

AMC 2023 Junior Solutions

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1. $6^2 - 4 - 4 = 28 \dots A$
2. $3 - 2 = 1 < 3 \div 2 = 1.5 < 3 + 2 = 5 < 3 \times 2 = 6 < 32 \dots B$
3. $21 - 5 - 2 - 5 - 7 = 2 \dots A$
4. $\frac{180-80}{2} = 50 \dots A$
5. 8 : 50 am to 3 : 10 pm is 6 hours 20 minutes... *B*
6. $\frac{2+3+4}{7+8+9} = \frac{9}{24} = \frac{3}{8} \dots C$
7. $\frac{100 \times 50}{25 \times 25} = 4 \times 2 = 8 \dots E$
8. 44 kL - 12 kL = 32 kL ... *A*
9. $x + y + z = 32 \Rightarrow (x + 3) + (y + 3) + (z + 3) = x + y + z + 9 = 32 + 9 = 41 \dots D$
10. $PQ \times PT = 10 \times PT = 60\text{cm}^2 \therefore PT = 6\text{cm} \dots D$
11. Direct = x , scenic = $y \Rightarrow y = x + 5$ and $x + y = 35$

$$\therefore \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 35 \end{pmatrix} \therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 35 \end{pmatrix} = \begin{pmatrix} 15 \\ 20 \end{pmatrix} \therefore x = 15 \dots C$$

12. Numbers are 320, 322, 324, 326, 327, 328, 329... *C*

13.

$$\begin{aligned} x + y &= 48, y + z = 34 \text{ and } x + z = 46 \\ (x + y) + (y + z) + (x + z) &= 2(x + y + z) \\ &= 48 + 34 + 46 \\ &= 128 \\ \therefore x + y + z &= \frac{128}{2} \\ &= 64 \dots D \end{aligned}$$

14. $0.6x - 40 = 0.4x \therefore 0.2x = 40 \therefore x = 200\text{L} \dots D$

15. Red is $1 + 3 + 5$

Green maximum is $2 + 4 + 6$

Blue maximum is $1 + 4 + 5$ since 6 is on the bottom.

Yellow maximum is $4 + 5 + 6$

\therefore total maximum = $1 + 3 + 5 + 2 + 4 + 6 + 1 + 4 + 5 + 4 + 5 + 6 = 46 \dots B$

16.

$$9 : 55 \pm 3 \text{ min} = 9 : 52, 9 : 58$$

$$9 : 55 \pm 7 \text{ min} = 9 : 48, 10 : 02$$

$$9 : 58 \pm 3 \text{ min} = 9 : 55, 10 : 01$$

$$9 : 58 \pm 7 \text{ min} = 9 : 51, 10 : 05$$

$$10 : 00 \pm 3 \text{ min} = 9 : 57, 10 : 03$$

$$10 : 00 \pm 7 \text{ min} = 9 : 53, 10 : 07$$

$$10 : 04 \pm 3 \text{ min} = 10 : 01, 10 : 07$$

$$10 : 04 \pm 7 \text{ min} = 9 : 57, 10 : 11$$

$$10 : 07 \pm 3 \text{ min} = 10 : 04, 10 : 10$$

$$10 : 07 \pm 7 \text{ min} = 10 : 00, 10 : 14$$

The only ones common are 10 : 07 and 9 : 57 but 10 : 07 is answer *E*
Hence correct time is 9 : 57 and closest one to 9 : 57 is 9 : 58 ... *B*

17. Last multiple of 7 which is less than 500 is $\lfloor \frac{500}{7} \rfloor \times 7 = 71 \times 7 = 497$. Last digits are 7, 4, 1, 8, 5, 2, 9, 6, 3, 0 which repeats every 10 numbers. Since there are 71 numbers, all the other digits occur 7 times and 7 occurs 8 times ... *C*

$$18. \frac{\frac{1}{2} \times 2 \times 6 + 2 \times \frac{1}{2} \times 2 \times 4}{4 \times 6} = \frac{14}{24} = \frac{7}{12} \dots E$$

19. Code is 9856073214 and 5th digit is 0 ... *A*

20. If at the beginning there are B blue, G gold balloons and x blue blow away and y gold blow away. Then $G = 3B$ and $B - x = 5(G - y)$ so $B = \frac{5y-x}{14}$

Not all blue or gold blow away so $0 \leq x \leq B - 1$ and $0 \leq y \leq G - 1$

For $B = \frac{5y-x}{14}$ to be a minimum integer we have that $x = 0, y = 14$ and $B = 5$ and $G = 15$ and so minimum total is 20 ... *B*

$$21. \frac{189}{63-36} \times 36 = 252 \dots D$$

22. If the constant is x then $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 2x = 5x$ and hence $x = 26$... *A*

$$23. \frac{1 \frac{40}{60}}{\frac{1}{3} - \frac{1}{4}} = 20 \text{ min} \dots B$$

24. codes:

077770,177771,277772,377773,477774,577775,677776,777777,877778,977779
777759,777768,777786,777795
597777,687777,867777,957777

which is 18 codes ... *B*

25. H is opposite M and A is opposite T and so C is opposite S hence C is on the bottom.

A and M are same direction ... *E*

26. $10x - 9y = 3547$ with $9y < 100$ and so $0 \leq y \leq 11$ but then for $x = \frac{5347+9y}{10}$ to be an integer, $y = 7$ and so $x = 541$

$$27. I = M + A \text{ and } E = L + A$$

$$\begin{aligned} A + M + E + L + I + A &= I + E + I + E \\ &= 2(I + E) \\ &\leq 2(8 + 9) \\ &= 34 \end{aligned}$$

Hence maximum is 34

$$28. 3^3 \times (1 + 4 + 3 + 4 + 1) = 351$$

29. If the number is $10x + y$ then $10x + y = 4(x + y) + 3$ and so $y = 2x - 1$. It can't be 11 because remainder upon division by 2 is 1 not 3 and hence the numbers are 23, 35, 47, 59. All of these have remainder 3 upon division by sum of their digits.

$$23+35+47+59=164$$

$$30. 6^4 - 3^4 \times \frac{4!}{(2!)^2} = 810$$