Student No: ....

Barker College

2014
TRIAL
HIGHER SCHOOL CERTIFICATE

Mathematics

PM Friday 1 August

Section 1 – Multiple Choice

Select the alternative A, B, C or D that best answers the question.
Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

BLANK PAGE

Student Number: …………………………………

Barker College

2014
TRIAL
HIGHER SCHOOL CERTIFICATE

Mathematics

Staff Involved:

- JGD*
- LMD*
- BJB
- ASC
- PJR
- AJD
- DZP
- GPF
- WMD

PM Friday 1 August

135 copies

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Write your Barker Student Number on all pages of your solutions
- In Questions 11 – 16, show all relevant mathematical reasoning and/or calculations

Total marks – 100

Section I **Pages 2 - 4**

10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II **Pages 5 - 12**

90 marks

- Attempt Questions 11 - 16
- Allow about 2 hours and 45 minutes for this section

Section 1 - Multiple Choice (10 marks)

Attempt Questions 1 - 10

Use the multiple-choice answer sheet for Questions 1 - 10

1 What is $\dfrac{2\sqrt{5}}{\sqrt{2} - \sqrt{5}}$ as a fraction with a rational denominator?

(A) $\dfrac{-5 - \sqrt{10}}{3}$ (B) $\dfrac{-5 + \sqrt{10}}{3}$ (C) $\dfrac{-10 + 2\sqrt{10}}{3}$ (D) $\dfrac{-10 - 2\sqrt{10}}{3}$

2 What is the value of $\sqrt{\dfrac{a^2 + b^2}{c^2}}$, if a =1.23, b = 0.85 and c = 4.81?

Answer correct to three significant figures.

(A) 3.11 (B) 0.311 (C) 3.10 (D) 0.310

3 Given $\log_a 3 = 0.6$ and $\log_a 2 = 0.4$, find $\log_a 18$.

(A) 1.8 (B) 1.6 (C) 3.0 (D) 0.74

4 Evaluate $\sum_{r=3}^{10} 2r+1$.

(A) 120 (B) 91 (C) 122 (D) 112

5 For $f(x) = \begin{cases} x+1 & x \geq 3 \\ x^2 + 2x - 1 & -1 < x < 3 \\ 3^x & x \leq -1 \end{cases}$, find the value of $f(2) + f(-2) - f(6)$.

(A) $\dfrac{1}{9}$ (B) $14\dfrac{1}{9}$ (C) 9 (D) $3\dfrac{8}{9}$

6 Evaluate $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$.

(A) 2 (B) 1 (C) undefined (D) 0

7 What is the domain and range of the function $f(x) = |x-2| + 1$?

(A) Domain: $x \geq 0$, Range: $y \geq 0$

(B) Domain: $x < 2$, Range: $y \geq 1$

(C) Domain: All real x, Range: $y \geq 1$

(D) Domain: All real x, Range: $y \geq -1$

8 A primitive function of $4 + \sqrt{x}$ is

(A) $4x + \dfrac{2\sqrt{x^3}}{3}$ (B) $4x + \dfrac{3\sqrt{x^2}}{2}$ (C) $4x + \dfrac{1}{2\sqrt{x}}$ (D) $\dfrac{1}{2\sqrt{x}}$

9 Given $\sin A = \dfrac{20}{29}$ and $\dfrac{\pi}{2} \leq A \leq \pi$, what is $\cos A$ in exact form?

(A) $\dfrac{21}{29}$ (B) $\dfrac{-21}{29}$ (C) $\dfrac{21}{20}$ (D) $\dfrac{-21}{20}$

10 The derivative of $e^{\cos x}$ is

(A) $\cos x . e^{\sin x}$ (B) $-\cos x . e^{\sin x}$ (C) $\sin x . e^{\cos x}$ (D) $-\sin x . e^{\cos x}$

End of Section I

BLANK PAGE

Section II – Extended Response (90 marks)

Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question on a separate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) [START A NEW BOOKLET] **Marks**

(a) Simplify $\dfrac{y^3-8}{y-2}$. **2**

(b) Find the perpendicular distance from the line $y=5x-1$ to the point $(-3, 2)$. **2**

(c) Solve $x^2-8x<0$. **2**

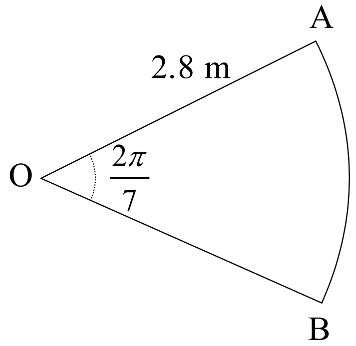
(d) Let α and β be the roots of the equation $x^2 - 2x - 1 = 0$. **2**

What is the value of $\dfrac{1}{\alpha}+\dfrac{1}{\beta}$?

(e) In the diagram, AB is an arc of a circle with centre O. **2**

The radius OA is 2.8 m and the angle AOB is $\dfrac{2\pi}{7}$ radians.

Find the area of the sector AOB.



(f) Find the exact value of $\cos\dfrac{\pi}{4}+\sin\dfrac{5\pi}{6}$. **2**

(g) Solve $\ln 4 = 2\ln x$. **3**

End of Question 11

Question 12 (15 marks) [START A NEW BOOKLET] **Marks**

(a) Differentiate with respect to x.

(i) $\sin(4x^2 + 1)$ **2**

(ii) $x \ln x$ **2**

(iii) $\dfrac{4x+5}{2x^2+5x}$ **2**

(b) Find

(i) $\displaystyle\int \sec^2 \frac{x}{2}\, dx$ **2**

(ii) $\displaystyle\int_0^1 e^{2y+1}\, dy$ **2**

(c) For the function $y = 3 - \cos 2x$

(i) State the period of the function. **1**

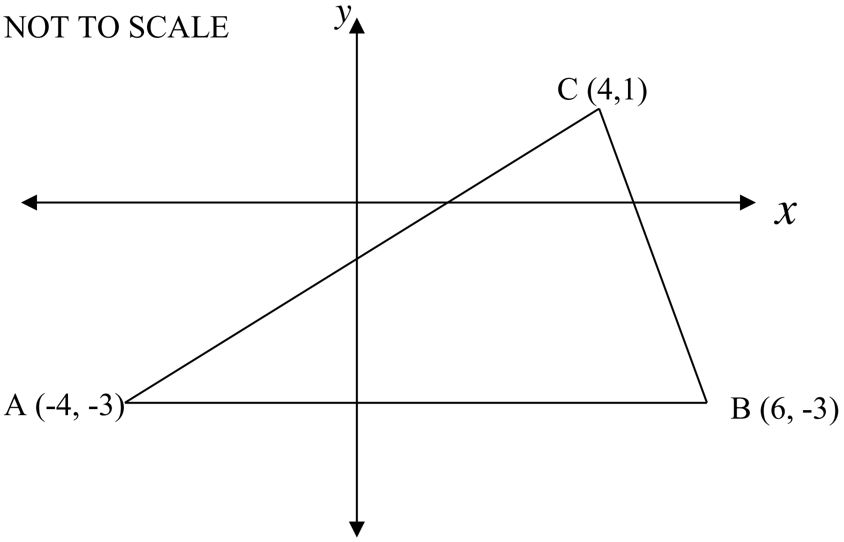
(ii) State the amplitude of the function. **1**

(iii) Sketch the graph of the function for $0 \le \theta \le 2\pi$. **3**

End of Question 12

Question 13 (15 marks) [START A NEW BOOKLET] **Marks**

(a) The diagram below shows ΔABC where the vertices are $A(-4,-3)$, $B(6,-3)$ and $C(4,1)$.



(i) Find the gradient of the interval BC. **1**

(ii) Prove A, B and C are the vertices of a right-angled triangle. **2**

(iii) Find the distance BC in exact form. **1**

(iv) Find the size of $\angle CAB$ to the nearest minute. **2**

(v) Show that the coordinates of M, the midpoint of AB are $(1,-3)$. **1**

(vi) Show that the equation of the circle centre M, diameter AB is $x^2+y^2-2x+6y-15=0$ **3**

(b) (i) Sketch the curve $y=\ln(x+1)$ showing essential features. **2**

(ii) Using the Trapezoidal Rule with 4 sub-intervals, evaluate **2**

$\int_1^2 \ln(x+1)\,dx$ correct to 3 decimal places.

.

(iii) Would the Trapezoidal Rule used in Part (ii) above provide a value that is greater or lesser than $\int_1^2 \ln(x+1)\,dx$? Explain your answer. **1**

End of Question 13

Question 14 (15 marks) [START A NEW BOOKLET] **Marks**

(a) (i) For what values of x can the geometric series $1+2x+4x^2+8x^3+...$ have a limiting sum? **1**

(ii) For what values of x does $\sum_{r=0}^{\infty}(2x)^r = \frac{5}{9}$? **2**

(b) Simplify $\frac{2\sec^2 A - 2}{4\tan A}$. **2**

(c) What are the coordinates of the focus of the parabola defined by the equation $y = -16x^2$? **2**

(d) For the curve $f(x) = \frac{2}{3}x^3 - 8x + 1$,

(i) Find the coordinates of the stationary point(s) and determine their nature. **3**

(ii) Determine any point(s) of inflection. **2**

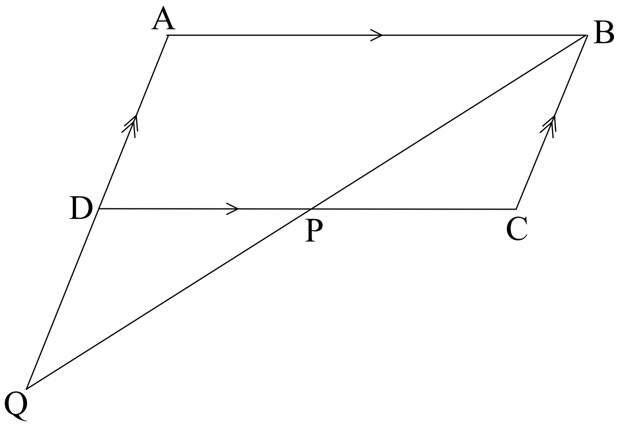
(iii) Sketch the curve, showing all of the above features. **2**

(iv) State the domain of x for which $f(x)$ is monotonic increasing. **1**

End of Question 14

Question 15 (15 marks) [START A NEW BOOKLET] **Marks**

(a) ABCD is a parallelogram. DP = PC and P lies on DC. BP produced, meets AD produced at Q.



(i) Prove that $\Delta DPQ \equiv \Delta CPB$. **2**

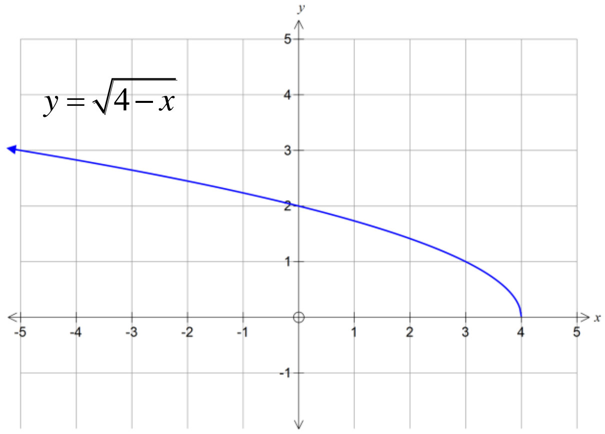
(ii) Hence, or otherwise, show that BP = PQ. **1**

(b) (i) Find the points of intersection of the curves $y = \dfrac{1}{2-x}$ and $y = 2 - x$. **3**

(ii) On the same large diagram, sketch and label these two curves showing any asymptotes and intercepts with the coordinate axes. **3**

(iii) Find the area bounded by the two curves and the y-axis. **3**

(c) Find the volume of the solid formed when the area bounded by $y=\sqrt{4-x}$, $y=0$ and the y-axis is rotated around the x-axis. **3**



End of Question 15

Question 16 (15 marks) [START A NEW BOOKLET] **Marks**

(a) A Year 12 Biology student tested to see how much bacteria was present in a variety of food samples left in the classroom.
It is known that after t hours the number of bacteria *(N)* present in a particular type of food is given by the formula $N = N_0 e^{kt}$.

(i) If initially there were 20 000 bacteria present and after three hours there were 45 000 bacteria present, calculate the value of k (correct to 2 decimal places). **1**

(ii) How long would it take for the initial number of bacteria to triple in quantity? **2**

(iii) What will be the rate of increase of the bacteria after $4\frac{1}{2}$ hours? **2**

(b) Jessica plans to work for the next ten years.

During that time she wants to save \$50 000.

She decides to invest a fixed amount of money at the beginning of each month during this time.

Interest is paid at a fixed rate of 6% p.a. **compounded monthly.**

(i) Let M be the monthly investment in dollars. **2**

Show that the total investment R after 10 years is given by

$$R = M\left[(1.005) + (1.005)^2 + (1.005)^3 + ... + (1.005)^{120}\right].$$

(ii) Find the amount M needing to be deposited each month to reach her goal. **3**

(c) A 6 metre piece of wire is cut into three pieces to form two congruent squares and a circle of radius *r* metres.

(i) Show that the total combined area of these three shapes is given by **2**

$$A = \frac{(3-\pi r)^2}{8} + \pi r^2.$$

(ii) Find the exact value of *r* which will make this total combined area a minimum. **3**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1\,; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

HSC
2014 Year 12 Mathematics
Trial Examination.

1. $\frac{2\sqrt{5}}{\sqrt{2}-\sqrt{5}} \times \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}+\sqrt{5}} = \frac{2\sqrt{10}+10}{2-5} = \frac{2\sqrt{10}+10}{-3}$

$= \frac{-2\sqrt{10}-10}{3}$ (D).

2. $\sqrt{\frac{1\cdot23^2+0\cdot85^2}{4\cdot81^2}} = 0\cdot3108368842$

$= 0\cdot311$ (3 sig figs) (B)

3. $\log_a 18 = \log_a(3^2 \times 2)$
$= \log_a 3^2 + \log_a 2$
$= 2\log_a 3 + \log_a 2$
$= 2 \times 0\cdot6 + 0\cdot4 = 1\cdot6$ (B)

4. $\sum_{r=3}^{10} 2r+1 = 7+9+11+\ldots+21$
$= 112$ (D)

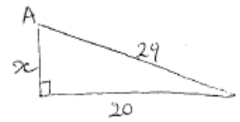
5. $f(2) = 2^2+2\times2-1 = 7$
$f(-2) = 3^{-2} = \frac{1}{9}$
$f(6) = 7$
$\therefore f(2)+f(-2)-f(6) = 7+\frac{1}{9}-7 = \frac{1}{9}$ (A)

6. $\lim_{x\to1} \frac{x^2-1}{x-1} = \lim_{x\to1} \frac{(x-1)(x+1)}{x-1}$
$= \lim_{x\to1} x+1$
$= 1+1 = 2$ (A)

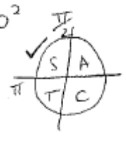
7. (C).

8. $y' = 4+\sqrt{x}$
$= 4+x^{\frac{1}{2}}$

$y = 4x + \frac{2x^{\frac{3}{2}}}{3}$
$= 4x + \frac{2\sqrt{x^3}}{3}$ (A)

9. 

$x^2 = 29^2 - 20^2$
$= 441$
$x = 21$.



$\therefore \cos A = -\frac{21}{29}$ (B)

10. $y = e^{\cos x}$
$y' = -\sin x \cdot e^{\cos x}$ (D)

11. a) $\frac{y^3-8}{y-2} = \frac{(y-2)(y^2+2y+4)}{y-2}$
$= y^2+2y+4$

b) $5x-y-1=0$ $(-3,2)$
$a=5$ $b=-1$ $c=-1$ $x_1=-3$ $y_1=2$.

$\perp d = \frac{|5\times-3+-1\times2-1|}{\sqrt{5^2+1^2}}$
$= \frac{|-15-2-1|}{\sqrt{26}} = \frac{18}{\sqrt{26}}$ units.

11 c) $x^2 - 8x < 0$

$x(x-8) < 0$

$0 < x < 8$

d) $x^2 - 2x - 1 = 0$

$\alpha + \beta = 2 \qquad \alpha\beta = -1$

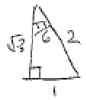
$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{2}{-1} = -2$

e) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 2.8^2 \times \frac{2\pi}{7}$

$= 3.52 \text{ m}^2$ OR $1.12\pi \text{ m}^2$

f) $\cos\frac{\pi}{4} + \sin\frac{5\pi}{6} = \frac{1}{\sqrt{2}} + \sin\frac{\pi}{6}$



$= \frac{1}{\sqrt{2}} + \frac{1}{2}$

$= \frac{\sqrt{2}}{2} + \frac{1}{2}$

$= \frac{1+\sqrt{2}}{2}$

g) $\ln 4 = 2\ln x$

$\ln 4 = \ln x^2$

$4 = x^2$

$x = \pm 2$

But $x > 0 \therefore x = 2$ is the only solution.

12. a) (i) $y = \sin(4x^2+1)$

$y' = 8x\cos(4x^2+1)$

(ii) $y = x\ln x$

$y' = \ln x + x \times \frac{1}{x}$

$= \ln x + 1$

(iii) $y = \frac{4x+5}{2x^2+5x}$

$y' = \frac{(2x^2+5)\times 4 - (4x+5)(4x+5)}{(2x^2+5x)^2}$

$= \frac{4(2x^2+5) - (4x+5)^2}{(2x^2+5x)^2}$

b) (i) $\int \sec^2\frac{x}{2}\,dx$

$= 2\tan\frac{x}{2} + C$

(ii) $\int_0^1 e^{2y+1}\,dy$

$= \left[\frac{e^{2y+1}}{2}\right]_0^1$

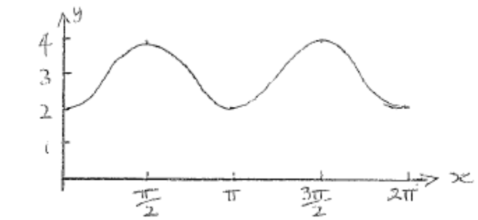
$= \frac{e^3}{2} - \frac{e}{2}$

$= \frac{e^3 - e}{2}$

c) (i) Period $= \frac{2\pi}{2} = \pi$

(ii) Amplitude $= 1$

12 c) (iii)



13 a) (i) $m_{BC} = \frac{-3-1}{6-4} = \frac{-4}{2} = -2$

(ii) $m_{AC} = \frac{-3-1}{-4-4} = \frac{-4}{-8} = \frac{1}{2}$

$m_{BC} \times m_{AC} = -2 \times \frac{1}{2} = -1$

$\therefore AC \perp BC$

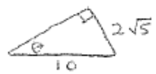
$\therefore \Delta ABC$ is right angled

(iii) $dist_{BC} = \sqrt{(6-4)^2 + (-3-1)^2}$

$= \sqrt{2^2 + 4^2}$

$= \sqrt{20} = 2\sqrt{5}$

(iv)



$\sin\theta = \frac{2\sqrt{5}}{10} =$

$\theta = 26°33'54.18"$

$= 26°34'$

(v) $mp_{AB} = \left(\frac{-4+6}{2}, \frac{-3-3}{2}\right)$

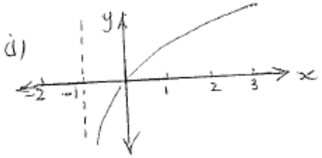
$= \left(\frac{2}{2}, \frac{-6}{2}\right) = (1, -3)$

(vi) $(x-1)^2 + (y+3)^2 = 25$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 25$

$x^2 + y^2 - 2x + 6y - 15 = 0$

13 b) (i)



(ii)

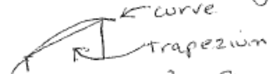
x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
y	ln 2	$\ln\frac{9}{4}$	$\ln\frac{5}{2}$	$\ln\frac{11}{4}$	ln 3
	1	2	2	2	1

$\int \ln(x+1)\,dx \doteq \frac{0.25}{2}\left(\ln 2 + 2\times\ln\frac{9}{4} + 2\times\ln\frac{5}{2} + 2\times\ln\frac{11}{4} + \ln 3\right)$

$= 0.9086753986$

$= 0.909$ units^2

(iii) Less because the curve is concave down and the straight line cuts below



14 a) (i) $1 + 2x + 4x^2 + 8x^3 + \ldots$

$a = 1 \quad r = 2x$

For a limiting sum $-1 < r < 1 \quad r \neq 0$

$\therefore -1 < 2x < 1$

$-\frac{1}{2} < x < \frac{1}{2} \quad x \neq 0$

(ii) $S_\infty = \frac{a}{1-r}$

$\frac{5}{9} = \frac{1}{1-2x}$

$5(1-2x) = 9$

$5 - 10x = 9$

$-10x = 4$

$x = -\frac{2}{5}$

14 b) $\frac{2\sec^2 A - 2}{4\tan A} = \frac{2(\sec^2 A - 1)}{4(\tan A)}$

$= \frac{2\tan^2 A}{4\tan A}$

$= \frac{\tan A}{2}$

c) $x^2 = \frac{1}{-16}y$

$4a = \frac{1}{16}$

$a = \frac{1}{64}$ Focus is at $(0, -\frac{1}{64})$

d) $f(x) = \frac{2}{3}x^3 - 8x + 1$

(i) $f'(x) = 2x^2 - 8$

stationary points when $f'(x) = 0$

$0 = 2x^2 - 8$

$= x^2 - 4$

$= (x-2)(x+2)$

$\therefore x = \pm 2$. $\therefore (-2, \frac{35}{3})$ max T.P.

$(2, -\frac{29}{3})$ min T.P.

x	-3	-2	0	2	3
$f'(x)$	10	0	−8	0	10
	/	—	\	—	/

OR

$f''(x) = 4x$

when $x = -2$ $f''(x) = -8$ $\therefore$ concave down ∩ max T.P

when $x = 2$ $f''(x) = 8$ $\therefore$ concave up ∪ min T.P.

$(-2, \frac{35}{3})$ max TP

$(2, -\frac{29}{3})$ min T.P.

14 d) (ii) Possible POI when $f''(x) = 0$

$f''(x) = 4x$

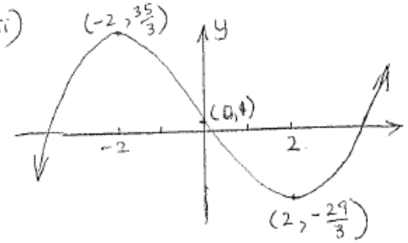
$4x = 0$

$x = 0$

x	-1	0	1
$f''(x)$	-4	0	4
	∩		∪

$\therefore (0, 1)$ is a point of inflexion.

(iii)



(iv) $x < -2$ or $x > 2$

15 a) (i) In $\triangle DPQ$ and $\triangle CPB$

$\angle QDP = \angle BCP$ (alternate angles on parallel lines $AQ \parallel BC$)

$DP = PC$ (given)

$\angle DPQ = \angle CPB$ (vertically opposite angles)

$\therefore \triangle DPQ \equiv \triangle CPB$ (AAS)

(ii) $BP = PQ$ (corresponding sides in congruent $\triangle$'s)

b) $y = \frac{1}{2-x}$, $y = 2 - x$

$\frac{1}{2-x} = 2 - x$

$1 = (2-x)(2-x)$

$1 = 4 - 4x + x^2$

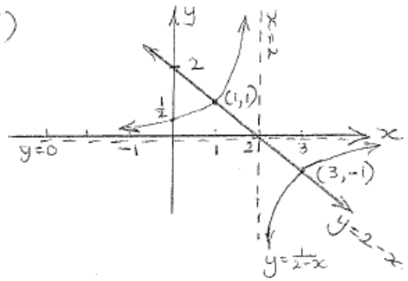
$x^2 - 4x + 4 - 1 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\therefore (3, -1)$ $(1, 1)$

15 b) (ii)



(iii) $A = \frac{1}{2}(2+1)\times 1 - \int_0^1 \frac{1}{2-x}\,dx$

$= \frac{3}{2} + 1\left[\ln(2-x)\right]_0^1$

$= \frac{3}{2} + \left[\ln 1 - \ln 2\right] = \frac{3}{2} + \ln\frac{1}{2}$

$= \frac{3}{2} - \ln 2$

$= \frac{3}{2} - 0.6931471806$

$= 0.8068528194$ units^2

c) $V = \int_0^4 \pi y^2\,dx$

$V = \pi\int_0^4 4 - x\,dx$.

$= \pi\left[4x - \frac{x^2}{2}\right]_0^4$

$= \pi\left[(16-8) - (0-0)\right]$

$= 8\pi$ units^3 $= 25.13274123$ units^3

16. a) (i) $N = N_0 e^{kt}$

when $t = 0$, $N = 20000$ $\therefore N_0 = 20\,000$

when $t = 3$, $N = 45000$

$45000 = 20000\,e^{3k}$

$\frac{45}{20} = e^{3k}$

$\ln\left(\frac{9}{4}\right) = 3k$ $\qquad k = \frac{\ln\frac{9}{4}}{3} = 0.2703100721$

≈ 0.27

(ii) $60000 = 20000e^{0.27t}$

$3 = e^{0.27t}$

$\ln 3 = 0.27t$

$t = \frac{\ln 3}{0.27} = 4.068934402$ hours

$= 4$ hours 4 mins 8.16 seconds

(iii) When $t = 4.5$ $\quad \frac{dN}{dt} = 0.27\times 20000\,e^{0.27t}$

$\frac{dN}{dt} = 0.27\times 20000\times e^{0.27\times 4.5}$

$= 18199.58795$

$= 18200$ bacteria/hr at the 4.5 hour mark.

b) 50000 $\quad$ 6% pa = 0.5% per month

(i) $A_1 = M\times 1.005$

$A_2 = (A_1 + M)\times 1.005$

$= (1.005M + M)\times 1.005$

$= 1.005^2 M + 1.005M$

$A_3 = (A_2 + M)\times 1.005$

$= ((1.005^2 M + 1.005M) + M)\times 1.005$

$= 1.005^3 M + 1.005^2 M + 1.005M$

$A_{120} = R = 1.005^{120}M + 1.005^{119}M + \dots + 1.005M$

$= M(1.005 + 1.005^2 + \dots + 1.005^{119} + 1.005^{120})$

16 b) (ii) $1.005 + 1.005^2 + \ldots + 1.005^{120}$

$a = 1.005 \quad r = 1.005 \quad n = 120.$

$R = M \times \left(\frac{1.005\,(1.005^{120} - 1)}{1.005 - 1} \right)$

$50000 \times (1.005 -) = M\,(1.005(1.005^{120} - 1))$

$M = \frac{50000 \times (1.005 - 1)}{1.005(1.005^{120} - 1)}$

$= 303.5845868 = \$303.58$

c)

$C = 2\pi r \quad A = \pi r^2$

$\frac{6 - 2\pi r}{2}$ = one square perimeter

$P = 3 - \pi r$

side of each square $= \frac{3 - \pi r}{4}$

area of each square

$= \left(\frac{3 - \pi r}{4} \right)^2$

$\therefore$ Total area $= 2 \times \left(\frac{3 - \pi r}{4} \right)^2 + \pi r^2$

$= \frac{2 \times (3 - \pi r)^2}{16\ 8} + \pi r^2$

$= \frac{(3 - \pi r)^2}{8} + \pi r^2$

Minimum when $A' = 0$.

$A' = \frac{2(3 - \pi r)}{8\ 4} \times -\pi + 2\pi r$

$0 = \frac{-\pi(3 - \pi r)}{4} + 2\pi r$

$0 = -3\pi + \pi^2 r + 8\pi r$

$= -3 + \pi r + 8r$

$3 = (\pi + 8) r \qquad r = \frac{3}{\pi + 8} \approx 0.269.$

x	0	$\frac{3}{\pi+8}$	1
A'	$\frac{-3\pi}{4}$	0	$\frac{-\pi(3-\pi)}{4} + 2\pi \approx 6.39$

$\therefore$ minimum when $r = \frac{3}{\pi + 8}$