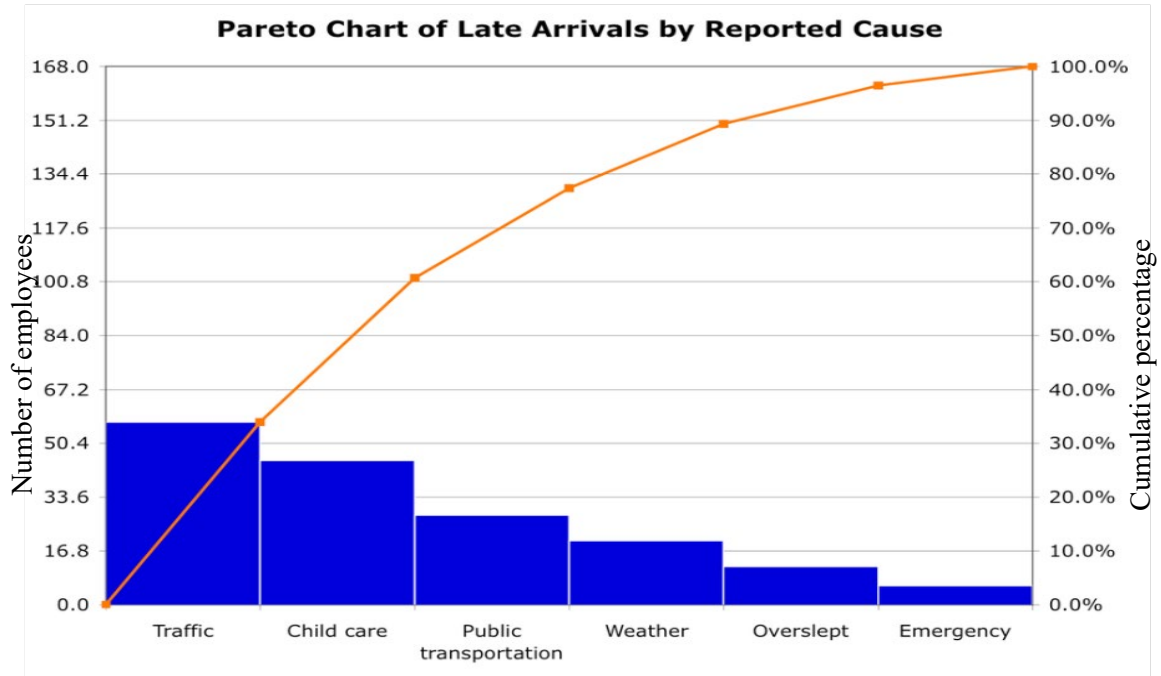
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**Question 14 (1 marks)**

A company owner collected data related to the reasons given by employees for being late to work.

The Pareto chart shows the data collected.



Estimate the percentage of employees that were late because of public transport. (1)

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**Question 15 (2 marks)**

Differentiate  $e^{4x} \ln(x^2)$ . (2)

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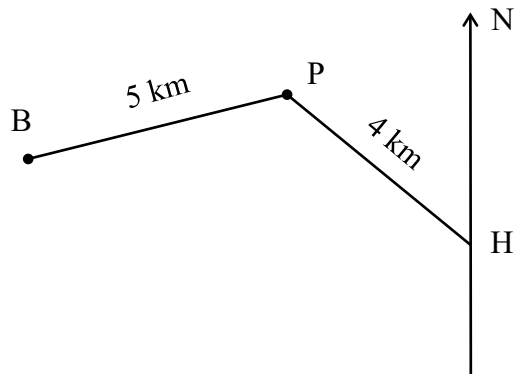
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**Question 16 (5 marks)**

Kate starts riding her bike from her house H. She rides 4 km on a bearing  $320^\circ$  to the park P then rides 5 km on a bearing  $250^\circ$  to the beach B.



i) Show that the angle HPB is  $110^\circ$ . (1)

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b) Find the distance BH correct to three decimal places. (2)

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c) Find the bearing of Kate's house H from the beach B. (2)

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**Question 17 (3 marks)**

If  $0 \leq \theta \leq 2\pi$ , solve  $\sqrt{2} \cos\left(2\theta - \frac{\pi}{3}\right) - 1 = 0$ . **(3)**

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**Question 18 (2 marks)**

Explain why the geometric series shown below does not have a limiting sum. **(2)**

$$3 + \frac{3}{\sqrt{3}-1} + \frac{3}{(\sqrt{3}-1)^2} + \dots$$

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**Question 19 (2 marks)**

The future value of an annuity when \$1 is invested at the start of each period is shown in the table below.

Future value of \$1						
Number of periods	Interest rate per period					
	1%	2%	3%	4%	5%	6%
1	\$1.0100	\$1.0200	\$1.0300	\$1.0400	\$1.0500	\$1.0600
2	\$2.0301	\$2.0604	\$2.0909	\$2.1216	\$2.1525	\$2.1836
3	\$3.0604	\$3.1216	\$3.1836	\$3.2465	\$3.3101	\$3.3746
4	\$4.1010	\$4.2040	\$4.3091	\$4.4163	\$4.5256	\$4.6371
5	\$5.1520	\$5.3081	\$5.4684	\$5.6330	\$5.8019	\$5.9753
6	\$6.2135	\$6.4343	\$6.6625	\$6.8983	\$7.1420	\$7.3938

Jamie deposits \$800 into a savings account at the start of each month for 6 months. After the 6th deposit, Jamie stops making deposits but leaves the money in the savings account until exactly 12 months from the first deposit.

The interest rate on her savings account is 12% per annum, compounded monthly.

What is the balance of Jamie's savings account at the end of the 12 months? (2)

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**Question 20 (6 marks)**

Given a function:

$$f(x) = \begin{cases} \frac{3x}{4}(2-x) & \text{for } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- i) Prove that  $f(x)$  represents a probability density function. **(2)**

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- ii) State the mode of the distribution. **(1)**

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- iii) Explain why the median is equal to the mode in this distribution. **(1)**

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- iv) Find  $P(X > 1.5)$ . **(2)**

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# Mathematics Advanced

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## Section II

### Answer Booklet 2

#### Booklet 2 - Attempt Questions 21 – 37

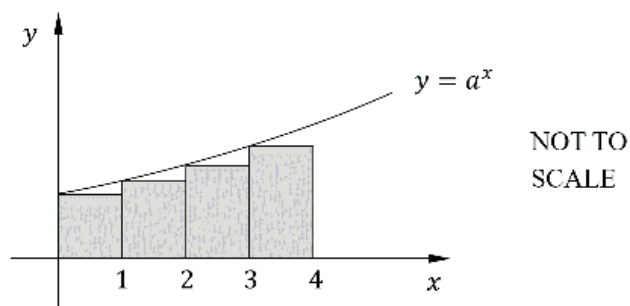
**Instructions**

Answer the questions in the spaces provided.

- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the question paper. If you use this space, clearly indicate which question you are answering.

**Question 21 (4 marks)**

The diagram shows the graph of  $y = a^x$ , where  $a > 1$ . Also shown on the diagram are 4 inner rectangles of width 1 unit.



- i) Show that the area of the 4 rectangles is  $\frac{a^4 - 1}{a - 1} u^2$ . (2)

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- ii) By finding the area under the curve  $y = a^x$  between  $x = 0$  and  $x = 4$ , and using the result in part (i), prove that  $\ln a < a - 1$ . (2)

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**Question 22 (4 marks)**

A continuous random variable  $X$  has the following probability density function:

$$f(x) = \begin{cases} 3^{-x} \ln 3 & \text{for } x \geq 0 \\ 0 & \text{for all other values of } x \end{cases}$$

- i) Find the cumulative distribution function  $F(x)$ . (2)

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- ii) Find the exact value of the third quartile. (2)

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**Question 23 (2 marks)**

Evaluate  $\int_1^6 |2x - 6| dx$ . (2)

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**Question 25 (2 marks)**

Differentiate  $\ln\left(3\sin\frac{x}{2}\right)^3$ . **(2)**

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**Question 26 (2 marks)**

If  $\int_0^3 g(x) dx = 11$ , find  $\int_0^3 \left(\frac{1}{2}g(x) + 3x\right) dx$ . **(2)**

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**Question 27 (3 marks)**

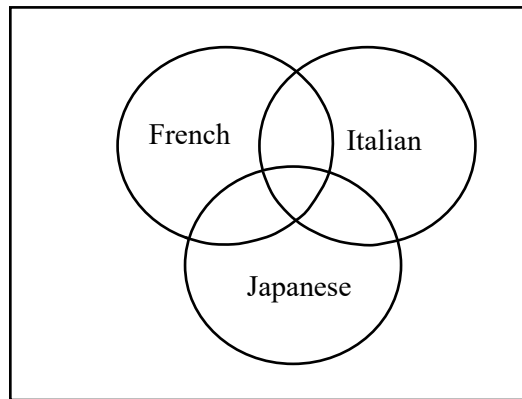
Last Sunday, 45 people were surveyed about three types of restaurants that they prefer to attend on their next birthday.

Of them, 26 chose French, 22 chose Italian, 18 Japanese.

It is also found that 11 chose French and Italian, 10 chose French and Japanese and 7 chose Italian and Japanese.

Only 4 people chose all three types of restaurants.

a) Represent this information in the Venn Diagram below. (1)



i) A person from those who were surveyed last Sunday is to be selected at random.  
What is the probability that this person chose French or Italian but not Japanese? (1)

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ii) Two people are chosen at random, what is the probability of both chose French only? (1)

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**Question 28 (3 marks)**

Prove that  $(\cot \theta + \operatorname{cosec} \theta)^2 = \frac{1 + \cos \theta}{1 - \cos \theta}$ . (3)

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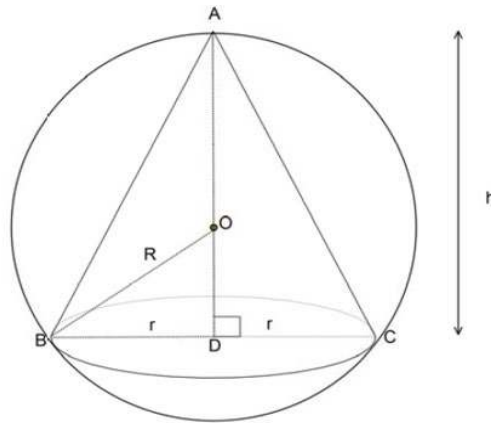
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**Question 29 (6 marks)**

Consider the following diagram with the cone inscribed in a sphere.



- i) If  $r$  is the radius of the cone show that  $r^2 = 2hR - h^2$ . (1)

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- ii) Show that the volume of the cone that can be inscribed in a sphere of radius  $R$  is given by  $V = \frac{1}{3}\pi(2h^2R - h^3)$  where  $h$  is the height of the inscribed cone. (1)

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Question 29 continued on the next page

***Question 29 continued***

iii) Show that the volume of the largest cone is  $\frac{8}{27}$  of the volume of the sphere. (4)

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**Question 30 (2 marks)** (The table on page 27 can be used for this question)

Madelyn is a long jumper training for the national age championships. To be able to compete at the national age championships her longest jump must be able to be beaten by less than 0.17% of the general population.

The distance jumped by the general population is normally distributed with a mean of 3.48 metres and standard deviation of 1.08 metres.

What is the minimum distance that Madelyn needs to jump, correct to the nearest centimetre?

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**Table : The standard normal distribution**

The table below provides some values of the probabilities for the standard normal distribution.

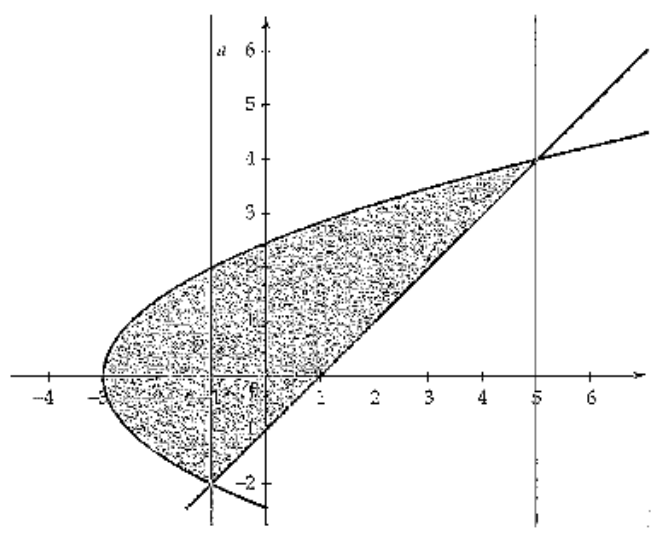
$$\text{i. e. } \Phi(z) = P(Z \leq z) = \int_{-\infty}^z \phi(t)dt$$

<b>z</b>	<b>+0.00</b>	<b>+0.01</b>	<b>+0.02</b>	<b>+0.03</b>	<b>+0.04</b>	<b>+0.05</b>	<b>+0.06</b>	<b>+0.07</b>	<b>+0.08</b>	<b>+0.09</b>
<b>0.0</b>	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
<b>0.1</b>	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56360	0.56749	0.57142	0.57535
<b>0.2</b>	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
<b>0.3</b>	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
<b>0.4</b>	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
<b>0.5</b>	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
<b>0.6</b>	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
<b>0.7</b>	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
<b>0.8</b>	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
<b>0.9</b>	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
<b>1.0</b>	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
<b>1.1</b>	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
<b>1.2</b>	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
<b>1.3</b>	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
<b>1.4</b>	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
<b>1.5</b>	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
<b>1.6</b>	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
<b>1.7</b>	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
<b>1.8</b>	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
<b>1.9</b>	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
<b>2.0</b>	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
<b>2.1</b>	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
<b>2.2</b>	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
<b>2.3</b>	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
<b>2.4</b>	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
<b>2.5</b>	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
<b>2.6</b>	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
<b>2.7</b>	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
<b>2.8</b>	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
<b>2.9</b>	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
<b>3.0</b>	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
<b>3.1</b>	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
<b>3.2</b>	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
<b>3.3</b>	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
<b>3.4</b>	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
<b>3.5</b>	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983



**Question 31 (4 marks)**

Determine the area of the region enclosed by  $y^2 = 2x + 6$  and  $y = x - 1$ .



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**Question 32 (4 marks)**

Madison is learning to drive. Her first lesson is 10 minutes long. Her second lesson is 15 minutes long. Each subsequent lesson is 5 minutes longer than the previous lesson.

- i) How long will Madison's fifteenth lesson be? **(1)**

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- ii) How many hours of lessons will Madison have completed after her fifteenth lesson? **(1)**

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- iii) During which lesson will Madison have completed a total of 40 hours of driving lessons? **(2)**

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**Question 33 (2 marks)**

Solve the equation  $\log_2(2 - 2x) = \log_{\sqrt{2}}(1 - x)$ . (2)

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**Question 34 (2 marks)**

Differentiate  $y = (\sin^3 2x^\circ)$ . (2)

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**Question 35 (7 marks)**

Marge borrows \$50 000 in order to buy a car. The loan attracts an interest of just 0.5% per month. The company also offers an interest free period for the first six months. However, the first payment is due at the end of the first month. Marge agrees to repay the loan over 10 years, by making 120 equal monthly repayments of \$ $M$  . Let  $A_n$  be the amount owing at the end of  $n$ th repayment, then

i) Show that  $A_6 = 50\,000 - 6M$  (1)

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ii) Show that  $A_8 = (50\,000 - 6M) \times 1.005^2 - M(1.005 + 1)$  (2)

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iii) Hence derive the expression for  $A_{120}$  in simplest form. (2)

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iv) Find the value of the monthly repayment to the nearest cent. (2)

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**Question 36 (5 marks)**

A particle is moving in a straight line. It's displacement,  $x$  metres, from the origin,  $O$  at time  $t$  seconds, where  $t \geq 0$ , is given by  $x = 4 + 3te^{-2t}$

i) Show the velocity of the particle is given by  $\frac{dx}{dt} = 3e^{-2t}(1 - 2t)$ . **(2)**

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ii) Show that the particle is at rest when  $t = \frac{1}{2}$ . **(1)**

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iii) Find the greatest possible total distance the particle could travel. **(2)**

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**Question 37 (3 marks)**

Sally contributes \$7200 each year into a superannuation fund for the first 15 years of her working life. For the next 20 years until retirement, she decides to increase this, and invest a total of \$10000 each year. Each contribution is paid at the beginning of the year.

If the investment earns 7% per annum paid yearly over whole period, how much will her investment be upon retiring? (Show all your working) **(3)**

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