EAST HILLS BOYS TECHNOLOGY HIGH

STUDENT: ____________________

CLASS: ____________________

2012
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time - 5 minutes
- Working Time - 2 hours
- Write using a blue or black pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11 - 14
- Begin each question on a new sheet of paper.

Total marks (70)

Section I

10 marks

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section

Section II

60 marks

- Attempt questions 11 – 14
- Answer on the blank paper provided, unless otherwise instructed. Start a new page for each question.
- Allow about 1 hour 45 minutes for this section

Section I

Total marks (10)

Attempt Questions 1-10

Allow about 15 minutes for this section

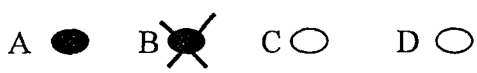
Use the multiple choice answer sheet.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample

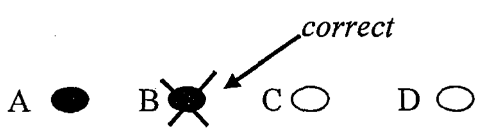
$2+4=?$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



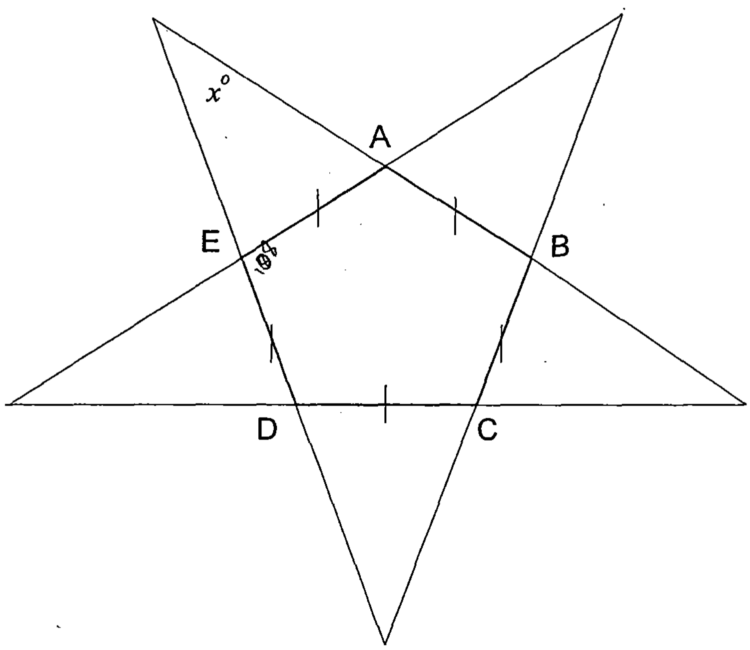
If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:



1. Calculate to 3 significant figures $\sqrt[5]{\dfrac{18.7+3.65}{\sqrt{(4.25)^3}}} =$

(A) 1.20 (B) 1.206 (C) 1.21 (D) 12.6

2. In the diagram, ABCDE is a regular pentagon. The value of x is:



(A) 90° (B) 36° (C) 108° (D) 72°

3. Find $\lim_{x \to 0} \dfrac{\sin 7x}{5x}$

(A) $\dfrac{7}{5}$

(B) 1

(C) $\dfrac{5}{7}$

(D) 0

4. $\frac{d}{dx}[\cos(\ln x)] =$

(A) $-\sin(\ln x)$

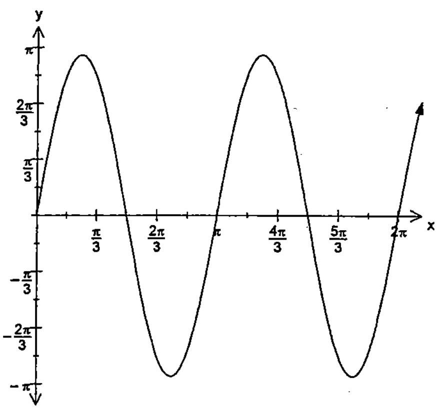
(B) $\frac{\cos(\ln x)}{x}$

(C) $\sin(\ln x)$

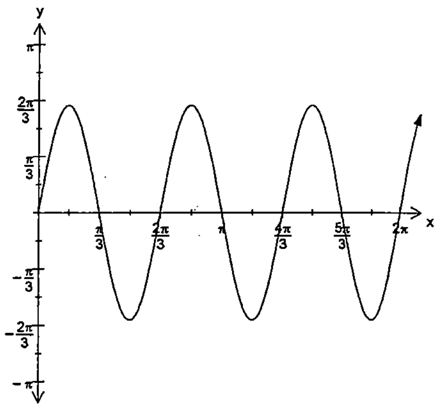
(D) $\frac{-\sin(\ln x)}{x}$

5. Which graph represents the curve $y = 3\sin 2x$

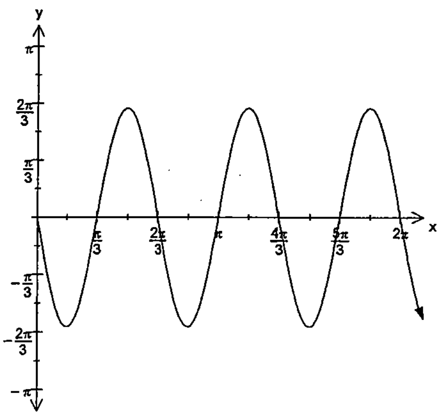
(A)



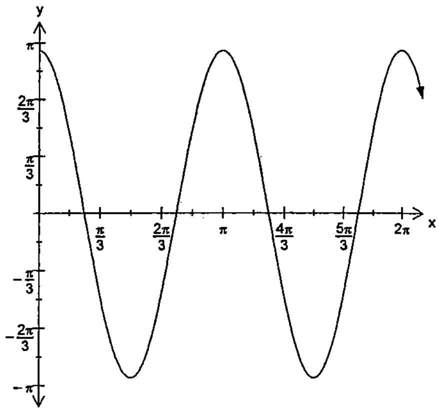
(B)



(C)



(D)



6. Find an approximation of the root of $y = e^x - 3x^2$ by using Newton's Method once and substituting with an approximation of $x = 3.8$ (answer correct to 2 decimal places)

(A) 3.74
(B) 4.22
(C) -12.06
(D) 3.7

7. $4 + \frac{3}{x^2} - \frac{2}{x^3} + \frac{7}{x^5}$ can also be written as:

(A) $\frac{4x + 3x - 2x + 7}{4x}$

(B) $\frac{4x^5 + 3x^3 - 2x^2 + 7}{x^5}$

(C) $\frac{4x^2 + 3x^2 - 2x^3 - 7x^4}{x^2}$

(D) $4 + \frac{3x^5 - 2x^3 + 7x^2}{x^{10}}$

8. Two dice are rolled and the sum of the numbers is written down. Find the probability of rolling a total less than 6.

(A) $\frac{1}{4}$

(B) $\frac{5}{36}$

(C) $\frac{5}{12}$

(D) $\frac{5}{18}$

9. The correct factorisation of $8x^3-27y^3$ is:

(A) $(x-y)(x^2+xy+y^2)$

(B) $(2x-3y)(4x^2+6xy+9y^2)$

(C) $(4x-9y)(x^2+xy+y^2)$

(D) $(2x-3y)(4x^2-6xy+9y^2)$

10. We can express $\sin x$ and $\cos x$ in terms of $\tan\frac{x}{2}$, for all values of x except.......

(A) $x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4}$........

(B) $x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2}$........

(C) $x=\pi,3\pi,5\pi$........

(D) $x=2\pi,6\pi,8\pi$........

End of Section 1

Student name / number ....

Marks

Section II

60 marks
Attempt Questions 11-14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Question 11 **Begin a new booklet**

(a) Find the number of ways in which 3 boys and 3 girls can be arranged in a straight line so that the girls are all next to each other, but the boys are not all next to each other. **2**

(b) Find the coordinates of the point $P(x, y)$ which divides the interval joining the points $A(-2, 5)$ and $B(4, 1)$ externally in the ratio 3 : 1. **2**

(c) Solve the inequality $\frac{1}{x-1} < 1$. **2**

(d) Consider the function $f(x) = x + e^{-x}$.

(i) Find the coordinates and nature of the stationary point on the curve $y = f(x)$. **2**

(ii) Find the equation of the asymptote on the graph of the curve $y = f(x)$. **1**

(e) $P(2at, at^2)$ is a point on the parabola $x^2 = 4ay$ with focus $F(0, a)$.

(i) Use differentiation to show that the normal to the parabola at P has gradient $-\frac{1}{t}$. **1**

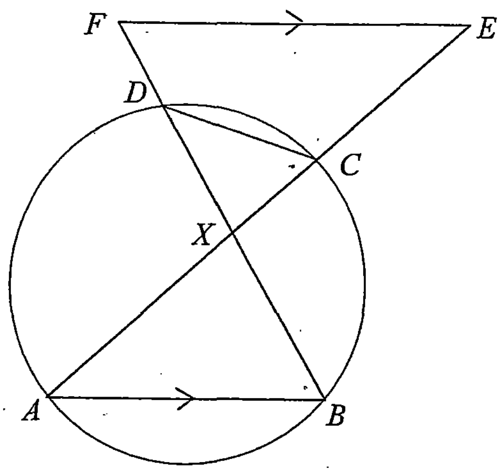
(ii) If θ is the acute angle between the normal to the parabola at P and the line PF show that $\tan\theta = |t|$. **2**

Student name / number ....

Marks

Question 11 (cont)

(f)



AC and BD are two chords of a circle which intersect at point X inside the circle. E is a point on AC produced and F is a point on BD produced such that $FE \parallel AB$. Show that $DCEF$ is a cyclic quadrilateral. **3**

Question 12 (15 Marks) Use a Separate Sheet of paper **Marks**

a) Prove by induction, that **3**

$4^n > 1 + 3n$ for $n > 1$, where n is an integer..

b) A mobile phone company has a success rate of 65% when signing up new customers who enter a particular store. If 10 new customers walk into the store:

i) Find the probability as a percentage that 9 of these people sign up. **1**

ii) What is the most likely number of customers to sign up? **2**

c) Evaluate $\int_0^{\frac{\pi}{4}} \sin x \cos^2 x \, dx$. **2**

d) ABCDE are points on a circle radius 4 cm and $\angle DBC = \angle DAE$

i) Draw a diagram to represent this information. **1**

ii) Prove that the triangle formed by the points CDE is isosceles. **2**

e) Sketch the graph of $y = \cos^{-1}(x+3)$. **2**

f) Find the 6$^{\text{th}}$ term in the expansion of $\left(3x - \frac{4}{5x^2}\right)^9$. **2**

End of Question 12

Question 13 (15 Marks) Use a Separate Sheet of paper **Marks**

a) Evaluate $\int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ in exact form. 2

b) Storm is making a toffee dessert. The rate at which the toffee cools is proportional to the difference between the temperature of the toffee (T) and room temperature (R).

$$\text{ie. } \frac{dT}{dt} = -k(T-R)$$

i) Show that $T = R + Ce^{-kt}$, where C is a constant, is a solution of this differential equation. 1

ii) Storm notices that a 2L pot of toffee initially cools from 540°C to 100°C in 50 minutes in a room whose temperature is 20°C. Storm can not put the toffee into the dessert until it reaches 40°C. How much longer does Storm need to wait to be able to add the toffee and finish her dessert (to the nearest minute)? 3

iii) Explain or show by calculations, if it would take more or less time to create this dessert if the room temperature was 25°C. Assuming k and C remain the same. 2

c) Calculate the exact volume generated by the solid formed when $y = \ln x - 1$ is rotated about the y-axis between $y = 0$ and $y = 1$. 2

d) $P(x) = x^4 - 2x^3 + 5x^2 - 16x + 12$

i) Show that $(x-1)(x-2)$ is a factor of $P(x)$. 1

ii) Hence find the remaining factor of $P(x)$. 1

e) i) Show that the area of an equilateral triangle of side length (x-2) is given by: 1

$$A = \frac{\sqrt{3}(x-2)^2}{4}$$

ii) The sides of the equilateral triangle are increasing at the rate of 5mm/s. At what rate is the area increasing at the instant when the sides are 10cm long? 2

End of Question 13

Student name / number ....

Marks

Question 14 (15 Marks) **Begin a new booklet**

(a) A particle is moving in a straight line and performing Simple Harmonic Motion. At time t seconds it has displacement x metres from a fixed point O on the line, given by $x = 2\cos\left(2t - \frac{\pi}{4}\right)$, velocity $v \text{ ms}^{-1}$ and acceleration $\ddot{x} \text{ ms}^{-2}$.

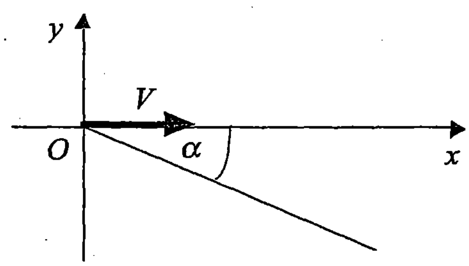
(i) Show that $v^2 - x\ddot{x} = 16$. **2**

(ii) Sketch the graph of x as a function of t for $0 \le t \le \pi$ showing clearly the coordinates of the endpoints. **2**

(iii) Show that the particle first returns to its starting point after one quarter of its period. **1**

(iv) Find the time taken by the particle to travel the first 100 metres of its motion. **2**

(b)



A particle is projected horizontally from a point O with speed $V \text{ ms}^{-1}$ down a slope which is inclined at an angle $\alpha = \tan^{-1}\frac{1}{2}$ below the horizontal. The particle moves in a vertical plane under gravity where the acceleration due to gravity is $g \text{ ms}^{-2}$. At time t seconds the horizontal and vertical displacements from O, x metres and y metres respectively, are given by $x = Vt$ and $y = -\frac{1}{2}gt^2$. (DO NOT PROVE THESE RESULTS.)

(i) Show the particle hits the slope after time $\dfrac{V}{g}$ seconds. **2**

(ii) Show that the particle hits the slope with velocity $V\sqrt{2} \text{ ms}^{-1}$ at an angle of $45°$ to the vertical. **2**

(c)(i) Show that $\sum_{r=1}^{n}(1+x)^{r-1} = \sum_{r=1}^{n} {}^{n}C_r\, x^{r-1}$ **2**

(ii) Hence show that for $n \ge 3$, $\sum_{r=2}^{n-1} {}^{r}C_2 = {}^{n}C_3$. **2**

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$