Correcting 2025 HSC Exams

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It would appear that there are some questions in the 2025 HSC maths exams which were written in a way that did not convey the intent of the question. Herein I explain how they can be changed to more accurately reflect the intent of the questions.

Extension 1 Question 12d

As written it asks for "the solution" to $\frac{dy}{dx} = \sqrt{(2-y)(2+y)}$ passing through $(0,1)$. NESA's solution has the answer $y = 2\sin(x + \frac{\pi}{6})$ and whilst that may well be the solution to the resulting integral nevertheless there are problems with it.

Firstly there is no consideration whatsoever for the domain in the question or solution. If we accept the domain of the solution should be the same as that of the question then the maximal domain of both is $\mathbb{R}$.

Secondly it seems from the way it was marked there is an assumption that solving the differential equation is synonymous with finding the integral and it is clear from what follows that it isn't.

Thirdly with $\frac{dy}{dx} = \sqrt{(2-y)(2+y)}$ we have that $\frac{dy}{dx} \geq 0$ but alas this is not always the case with $y = 2\sin(x + \frac{\pi}{6})$ and so this cannot possibly be "the solution" as asked in the question, and not if we want the domain in the solution to be the same as the domain in the question.

Nevertheless NESA's solution can be modified to procure "the solution" thusly

$$y = \begin{cases} -2 & \text{for } x \leq -\frac{2\pi}{3} \\ 2\sin(x + \frac{\pi}{6}) & \text{for } -\frac{2\pi}{3} < x < \frac{\pi}{3} \\ 2 & \text{for } x \geq \frac{\pi}{3} \end{cases}$$

It is clear from the marking guidelines that this was not the intent of the question and so rather than modify NESA's solution we now proceed to modify the question instead, with changes in red.

(d) Find the solution of $\frac{dy}{dx} = \sqrt{(2-y)(2+y)}$ for $-\frac{2\pi}{3} < x < \frac{\pi}{3}$, given that $y = 1$ when $x = 0$. **3**

and now NESA's solution becomes "the solution" with the same domain as the question.

Extension 2 Question 13a

In this question we are asked to show there exists a constant k such that for a constant m then $\int_{m-4}^{m-2} \frac{e^{-x}}{x-m+1}\, dx = kA$ where $A = \int_2^4 \frac{e^x}{x-1}\, dx$.

From the way it was marked there is an assumption that an existence proof for k is synonymous with finding a formula for k in terms of m. However there are many non-constructive existence proofs and from what follows we can see that this can indeed be applied in this case without finding a formula for k in terms of m.

The *closure of non-zero reals under division* states that for non-zero reals a, b there exists a non-zero real $k = \frac{a}{b}$ such that $a = kb$.

Now with $A = \int_2^4 \frac{e^x}{x-1}\, dx$ a positive real number and $\int_{m-4}^{m-2} \frac{e^{-x}}{x-m+1}\, dx$ a negative real number for a constant m we invoke the closure of non-zero reals under division to conclude $\int_{m-4}^{m-2} \frac{e^{-x}}{x-m+1}\, dx = kA$ for a constant k.

So this does answer the question and shows such a constant k exists without having to find a formula for k in terms of m. From the marking guidelines it is clear this is not the intent of the question. We therefore modify the question and in so doing more accurately reflect the intent of the question.

(a) It is given that $A = \int_2^4 \frac{e^x}{x-1}\, dx$. **3**

If $\int_{m-4}^{m-2} \frac{e^{-x}}{x-m+1}\, dx = kA$, where k and m are constants, find k in terms of m.

It could also be turned into a multiple choice question like this:

If $A = \int_2^4 \frac{e^x}{x-1}\, dx$ and $\int_{m-4}^{m-2} \frac{e^{-x}}{x-m+1}\, dx = kA$ then $k =$

(A) e^m

(B) $-e^m$

(C) e^{-m}

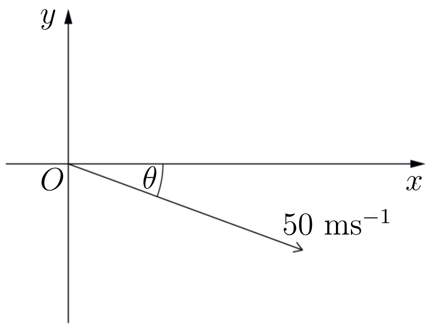
(D) $-e^{-m}$

Although this version of it isn't quite the intent of the question one suspects this may indeed have been the *original* intent.

Extension 2 Question 16b

This question failed to specify (but assumed) that the initial vertical speed is less than the terminal speed. Removing that assumption makes the solution longer because then one will have to consider 3 cases with it less than, equal to, and greater than the terminal speed for failure to do so results in dividing by 0, taking the logarithm of 0 and taking the logarithm of a negative. Therefore we add this information so that this no longer becomes an issue.

(b) A particle of mass 1 kg is projected from the origin with a speed of 50 ms^{-1} at an angle of θ below the horizontal into a resistive medium. **5**
The initial vertical speed is less than the terminal speed.



The position of the particle t seconds after projection is (x, y), and the velocity of the particle at that time is $\underset{\sim}{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$.

The resistive force, $\underset{\sim}{R}$, is proportional to the velocity of the particle, so that $\underset{\sim}{R} = -k\underset{\sim}{v}$, where k is a positive constant.

Taking the acceleration due to gravity to be 10 ms^{-2}, and the upwards vertical direction to be positive, the acceleration of the particle at time t is given by:

$$\underset{\sim}{a} = \begin{pmatrix} -k\dot{x} \\ -k\dot{y} - 10 \end{pmatrix} \qquad \text{(Do NOT prove this.)}$$

Derive the Cartesian equation of the motion of the particle, given $\sin\theta = \frac{3}{5}$.

The 110 year history of excellence in external final examinations in NSW behoves one to uphold the recommendation of the Stacey Review [1] that the NSW syllabus maintains rigour. It seems this has gone awry with some questions in the 2025 HSC exams. This document has gone some way towards bringing this rigour back and it is hoped that NESA will adjust their processes so that this kind of document will not be necessary in the future.

References

[1] Stacey, K., Dowsey, J., McCrae, B., & Stephens, M. (1998). Review of senior secondary mathematics curriculum. ("Stacey Review") Sydney: NSW Board of Studies.