EPPING BOYS HIGH SCHOOL

2022 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a black/blue pen
- Calculators approved by NESA may be used
- Reference sheets are provided separately
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write NESA student number on the question booklet and also on every answer booklet

Total marks: 70

SECTION I – 10 marks (pages 2 – 6)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

SECTION II – 60 marks (pages 7 – 11)

- Attempt Questions 11 - 14
- Allow about 1 hour and 45 minutes for this section

Student NESA number:____

Please tick:

	12MX1-1	**Mr.Param**
	12MX1-2	**Mr.Xu**
	12MX1-3	**Mr.Yuen**

MCQ	**Q11**	**Q12**	**Q13**	**Q14**	**TOTAL**	**100**
10	**15**	**15**	**15**	**15**	**70**	

SECTION I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. Which of the following set of numbers could be the roots of the polynomial equation

$x^3 + kx^2 - 45x + 36 = 0$?

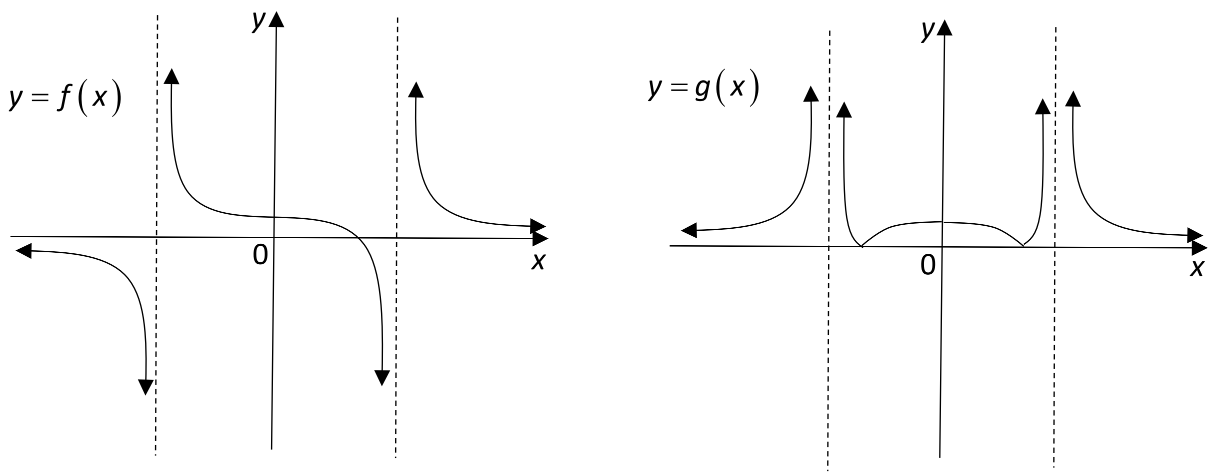
A. $3, -12, 1$

B. $6, -6, 1$

C. $4, -9, -1$

D. $18, -2, -1$

2. Consider the following graphs $y = f(x)$ and $y = g(x)$.



What could be the possible relationship between the two graphs?

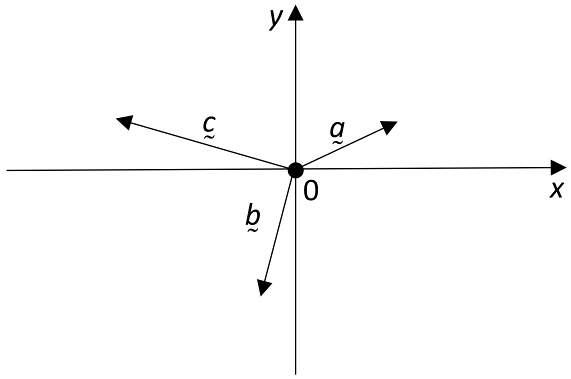
A. $g(x) = |f(x)|$

B. $g(x) = |f(|x|)|$

C. $g(x) = \sqrt{f(x)}$

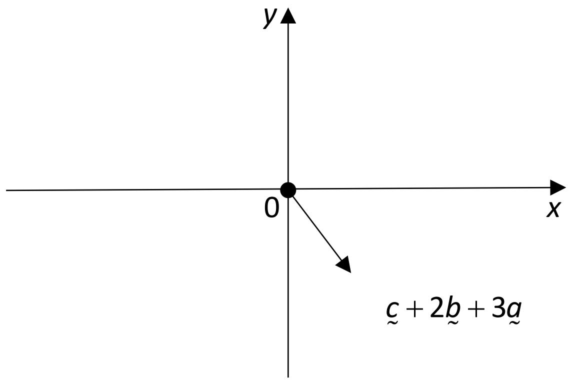
D. $g(x) = \dfrac{1}{|f(x)|}$

3. Three vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are shown in the following diagram.

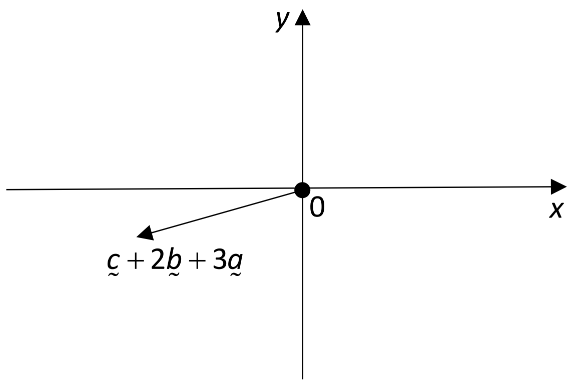


Which of the following diagrams best represents the vector $\underset{\sim}{c} + 2\underset{\sim}{b} + 3\underset{\sim}{a}$?

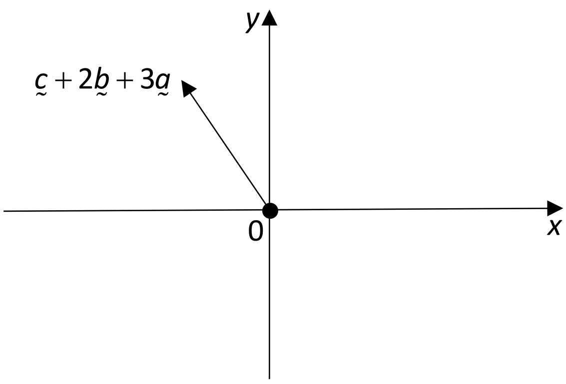
A.

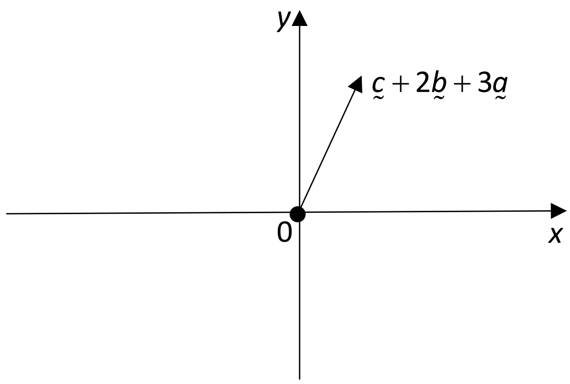


B.



C.





4. The exact value of $\int_2^4 \frac{6}{\sqrt{16 - x^2}}\, dx$ is

A. π

B. 2π

C. 3π

D. $\frac{\pi}{3}$

5. What is the constant term in the binomial expansion $\left(3x^2 - \frac{4}{x}\right)^{12}$?

A. ${}^{12}C_8$

B. $-\ {}^{12}C_8 \times 3^4 \times 4^8$

C. ${}^{12}C_8 \times 3^4 \times 4^8$

D. ${}^{12}C_6 \times 3^6 \times 4^6$

6. A projectile is fired from a point with initial velocity v m/s at an angle θ to the horizontal.

Its position vector after time t seconds is given by $\underset{\sim}{r}(t) = (36t)\underset{\sim}{i} + \left(50 + 48t - 5t^2\right)\underset{\sim}{j}$.

The initial velocity v and the angle of projection θ, respectively are:

A. $v = 30$ and $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

B. $v = 50$ and $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

C. $v = 60$ and $\theta = \tan^{-1}\left(\frac{3}{4}\right)$

D. $v = 60$ and $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

7. A fancy solid is formed by rotating the graph $y = \frac{1}{\pi}\cos^{-1}(x-1)$ about the y-axis.

Which of the following integral expressions will give the correct volume of the solid?

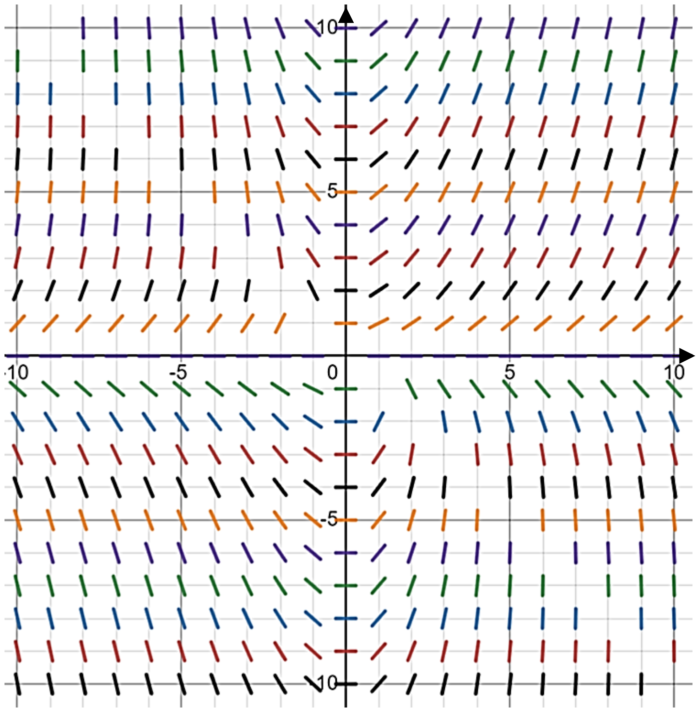
A. $\pi \int_0^{\pi} \left[\cos(\pi y) + 1\right]^2 \, dy$

B. $\pi \int_0^{1} \left[\cos(\pi y) + 1\right]^2 \, dy$

C. $\pi \int_0^{1} \left[\cos(\pi y) + 1\right] \, dy$

D. $\pi \int_0^{1} \left[\cos^2(\pi y) + 1\right] \, dy$

8. Consider the slope field below.



Which of the following differential equations is represented by the above slope field?

A. $\frac{dy}{dx} = \frac{x}{y}$

B. $\frac{dy}{dx} = \frac{x-y}{x+y}$

C. $\frac{dy}{dx} = \frac{x+y}{x-y}$

D. $\frac{dy}{dx} = \frac{xy}{x+y}$

9. What is the vector projection of $\underset{\sim}{a} = \begin{pmatrix} -8 \\ -6 \end{pmatrix}$ onto $\underset{\sim}{b} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$?

A. $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$

B. $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

C. $\begin{pmatrix} 8 \\ -18 \end{pmatrix}$

D. $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$

10. A bag contains 10 red, 12 blue, 20 yellow and 24 green marbles. They are all identical except for their colour. What is the minimum number of marbles that must be picked at random to ensure that there 15 marbles of the same colour?

A. 60

B. 50

C. 51

D. 61

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a separate answer booklet.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

a) i) Show that $(x-2)$ is a factor of the polynomial $P(x)=x^3-3x^2+4$. 1

ii) Hence, express $P(x)$ as a product of linear factors. 2

iii) Using the result above, solve $x^3-3x^2+4\leq 0$. 1

b) The letters of the word **INTEGRAL** are arranged in a line.

If one of these arrangements is selected at random, what is the probability that

i) the vowels are in the same position? 1

ii) the vowels are together? 2

c) Sketch the graphs of $y=e^x$ and $y=\cos x$ on the same set of axes for $0\leq x\leq\frac{\pi}{2}$.

i) Calculate the exact area bounded between the two curves and the line $x=\frac{\pi}{2}$. 2

ii) Calculate the exact volume of the solid when this area is rotated about the $x-$axis. 2

d) If $f(x)=g(x)-\ln\left[g(x)+1\right]$ 4

i) Show that $f'(x)=\dfrac{g(x)g'(x)}{1+g(x)}$

ii) Hence or otherwise evaluate $\displaystyle\int_0^{\frac{\pi}{2}}\frac{\cos x\sin x}{1+\sin x}\,dx$

Question 12 (15 marks)

a) Find the solution of the differential equation $\frac{dy}{dx} = 1 + 4y^2$, given that $y(\pi) = -1$. 2

b) Using mathematical induction, prove that, for all positive integers n, 3

$$\frac{5}{6} + \frac{1}{4} + \frac{7}{60} + \ldots\ldots\ldots\ldots\ldots + \frac{n+4}{n(n+1)(n+2)} = \frac{3}{2} - \frac{n+3}{(n+1)(n+2)}$$

c) i) Show that $\tan 2x + \tan x = \frac{\sin 3x}{\cos 2x \cos x}$ 2

ii) Hence solve the equation $\tan 2x + \tan x = 0$ for $0 < x < \frac{\pi}{2}$. 1

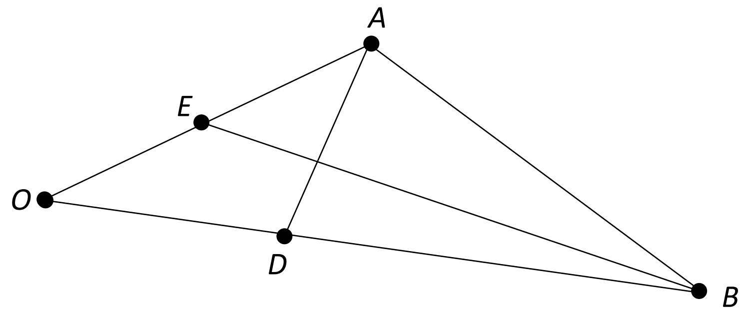
d) Express $\sqrt{7}\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. 2

e) In ΔOAB, the length of OB is twice the length of OA. E is the midpoint of OA.

D is a point on OB such that $OD : DB$ = 2:3.

It is also given that AD is **perpendicular** to BE.

Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.



i) Show that $\overrightarrow{DA} = \underset{\sim}{a} - \frac{2}{5}\underset{\sim}{b}$ and $\overrightarrow{EB} = \underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$. 2

ii) Hence, by considering the dot product of $\overrightarrow{DA}$ and $\overrightarrow{EB}$, or otherwise, 3

show that $\cos\angle AOB = \frac{7}{8}$

Question 13 (15 marks)

a) Let $f(x) = x - \frac{1}{2}x^2$ for $x \le 1$

This function has an inverse, $f^{-1}(x)$.

i) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes. 2

ii) Find an expression for $f^{-1}(x)$. 2

iii) Evaluate $f^{-1}\left(\frac{3}{8}\right)$. 1

b) Find y in terms of x if $\frac{dy}{dx} = (1+y)^2 \tan^2 x \sec^2 x$, given that when $x = \frac{3\pi}{4}$, $y = 2$. 3

c) An aeroplane needs to fly due north from one city to another at a speed of 500 km/h.

However, a strong wind blows constantly at 60 km/h from a direction $S30°W$.

Assume that the aeroplane had to fly at *v* km/h to compensate for the wind.

i) By drawing a vector diagram, find the value of *v* to the nearest km/h. 2

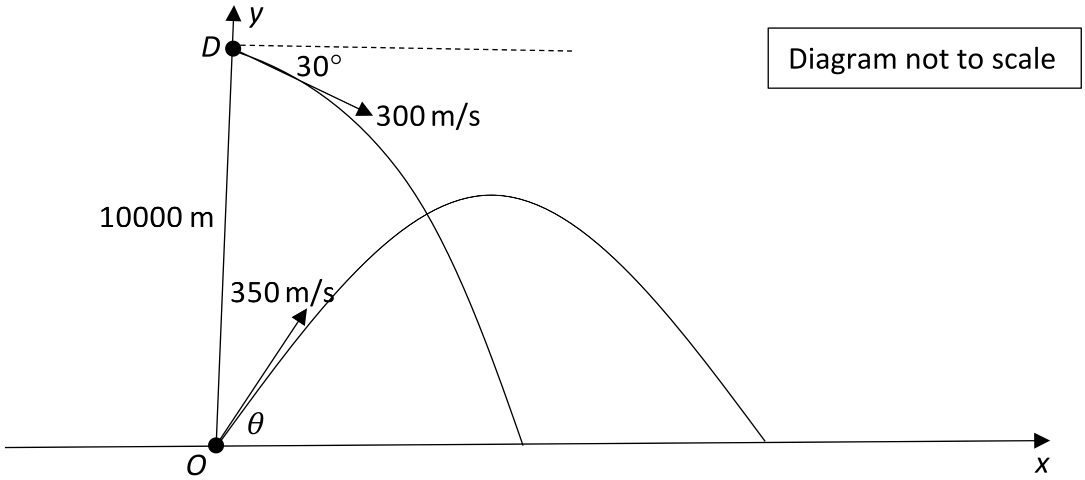
iii) Find, also, the true bearing, to the nearest whole number, at which the aeroplane needs to fly. 1

d) i) Show that $\frac{u^2}{1+u^2} = 1 - \frac{1}{1+u^2}$ 1

ii) Use the substitution $u = \sqrt{x}$, to find $\int \frac{\sqrt{x}}{1+x}\,dx$ 3

Question 14 (15 marks)

a) During an army operations exercise, a surface to air missile is launched from the point *O* in order to intercept a dummy bomb, which is released from a point *D*. The point *D* is 10000 m directly above *O*.



The dummy bomb is released at an angle of $30°$ below the horizontal, with the velocity of 300 m/s.

The position vector of the dummy bomb at any time *t* is given by

$\underset{\sim}{r}_B = \left(150\sqrt{3}\,t\right)\underset{\sim}{i} + \left(10000 - 150t - 5t^2\right)\underset{\sim}{j}$ (Do **NOT** prove this)

i) Calculate how long it would take the dummy bomb to reach the ground, to the nearest second. **2**

ii) How far from *O* will the dummy bomb hit the ground? Give the answer to the nearest metre. **1**

The missile is launched at the same time as the dummy bomb is released.
It is launched with an initial velocity of 350 m/s at an angle of projection θ, above the horizontal.

The position vector of the missile at any time *t* is given by

$\underset{\sim}{r}_M = \left(350\cos\theta t\right)\underset{\sim}{i} + \left(350\sin\theta t - 5t^2\right)\underset{\sim}{j}$ (Do NOT prove this)

iii) Show that in order for the missile to intercept the dummy bomb, it must be launched with an angle of projection $\theta = \cos^{-1}\left(\frac{3\sqrt{3}}{7}\right)$ **1**

iv) If the dummy bomb is intercepted, how high above the ground will that happen? Give your answer to the nearest metre. **3**

b) A big tank initially contains 400 litres of water and it has x_0 grams of chemical already dissolved in it.

Pure water is added to the tank at the rate of 6 L/min.

The mixture is stirred consistently for the concentration of the chemical to be uniform.

However, the mixed solution is drained simultaneously at the rate of 8 L/min. through a hole at the bottom of the tank.

Let x be the amount of chemical in the tank after t minutes.

i) Show that $\dfrac{dx}{dt} = -\dfrac{4x}{200-t}$. 2

ii) Find the general solution of the above differential equation. 2

iii) What fraction of the original amount of the chemical will still be in the tank after 100 minutes? 1

c) By considering the expansion

$$(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots\ldots\ldots\ldots\ldots + {}^{2n}C_{2n} x^{2n},$$ 3

prove that $\displaystyle\sum_{r=2}^{2n} r(r-1)\,{}^{2n}C_r = 2n(2n-1)4^{n-1}$.

END OF PAPER