Girraween High School

2017

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time: 5 minutes
- Working time: 2 Hours
- Write using a black or blue pen
- Board approved calculators may be used
- Laminated reference sheets are provided
- Answer multiple choice questions by completely colouring in the appropriate circle on your multiple choice answer sheet on the front page of your answer booklet.
- In questions 11-15 start all questions on a separate page in your answer booklet and show all relevant mathematical reasoning and/or calculations.

Total Marks: 85

Section 1 **10 Marks**

- Attempt Q1 - Q10
- Allow about 15 minutes for this section

Section 2 **75 marks**

- Attempt Q11 - Q15
- Allow about 1 hour and 45 minutes for this section

MATHEMATICS

Trial Examination

For questions 1-10, fill in the response oval corresponding to the correct answer on your Multiple choice answer sheet.

1. What is the value of $\lim_{x\to\infty} \dfrac{\sin\left(\frac{1}{2}\right)x}{2x}$?

A) 0 B) $\frac{1}{4}$ C) 1 D) 4

2. Which of the following is a simplification of $\cot 2x + \tan x$?

A) $\sec 2x$ B) $\sec x$ C) $\cos ecx$ D) $\cos ec2x$

3. The equation $x^3 + bx^2 + cx + d = 0$ has roots $\alpha, \beta\ \gamma$. What is the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$?

A) $-b$ B) $\frac{-b}{d}$ C) $\frac{b}{d}$ D) b

4. Which of the following is a simplification of $4\log_e \sqrt{e^x}$?

A) $4\sqrt{x}$ B) $\frac{1}{2}x$ C) $2x$ D) x^2

5. Which of the following is an expression for $\int \sin^2 6x\, dx$?

A) $\frac{x}{2} - \frac{1}{12}\sin 6x + c$ B) $\frac{x}{2} + \frac{1}{12}\sin 6x + c$

C) $\frac{x}{2} - \frac{1}{24}\sin 12x + c$ D) $\frac{x}{2} + \frac{1}{24}\sin 12x + c$

6. Four female and four male students are to be seated around a circular table.

How many ways can this be done if the males and females must alternate?

A) $4!\times 4!$ B) $3!\times 4!$ C) $3!\times 3!$ D) $2\times 3!\times 4!$

7. The acute angle between the lines $2x-y=0$ and $kx-y=0$ is equal to $\frac{\pi}{4}$.

What is the value of k?

A) $k=3$ or $k=-\frac{1}{3}$ B) $k=-3$ or $k=\frac{1}{3}$

C) $k=-3$ or $k=-\frac{1}{3}$ D) $k=3$ or $k=\frac{1}{3}$

8. Which of the following is equivalent to $\int \frac{dx}{4x^2+9}$, ignoring the constant of integration?

A) $\tan^{-1}\frac{2x}{3}$ B) $\frac{1}{6}\tan^{-1}\frac{2x}{3}$ C) $\frac{2}{3}\tan^{-1}\frac{2x}{3}$ D) $\frac{3}{2}\tan^{-1}\frac{2x}{3}$

9. What is the term independent of x in the expansion of $\left(x^3+\frac{2}{x}\right)^{20}$?

A) $\binom{20}{10}2^{20}$ B) $\binom{20}{5}2^{15}$ C) $\binom{20}{4}2^{16}$ D) $\binom{20}{5}2^{25}$

10. Which of the following is an expression for $\frac{d}{dx}\sin^{-1}(2x-1)$?

A) $\frac{-1}{\sqrt{x(x-1)}}$ B) $\frac{-1}{2\sqrt{x(x-1)}}$ C) $\frac{1}{2\sqrt{x(1-x)}}$ D) $\frac{1}{\sqrt{x(1-x)}}$

Question11.(15 marks)- (show all necessary working) **marks**

a) $A(-3,1)$ and $B(1,-2)$ are two points. Find the coordinates of the point P that divides the interval AB externally in the ratio 3:1. **2**

b) Find $\int \frac{1+2x}{1+x^2}dx$. **2**

c) Use the substitution $x=u-2$ to evaluate $\int_{-1}^{2} \frac{3x+5}{\sqrt{x+2}}dx$. **3**

d) Use mathematical induction to prove that $3^{2n+4}-2^{2n}$ is divisible by 5, for $n \geq 1$. **4**

(e) i) Show that $\frac{\sin 2x}{1+\cos 2x} = \tan x$ **2**

ii) Hence show that $\tan 15^0 + \cot 15^0 = 4$ **2**

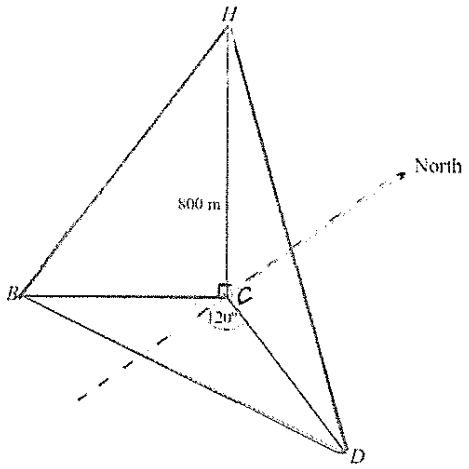
Question 12.(15 marks)

a)i) Find $\frac{d}{dx}\left(\tan^{-1}\frac{x}{3}\right)^2$ **2**

ii) Hence find the exact value of $\int_0^{\sqrt{3}} \frac{\tan^{-1}\frac{x}{3}}{x^2+9}dx$ **2**

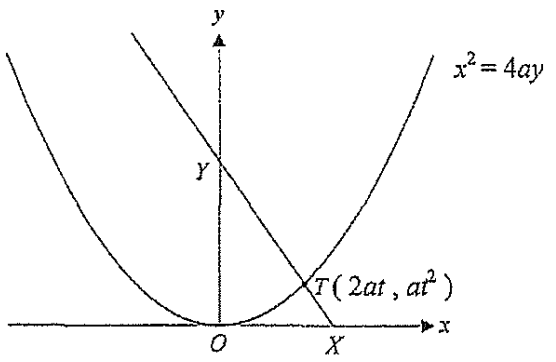
b) The region enclosed by the curve $y=\sin^{-1}x$ and the $y-$axis between $y=0$ and $y=\frac{\pi}{3}$ is rotated about the $y-$axis to form a solid. Find the exact volume of the solid of revolution formed. **3**

c)



The diagram above shows a hot air balloon at point H with altitude $800m$.
The passengers in the balloon can see a barn and a dam below, at points B and D respectively. Point C is directly below the hot airballoon. From the hot air balloon's position, the barn has a bearing of 250^0 and the dam has a bearing of 130^0, and $\angle BCD = 120^0$. The angles of depression to the barn and the dam are 50^0 and 30^0 respectively.
How far is the barn from the dam, to the nearest metre? **4**

d) In the diagram, $T(2at, at^2)$ is a point on the parabola $x^2 = 4ay$.



i) Show that the normal to the parabola at T has equation $x + ty = 2at + at^3$. **2**

ii) This normal cuts the x and y axes at X and Y respectively.

Show that $\frac{TX}{TY} = \frac{t^2}{2}$ **2**

Question 13.(15 marks)

a) A particle is performing Simple Harmonic Motion in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line given by $x = 6\cos^2 t - 2$.

i) Show that $\ddot{x} = -4(x-1)$. **2**

ii) Find the centre and period of the motion. **2**

b) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line. Its velocity $v\ m/s$ is given by $v = -\frac{1}{8}x^3$. The particle is initially 2 metres to the right of O.

i) Show that the acceleration a, is given by : $a = \frac{3}{64}x^5$. **2**

ii) Find an expression for x in terms of t. **3**

c) Consider the function $f(x) = (x+2)^2 - 9$, $-2 \le x \le 2$.

i) Find the equation of the inverse function $f^{-1}(x)$. **1**

ii) On the same diagram, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing clearly the coordinates of the end points and the intercepts on the coordinate axes. **3**

iii) Find the x-coordinate of the point of intersection of the curves $y = f(x)$ and $y = f^{-1}(x)$, giving the answer in simplest exact form. **2**

Question 14(15 marks).

a) The coefficients of x^2 and x^{-1} in the expansion of $\left(ax-\frac{b}{x^2}\right)^5$ are the same.

Show that $a+2b=0$, where a and b are positive integers. **3**

b) Show that $\tan^{-1}\left(\frac{3}{4}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\frac{\pi}{2}$ **2**

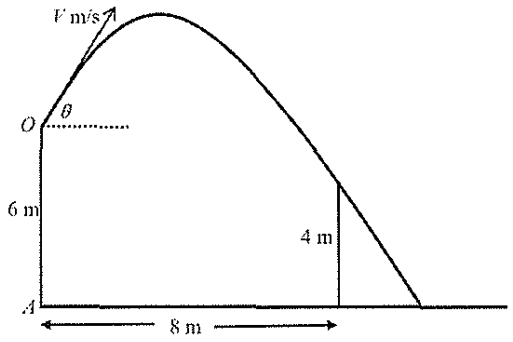
c) i) Neatly sketch the graph of $y=\sin^{-1}\left(\frac{x}{2}\right)$ clearly indicating the domain and range. **2**

ii) By considering the graph in part(i), find the exact value of:

$$\int_0^1 \sin^{-1}\left(\frac{x}{2}\right)dx$$ **2**

d) A projectile is fired from a point O, which is 6 metres above horizontal ground, with initial velocity Vm/s at an angle of θ to the horizontal.

There is a thin vertical post which is 4 metres high and 8 metres horizontally away from a point A, directly below O, as shown in the diagram below.



The equations of motion are given by:

$x=Vt\cos\theta$ and $y=Vt\sin\theta-4.9t^2$ (Do Not prove this)

i) If 2 seconds after projection, the projectile passes just above the top of the post, show that $\tan\theta=2.2$ **2**

ii) Show that the projectile hits the ground approximately 0.3 seconds after passing over the post. **2**

iii) Find the angle that the projectile makes with the ground when it hits the ground, correct to the nearest degree. **2**

Question 15.(15 marks)

a) $P(x) = ax^3 - 7x^2 + kx + 4$ has $x-4$ as a factor. When $P(x)$ is divided by $(x-1)$, the remainder is -6.

i) Determine the values of a and k. **2**

ii) Evaluate $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$. **1**

b) Consider the series $\log_e \frac{a^3}{\sqrt{b}} + \log_e \frac{a^3}{b} + \log_e \frac{a^3}{b\sqrt{b}} + \log_e \frac{a^3}{b^2} + ...$

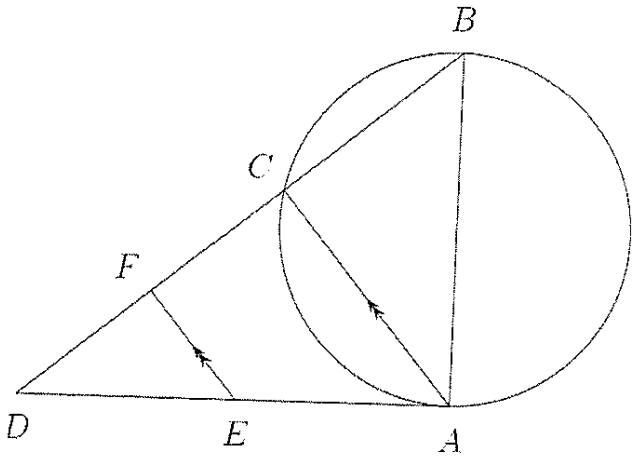
i) Prove that the series is an arithmetic series and state the common difference. **2**

ii) Find an expression for the sum of the first 23 terms of the series, giving your answer in the form $\log_e \frac{a^m}{b^n}$ where m and n are integers. **2**

c) Use the substitution $u = e^{4x} + 9$ to give the exact value of :

$$\int_0^{\ln 2} \frac{3e^{4x}}{\sqrt{e^{4x}+9}}dx$$ **2**

d) AB is a diameter of the circle and C is a point on the circle. The tangent to the circle at A meets BC produced at D. E is apoint on AD and F is a point on CD such that $EF \parallel AC$



i) Copy the diagram in your answer booklet and state why $\angle EAC = \angle ABC$ **1**

ii) Hence show that $EABF$ is a cyclic quadrilateral. **2**

iii) Show that BE is a diameter of the cicle through E, A, B and F. **1**

e) Four adults and four children are to be seated around a circular table.
A particular child cannot sit next to any adult and a particular adult cannot sit next to any child.
Find how many such arrangements are possible. **2**

End of examination!!!