GOSFORD HIGH SCHOOL

2008

Trial HSC

MATHEMATICS EXTENSION I

Time Allowed: 2 Hours + 5 minutes reading time

General Instructions:

- Reading Time – 5 minutes.
- Working time – 2 hours.
- Write using black or blue pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.

TOTAL MARKS – 84

- Attempt Questions 1 – 7
- All questions are of equal value.

QUESTION 1: (12 Marks) Use a **SEPARATE** writing booklet.

		Marks
a.	Solve $\dfrac{x^2-6}{x} \le 1$	2
b.	Two of the roots of $4x^3 - gx^2 + hx - 8 = 0$ are 3 and 7. Find the other root.	2
c.	Find the acute angle between the lines $4x-2y-1=0$ and $y=4x+1$ *(Answer correct to the nearest minute)*	2
d.	The point C(-2, -4) divides the interval AB externally in the ratio 2:1. If the co-ordinates of A are (6,3), find the co-ordinates of B.	2
e.	In how many ways can 8 people be seated at a circular table if two particular persons must be seated next to one another.	2
f.	Without the use of calculus, sketch the graph of the polynomial $P(x)=(x-3)(4-x)^2$ clearly indicating all x,y intercepts.	2

QUESTION 2: (12 Marks) Use a **SEPARATE** writing booklet.

Marks

a. Prove that: 2

$$\frac{\sin\theta}{1+\cos\theta} = t \quad \text{where} \quad t = \tan\frac{\theta}{2}$$

b. Use the substitution $u^2 = x+1$ to evaluate: 3

$$\int_0^3 \frac{x}{\sqrt{x+1}}dx$$

c. If $\alpha = \sin^{-1}\left(\frac{8}{17}\right)$ and $\beta = \tan^{-1}\left(\frac{4}{3}\right)$ 2

calculate the exact value of $\sin(\alpha - \beta)$

d. Find the general solution of: 2

$$2\cos x + 1 = 0$$

e. A mixed tennis team consisting of 3 men and 3 women is to be randomly chosen from 5 men and 6 women.

i. Find the number of possible teams. 1

ii. If one of the men is chosen as captain of the team, find the probability that his wife, who was one of the original 6 women, is also in the team. 2

QUESTION 3: (12 Marks) Use a **SEPARATE** writing booklet.

Marks

a. Find $\int \frac{dx}{5+4x^2}$ 2

b. A spherical balloon is being inflated so that the radius increases at a constant rate of 2mm/sec. Calculate the rate of change of the volume when the radius of the balloon is 6cm. (Answer in cm^3/sec, correct to the nearest whole number.) 2

c. If $y = x + \frac{4}{x}$

i. Show that $\frac{dy}{dx} = \frac{(x-2)(x+2)}{x^2}$ 1

ii. Hence, find the turning points of the curve and determine their nature. 2

d. The velocity, v m/s, of a particle moving in Simple Harmonic Motion is given by:

$$v^2 = 24 - 6x - 3x^2$$

i. Between which two points is the particle oscillating? 1

ii. Hence, or otherwise, find the maximum velocity of the particle. 1

e. Prove, by mathematic induction, that:

$$\sum_{k=1}^{n} \frac{1}{(4k-3)(4k+1)} = \frac{n}{4n+1} \quad \text{for all integers } n \geq 1$$ 3

QUESTION 4: (12 Marks) Use a **SEPARATE** writing booklet.

Marks

a. Given that $y = 3\cos^{-1}(2x-1)$

i. Show that $\dfrac{dy}{dx} = \dfrac{-3}{\sqrt{x-x^2}}$ 2

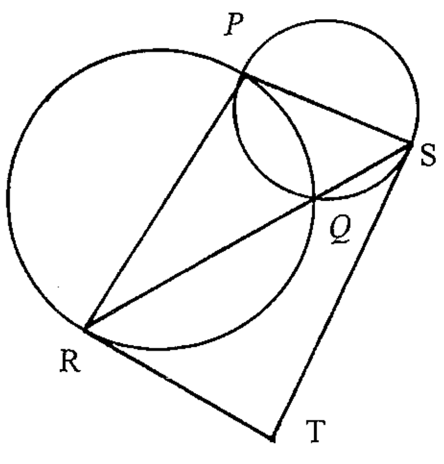
ii. Find the domain and range of $y = 3\cos^{-1}(2x-1)$ 2

iii. Find the value of y if $x = \dfrac{1}{2}$ 1

iv. Sketch the graph of $y = 3\cos^{-1}(2x-1)$ 1

b. Find the volume of the solid of revolution formed when the region bounded by the curve $y = 2 + \cos \pi x$, the x axis, the y axis and the line $x = 1$ is rotated about the x axis. 3

c. The circles intersect at P and Q. 3



RQS is a straight line.
TR and TS are tangents.

i. Copy or trace the diagram into your writing booklet, including a construction line from P to Q.

ii. Prove that PRTS is a cyclic quadrilateral

QUESTION 5: (12 Marks) Use a **SEPARATE** writing booklet.

Marks

a. An object moving in a straight line has an acceleration given by $a = 8x - 3x^2$, where x metres is its displacement from O. 2

If it has a speed of 4 m/s at the origin find its speed when it is 1 metre on the positive side of O

b. $P(4p, 2p^2)$ and $Q(4q, 2q^2)$ are points on the parabola $x^2 = 8y$

i. Show that the equation of PQ is given by the equation: 1

$$y - \frac{1}{2}(p+q)x + 2pq = 0$$

ii. Find the condition that PQ passes through the point (0,-2) 1

iii. If the focus of the parabola is S, prove that 2

$$\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{2}$$

c. i. Given that $f(x) = e^x - 2$, find the equation of the inverse function $f^{-1}(x)$ 1

ii. Sketch both y = f(x) and y= f^{-1}(x) on the same set of axes, clearly indicating all asymptotes and x and y intercepts. 2

iii. Given that the equation $e^x = x + 2$ has a solution close to x = 1.5 use one application of Newton's Method to find a better approximation, x_2 (Answer correct to 2 decimal places.) 2

iv. Clearly indicate the position of this x co-ordinate x_2 on your diagram in part (ii). 1

QUESTION 6: (12 Marks) Use a **SEPARATE** writing booklet.

Marks

a. The individual letters of the word RABBITS are selected at random to form a new 7 letter arrangement. Find the probability that the letters A,B,B and I occur together. 2

b. Stoneware pottery is fired in a closed kiln to a temperature of 1280°C. After reaching this temperature the kiln is turned off and to avoid cracking the cooling process is closely monitored in a surrounding environment whose temperature is maintained at 200°C. The rate of cooling is approximately given by:

$$\frac{dT}{dt} = -k(T - 200)$$

Where T is the temperature in °C, t is the time in hours and k is a positive constant.

i. Show that a solution of this equation is 1

$$T = 200 + Ae^{-kt}, \text{ where A is a constant.}$$

ii. If after 2 hours the pottery has cooled to 800°C, find A and k. (Answer for k correct to 3 decimal places.) 2

iii. The door to the kiln is safe to open when the temperature of the pottery drops below 240°C. How long after the kiln was turned off, to the nearest hour, can the door be opened. 2

c. A particle moves in a straight line. Its displacement, x metres from the origin after t seconds is given by $x = 3\cos 2t + \sqrt{3}\sin 2t$

i. Prove that the particle is moving in Simple Harmonic Motion. 1

ii. Express the displacement in the form 2

$$x = A\cos(2t - \alpha) \quad \text{where} \quad 0 < \alpha < \frac{\pi}{2}$$

iii. Hence, find all the times within the first 5 seconds when the particle is 3 metres to the right of the origin. 2

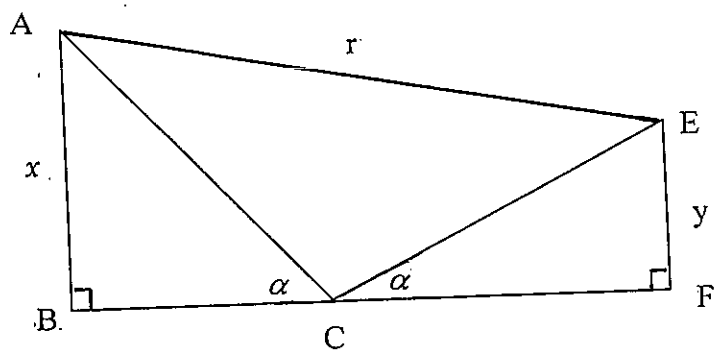
QUESTION 7: (12 Marks) Use a **SEPARATE** writing booklet.

Marks

a. By differentiating, or otherwise, prove that for $x > -1$:

$$\tan^{-1} x + \tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{\pi}{4}$$ 3

b. Given the diagram:

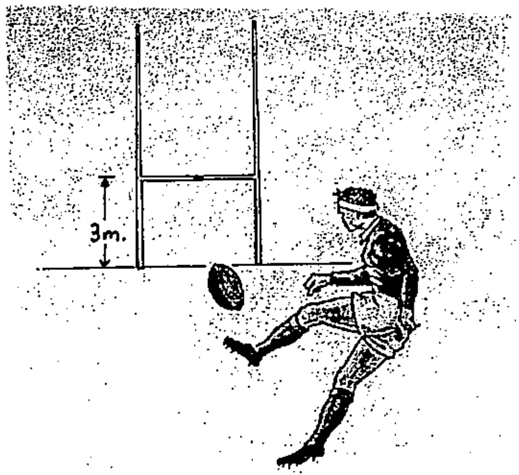


2

Prove that:

$$r^2 = (x+y)^2 \operatorname{cosec}^2\alpha - 4xy$$

c. The well known French Rugby Goal-kicker, Lucky Pierre, never misses a goal attempt.



He perfects his skills by practising relentlessly. At practice, by kicking the ball from O on level ground he aims to just clear the crossbar above the black mark.

He always kicks the ball with an initial velocity of 25m/s and with an angle of θ to the horizontal. After time t, the horizontal and vertical displacements of the ball from O, are x and y respectively. Ignoring wind resistance and taking $g = 10\text{m/s}^2$):

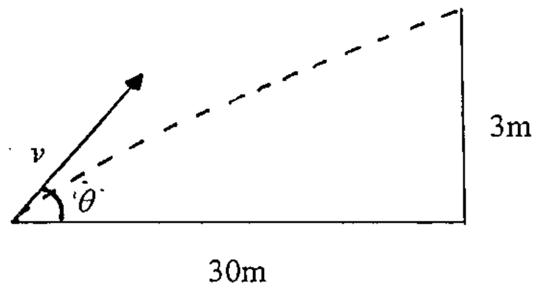
(Question 7 continues next page)

Question 7 *(continued)*

i. Show that the cartesian equation of the path of the ball is given by 2

$$y = x\tan\theta - \frac{1}{125}x^2\sec^2\theta$$

ii. On one occasion he kicked the ball from O on level ground to just clear the crossbar 30 metres away at a height of 3 metres. 3



Find the possible angles(s) of projection θ (to the nearest minute)

iii. Find the maximum height achieved by the ball given that it just clears the crossbar. Express your answer correct to the nearest metre. 2

END OF EXAM