# HORNSBY GIRLS' HIGH SCHOOL

## 2007
## TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics

**General Instructions**

- **Reading Time- 5 minutes**
- **Working Time – 3 hours**
- **Write using a black or blue pen**
- **Approved calculators may be used**
- **A table of standard integrals is provided at the back of this paper.**
- **All necessary working should be shown for every question.**
- **Begin each question on a fresh sheet of paper.**

Total marks (120)

- **Attempt Questions 1-10**
- **All questions are of equal value**

**Total Marks – 120**
**Attempt Questions 1-10**
**All Questions are of equal value**

Begin each question on a NEW SHEET of paper, writing your student number and question number at the top of the page. Extra paper is available.

**Question 1** (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) State the exact value of $\cos 135°$ **2**

(b) Solve for $x$: $2x - 11 \le 5x + 6$ **2**

(c) Find the arc length of a sector with central angle 60° and radius 12cm. Leave answer in exact form. **2**

(d) Evaluate $\log_3 5$ correct to 2 decimal places **2**

(e) Solve for $x$: $|2x-1| = 17$ **2**

(f) Jack's Jean Junction had a sale on its jeans. Susie paid $108 for a pair of jeans after a 20% discount. Calculate the original price of her jeans. **2**

**Question 2** (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Differentiate the following

(i) $y = 3x^3 - 4x^2 - 8x + 7$ **1**

(ii) $y = e^{4x}$ **1**

(iii) $y = x\,sinx$ **2**

(b) Evaluate: $\sum_{k=1}^{4} k^3$ **2**

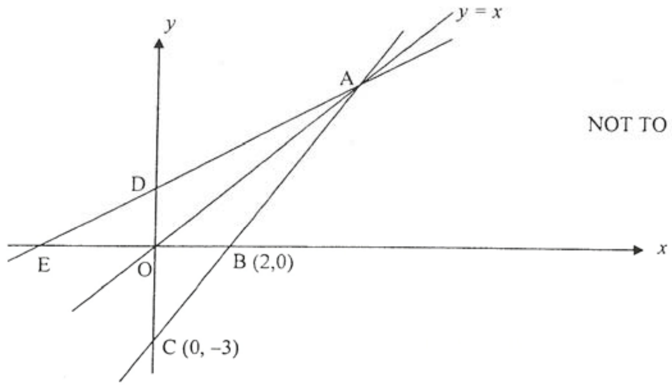
(c) Find the limiting sum of the geometric series: $8 - 4 + 2 - 1 \ldots.$ **2**

(d) State the domain and range of the function $y = 3\sqrt{x} + 1$ **2**

(e) Show that the centre and radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is $(2, -3)$ and 5cm respectively. **2**

**Question 3** (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a)




NOT TO SCALE

The diagram shows the line $y = x$ and the points $B(2, 0)$ and $C(0, -3)$. The line BC intersects the line $y = x$ at the point A. The points D and E lie on the $y$-axis and $x$-axis respectively.

(i) Show that the gradient of BC is $1\frac{1}{2}$ **1**

(ii) Find the equation of the line BC. **2**

(iii) Find the coordinates of the point A. **2**

The coordinates of the point E are $(-3,0)$.

(iv) Show that the equation of the line AE is $2x - 3y + 6 = 0$. **2**

(v) Find the coordinates of the point D. **1**

(vi) Prove that $\Delta AOB \equiv \Delta AOD$. **2**

(vii) Find the area of the quadrilateral OBAD. **2**

**Question 4** (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Find:

(i) $\int\left(\frac{4}{x}-\frac{1}{x^2}\right)dx$ **2**

(ii) $\int_{-1}^{1}(3x-5)^3\,dx$ **2**

(b) If the 4th term and 13th term of an arithmetic sequence are 16 and −2 respectively, find the first term and common difference. **2**

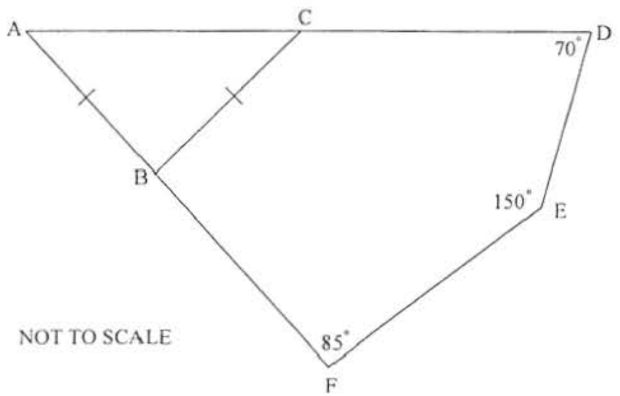
(c) For the quadratic equation $3x+5-2x^2=0$ find:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **2**

(d) Find the size of $\angle ABC$, giving reasons. **2**




**Question 5** (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The displacement of a particle is shown in the table below. The particle comes to rest at $t=3$. Calculate the distance travelled by the particle in the first 5 seconds. **1**

| t seconds | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $x$ metres | − 44 | − 12 | 8 | 23 | 10 | 2 |

(b) If $\log_x a = 4.2$ and $\log_x b = 3$ find:

(i) $\log_x ab$ **1**

(ii) $\log_x \sqrt{\frac{x}{a}}$ **2**

(c) Consider the curve given by $y=2x^3-3x^2-12x$

(i) Find $\frac{dy}{dx}$ **1**

(ii) Find the coordinates of the two stationary points and determine their nature **3**

(iii) Find the coordinates of the inflexion point **2**

(iv) Sketch the curve for $-2\le x\le 3$. **2**

**Question 6** (12 marks) Use a SEPARATE sheet of paper. **Marks**

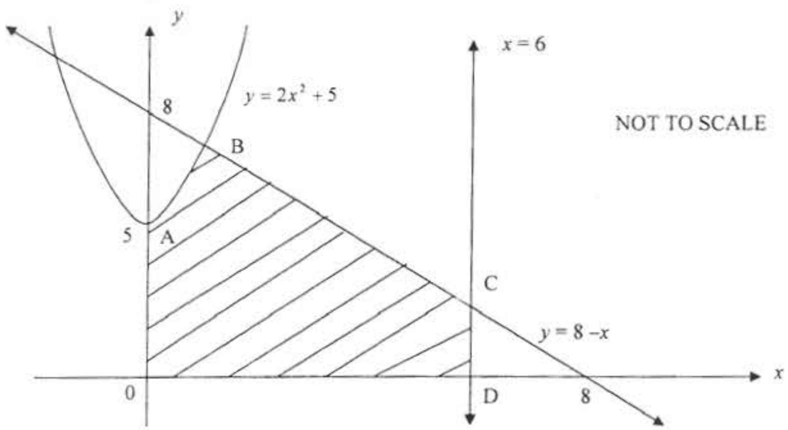
(a) A parabola has equation $y^2 + 8y - 3 = 1 - 20x$

(i) Show that the parabola can be written as $(y+4)^2 = -20(x-1)$ **2**

(ii) Find the coordinates of the vertex **1**

(iii) Find the coordinates of the focus **1**

(b)




In the diagram, the shaded region OABCD is bounded by the curve $y = 2x^2 + 5$, the lines $y = 8 - x$, $x = 6$ and the $x$ and $y$-axes.

(i) Show that B has coordinates (1,7) **2**

(ii) Calculate the area of the shaded region.
Leave your answer in exact form. **3**

(c) A die is rolled twice and the outcomes on the uppermost faces are added together.

(i) Find the probability of getting a score less than 10 **2**

(ii) If it is certain that a 2 will appear on one of the uppermost faces, find the probability of getting a score less than 5. **1**

**Question 7** (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) Solve for $x$ : $(2^x)^x = 4^{1-x}$ **2**

(b) Let A be the point (-2, 0) and B be the point (6, 0). At P $(x,y)$, PA meets PB at right angles.

(i) Show that the gradient of PA is $m_1 = \dfrac{y}{x+2}$ **1**

(ii) Find an equation for the locus of P **2**

(c) Boat A sails 15km for port P on a bearing of 055°.
Boat B sails from P for 25 km on a bearing of 135°

(i) Show the angle APB = 80° **1**

(ii) Calculate their distance apart to 1 decimal place. **2**

(d) The amount, A, of caffeine left in the blood t hours after consuming food or drink containing caffeine is calculated using the formula

$$A = Qe^{-0.17t}$$

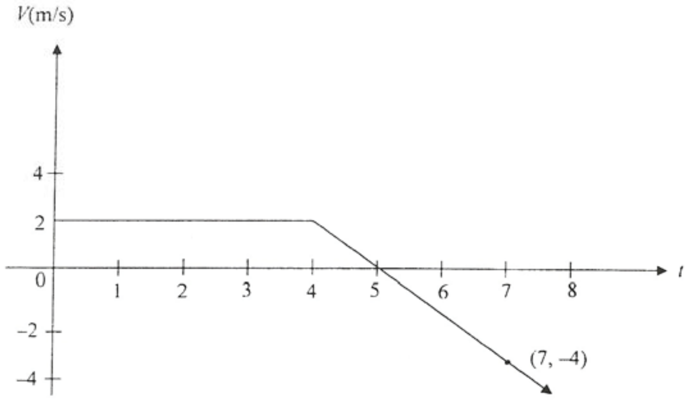
where Q is the original quantity of caffeine consumed and t is the time in hours after consuming the caffeine.

(i) Jane likes her coffee strong. At 8am she drank 2 cups of coffee containing a total of 25 mg of caffeine. How much caffeine will be in her blood at 12 noon? Answer correct to 2 decimal places. **2**

(ii) Calculate the time when 80% of the caffeine she had consumed had been removed from her blood. Answer correct to the nearest minute. **2**

**Question 8** (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The diagram shows the velocity, $V$m/s, of a particle moving in a straight line at time t seconds.




(i) When was the particle stationary? **1**

(ii) Between what times was the acceleration zero? **2**

(iii) What was the acceleration after 5 seconds? **2**

(iv) How far was the particle from its starting position and in what direction was it travelling 7 seconds after the start of the motion? **2**

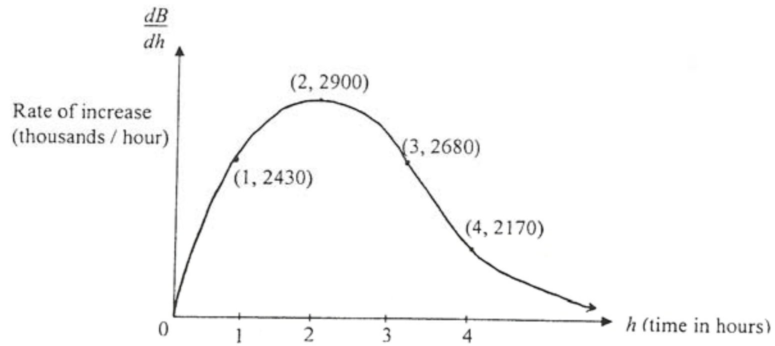
(b) (i) Prove the identity $\tan^2\theta + 1 = \sec^2\theta$ **2**

(ii) The section of the curve $y = \tan 2x$ which is between $x = \theta$ and $x = \frac{\pi}{8}$ is rotated about the $x$ – axis. Find the exact volume of the solid. **3**

**Question 9** (12 marks) Use a SEPARATE sheet of paper. **Marks**

(a) The number of bacteria ($B$) in an untreated infection is increasing. The diagram shows the birth rate of bacteria ($h$, in thousands per hour).

The equation of the rate of increase is $\frac{dB}{dh} = 4he^{-0.5h}$.




(i) Use the trapezoidal rule with 4 function values to estimate the total number of bacteria born in the first 3 hours. **2**

(ii) What value does the rate of increase in the number of bacteria approach as $h$ increases? **1**

(iii) Initially, the number of bacteria present is 1000. Without integrating, sketch a curve to represent the number of bacteria present in the first $h$ hours of the infection. **2**

(b) Richard borrowed \$100 000 to buy an apartment, which he is going to rent. He plans to repay \$500 every fortnight from his salary. On every 4th week he will repay an extra \$800 from the rent he receives. Interest on the loan is 6.5% pa reducing fortnightly.
Let $A_n$ = the amount owing on the loan after $n$ fortnights and R = 1.0025.

(i) Show that $A_2 = 100000R^2 - 500(R+1) - 800$ **1**

(ii) Show that $A_4 = 100000R^4 - 500(R^3 + R^2 + R + 1) - 800(R^2 + 1)$. **2**

(iii) Hence show that $A_{100}$ can be calculated by evaluating:

$$A_{100} = 100000R^{100} - 500 \times \frac{R^{100} - 1}{R - 1} - 800 \times \frac{R^{100} - 1}{R^2 - 1}$$ **2**

(iv) What percentage of the money repaid in the first 100 repayments will be interest? **2**

**Question 10** (12 marks) Use a SEPARATE sheet of paper. **Marks**

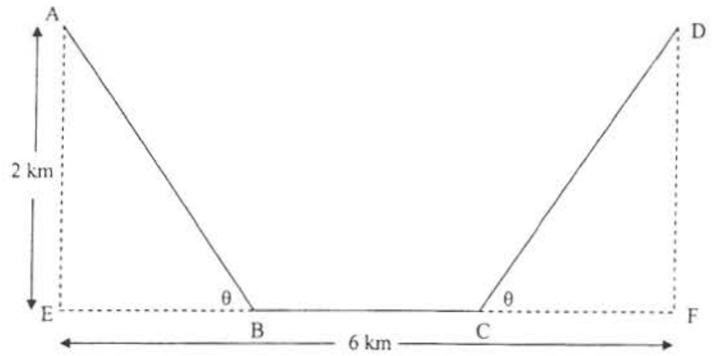
(a) (i) Show that the equation $y = \log_e(x-1)$ can be written as $x = 1 + e^y$ **1**

(ii) Find the size of the area enclosed by the curve $y = \log_e(x-1)$, the $x$-axis and the line $x = 5$. **2**

(b) Using the quotient rule or otherwise, show that the derivative of

$\frac{4 - 2Cos\theta}{Sin\theta}$ is $\frac{2 - 4Cos\theta}{Sin^2\theta}$ **2**

(c)




In the diagram above, A represents Claudia's home and D her school.
AEFD is a rectangle where AE = 2 km and EF = 6 km.
Claudia walks to school every morning along the route ABCD.
Along AB and CD she walks with a speed of 4km/hr but along BC she doubles her speed.

(i) Show that the total time needed for Claudia to walk to school, in minutes, is given by:

$$t = 15\left(3 + \frac{4 - 2Cos\theta}{Sin\theta}\right)$$ **4**

(ii) Using your answer to part (b) or otherwise, show that the minimum amount of time needed for Claudia to walk to school is approximately 97 minutes. **3**

**END OF EXAMINATION**

Hornsby Girls High School 2007 2U Trial HSC Solutions

12 - TRIAL - 2007 - MATHEMATICS

a) $\cos 135 = -\frac{\sqrt{2}}{2}$

$2x - 11 \leq 5x + 6$

$-3x \leq 17$

$x \geq -\frac{17}{3}$

$x \geq -5\frac{2}{3}$

$0° = \frac{\pi}{3}$

$l = r\theta$

$l = \frac{\pi}{3} \times 12$

$l = 4\pi$ cm

$g_3 5 = \frac{\log 5}{\log 3}$

$x - 1 = 17$ , $2x - 1 = -17$

$2x = 18$    $2x = -16$

$x = 9$    $x = -8$

108 represents 80%

$0.8x = 108$

$\therefore x = \$135$

a) i) $y' = 9x^2 - 8x - 8$   1

$= 4e^{4x}$   1

$' = x\cos x + \sin x$   2

$k^3 = 1 + 8 + 27 + 64$

$= 100$   2

$8$ , $r = -\frac{1}{2}$

$\infty = \frac{8}{\frac{3}{2}}$

$S_\infty = 5\frac{1}{3}$   2

ain : $\{x \in \mathbb{R}$ where $x \geq 0\}$   1

ge : $\{y \in \mathbb{R}$ where $y \geq 1\}$   1

$-4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 25$

centre is $(2, -3)$, $r = 5$

1    1

Q3) a) i) $m_{BC} = \frac{3}{2} = 1\frac{1}{2}$

ii) $y = \frac{3}{2}x - 3$ or $3x - 2y - 6 = 0$

iii) $3x - 2x - 6 = 0$    $3(x) - 2y - 6 = 0$

$x - 6 = 0$

$\therefore x = 6$

$\therefore A(6, 6)$

iv) $(6,6)$ & $(-3,0)$

$\frac{y}{x+3} = \frac{6}{9}$

$\frac{y}{x+3} = \frac{2}{3}$

$3y = 2x + 6$

$\therefore$ equation AE is $2x - 3y + 6 = 0$

v) For D sub $x = 0$

$-3y + 6 = 0$

$\therefore y = 2$   $\therefore$ D is $(0, 2)$

vi) In $\Delta$s AOB & AOD,

- OA is common
- OB = OD = 2 units
- $\angle AOB = \angle AOD = 45°$ (line $y = x$ makes 45° with the coordinate axes)

$\therefore \Delta AOB \equiv \Delta AOD$ (SAS)

vii) Area $\Delta AOB = \frac{1}{2} \times 2 \times 6$

$= 6$

$\therefore$ Area OBAD $= 2 \times 6 = 12$ u²

Q4) a) i) $\int \frac{4}{x} - x^{-2}\, dx$

$= 4\ln x + \frac{1}{x} + c$

ii) $\int_{-1}^{1} (3x-5)^3\, dx = \left[\frac{(3x-5)^4}{12}\right]_{-1}^{1}$

$= \frac{(-2)^4 - (-8)^4}{12}$

$= -340$

Q4 b) $a + 3d = 16$ —①

$a + 12d = -2$ —②

$-9d = 18$

$\therefore d = -2$

sub into ① $a + (3 \times -2) = 16$

$\therefore a = 22$

$\therefore$ first term is 22 & comm diff = −2

c) $-2x^2 + 3x - 5 = 0$

i) $\alpha + \beta = \frac{3}{2}$

ii) $\alpha\beta = -\frac{5}{2}$

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{3}{2}\right)^2 - 2\left(-\frac{5}{2}\right)$

$= \frac{29}{4}$

d) $\angle A = 55°$ ($\angle$ sum of quad = 360°)

$\angle ACB = 55°$ (base $\angle$s of isos $\Delta$ =)

$\therefore \angle ABC = 70°$ ($\angle$ sum of $\Delta$ = 180°)

Q5) a) $32 + 20 + 15 + 13 + 8 = 88$ m

b) $\log_x ab = \log_x a + \log_x b$

$= 7.2$

ii) $\log_x \sqrt{\frac{x}{a}} = \frac{1}{2}\left[\log_x x - \log_x a\right]$

$= \frac{1}{2}\left[1 - 4.2\right]$

$= -1.6$

c) $y = 2x^3 - 3x^2 - 12x$

i) $\frac{dy}{dx} = 6x^2 - 6x - 12$

ii) $6x^2 - 6x - 12 = 0$ for stat pts

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2$, $x = -1$

$y = -20$, $7$

$\frac{d^2y}{dx^2} = 12x - 6$

when $x = 2$, $\frac{d^2y}{dx^2} = 18 > 0$ $\therefore (2, -20)$ is a min TP

when $x = -1$, $\frac{d^2y}{dx^2} = -18 < 0$ $\therefore (-1, 7)$ is a max TP.

iii) $y'' = 12x - 6 = 0$ for POI

$12x = 6$

$\therefore x = \frac{1}{2}$

$y = -6\frac{1}{2}$

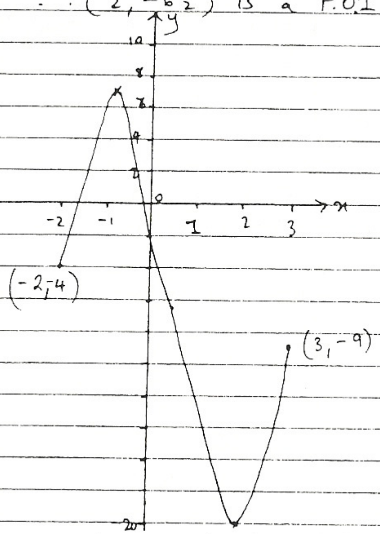
test

| $0$ | $\frac{1}{2}$ | $1$ |
|---|---|---|
| $-6$ | $0$ | $6$ |

change in concavity

$\therefore \left(\frac{1}{2}, -6\frac{1}{2}\right)$ is a P.O.I

iv)




when $x = -2$, $y = -4$

when $x = 3$, $y = -9$

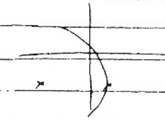
Q6) a) i) $y^2 + 8y + 16 = -20x + 1 + 3 + 16$

$y^2 + 8y + 16 = -20x + 20$

$(y+4)^2 = -20(x-1)$   2

ii) vertex $(1, -4)$ 1

iii)



$a = 5$

$\therefore$ focus $(-4, -4)$ 1

3)(i) ...

$2x^2 + x - 3 = 0$

$(2x+3)(x-1) = 0$

$x = -\frac{3}{2}, \quad x = 1$

$y = 7$

$\therefore$ B has coord $(1, 7)$ 2

ii) Area $= \int_0^1 2x^2 + 5 \, dx + \int_1^6 8 - x \, dx$

$= \left[\frac{2x^3}{3} + 5x\right]_0^1 + \left[8x - \frac{x^2}{2}\right]_1^6$

$= \left(\frac{2}{3} + 5\right) + \left((48 - 18) - \left(8 - \frac{1}{2}\right)\right)$

$= 5\frac{2}{3} + 30 - 7\frac{1}{2}$

$A = 28\frac{1}{6} u^2$ 3

)

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 6 | 16 | 26 | 36 | 46 | 56 | 66 |
| 5 | 15 | 25 | 35 | 45 | 55 | 65 |
| 4 | 14 | 24 | 34 | 44 | 54 | 64 |
| 3 | 13 | 23 | 33 | 43 | 53 | 63 |
| 2 | 12 | 22 | 32 | 42 | 52 | 62 |
| 1 | 11 | 21 | 31 | 41 | 51 | 61 |

$P(\text{sum} < 10) = \frac{30}{36}$

$= \frac{5}{6}$ 2

$P(\text{sum} < 5) = \frac{3}{11}$ 1

7)a) $(2^x)^x = 4^{1-x}$

$2^{x^2} = 2^{2-2x}$

$x^2 = 2 - 2x$

$x^2 + 2x - 2 = 0$

$x = \frac{-2 \pm \sqrt{12}}{2}$

$x = -1 \pm \sqrt{3}$

i) $P(x, y)$ $A(-2, 0)$ $B(6, 0)$

$M_{PA} = \frac{y}{x+2}$

$M_{PB} = \frac{y}{x-6}$

$\therefore \frac{y}{x+2} = \frac{6-x}{y}$

$\therefore y^2 = 6x - x^2 + 12 - 2x$

$y^2 = 4x - x^2 + 12$

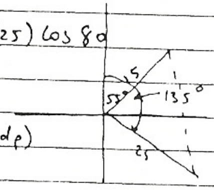
$\therefore x^2 + y^2 - 4x - 12 = 0$

c) i) $\angle APB = 80°$

ii) $AB^2 = 15^2 + 25^2 - 2(15)(25)\cos 80$

$AB = 26.8284\ldots$

$\therefore AB = 26.8$ km (1 dp)




d) $A = Qe^{-0.17t}$

i) $A = 25e^{-0.17(4)}$

$A = 12.665\ldots$

$A = 12.67$ mg.

ii) let the original amount $= 100$.

$20 = 100e^{-0.17t}$

$0.2 = e^{-0.17t}$

$\ln 0.2 = -0.17t$

$\therefore t = \frac{\ln 0.2}{-0.17}$

$\therefore t = 9.4672$

$t = 9$ hrs 28 min (nearest min)

$\therefore 8 + 9$ hrs 28 min $= 5:28$ pm.

Q8.i) $v = 0$ when $t = 5$ sec.

ii) acc$^n$ was zero between $t = 0$ & $t = 4$ sec.

iii) acc$^n$ $= \frac{-4}{2}$

$= -2$

$\therefore$ acc$^n$ was $2 m/s^2$

iv) distance = area under the curve

$= \frac{2}{2}(4 + 5) + \frac{1}{2}(2) \times 4$

$= 13$ metres.

$\therefore$ particle had travelled 13 metres.

It is travelling in the negative direction after 7 seconds.

8b) i) Prove $\tan^2\theta + 1 = \sec^2\theta$

LHS $= \tan^2\theta + 1$

$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$

$= \sec^2\theta$

$=$ RHS

$\therefore$ LHS = RHS QED.

ii) $V = \pi \int_0^{\frac{\pi}{8}} \tan^2 2x \, dx$

$= \pi \int_0^{\frac{\pi}{8}} \sec^2 2x - 1 \, dx$

$= \pi \left[\frac{1}{2}\tan 2x - x\right]_0^{\frac{\pi}{8}}$

$= \pi \left[\left(\frac{1}{2} - \frac{\pi}{8}\right) - 0\right]$

$V = \pi\left[\frac{4 - \pi}{8}\right] u^3$

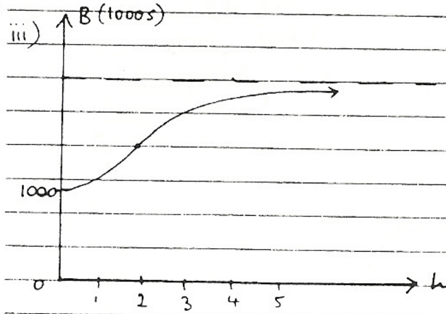
Q9.a) i) Number Bacteria $\doteqdot$

$\frac{1}{2} \times \left(0 + 2680 + 2(2430 + 2900)\right)$

$\doteqdot 6670\,000$

ii) As $h \to \infty$, $\frac{dB}{dh} \to 0$

iii)




b) Let $A_n$ = amount owing after n fortnights

$A_1 = 100000(1.0025) - 500$

$A_2 = [100\,000(1.0025) - 500](1.0025) - 500 - 800$

$\therefore A_2 = 100\,000(1.0025)^2 - 500(1 + 1.0025) - 800$

$\therefore A_2 = 100000R^2 - 500(1 + R) - 800$

$A_3 = [100\,000R^2 - 500(1 + R) - 800](R) -$

$A_3 = 100\,000R^3 - 500R(1 + R) - 800R\ldots$

$A_3 = 100\,000R^3 - 500(R^2 + R + 1) - 80\ldots$

$A_4 = [100\,000R^3 - 500(1 + R + R^2) - 800R]R -$

$= 100\,000R^4 - 500(R^3 + R^2 + R + 1) - 800\ldots$

$\therefore A_{100} = 100000R^{100} - 500(1 + R + \ldots$

$- 800(1 + R^2 + \ldots + R^{50})$

$= 100000R^{100} - 500\left(\frac{R^{100} - 1}{R - 1}\right)$

$- 800\left(\frac{(R^2)^{50} - 1}{R^2 - 1}\right)$

$= 100\,000R^{100} - 500\frac{(R^{100} - 1)}{R - 1} - 800(R\ldots$

iv) $A_{100} = 128\,362.49 - 56\,724.97\ldots$

$- 45\,323.328$

$A_{100} = \$26\,314.18$

Total repaid $= 500 \times 100 + 800 \times 50$

$= \$90\,000$

$\therefore$ Interest $= \$16\,314$

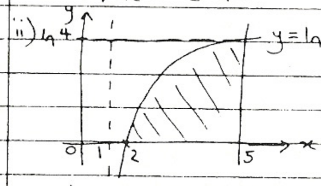
$\therefore$ percentage interest $\doteqdot 18.1\%$

Q10a) i) $y = \log_e(x - 1)$

$e^y = x - 1$

$\therefore x = e^y + 1$ 1

ii)




when $x = 5$, $y = \ln 4$

$\therefore$ Shaded Area $= 5\ln 4 - \int_0^{\ln 4} e^y + 1$

$= 5\ln 4 - [e^y + y]_0^{\ln 4}$

$= 5\ln 4 - [(4 + \ln 4) - 1]$

$= 5\ln 4 - 4 - \ln 4 + 1$

$= 4\ln 4 - 3$ 2

Q10 (cont.)

b) i) $\frac{d}{dx}\left(\frac{4-2\cos x}{\sin x}\right) = \frac{\sin x(2\sin x) - (4-2\cos x)\cos x}{\sin^2 x}$

$= \frac{2\sin^2 x - 4\cos x + 2\cos^2 x}{\sin^2 x}$

$= \frac{2\sin^2 x + 2\cos^2 x - 4\cos x}{\sin^2 x}$

$= \frac{2 - 4\cos x}{\sin^2 x}$ QED!

2

ii) $\sin\theta = \frac{2}{AB}$

$\therefore AB = \frac{2}{\sin\theta}$ & $CD = \frac{2}{\sin\theta}$

$\therefore$ time $AB = \frac{2}{4\sin\theta}$ time $CD = \frac{2}{4\sin\theta}$

$= \frac{1}{2\sin\theta}$ $= \frac{1}{2\sin\theta}$

$EB \Rightarrow \tan\theta = \frac{2}{EB}$

$\therefore EB = \frac{2}{\tan\theta}$

$\therefore BC = 6 - \frac{2}{\tan\theta} - \frac{2}{\tan\theta}$

$= 6 - \frac{4}{\tan\theta}$

$\therefore$ time $BC = \frac{6 - \frac{4}{\tan\theta}}{8} = \frac{3\tan\theta - 2}{4\tan\theta}$

$\therefore t = \frac{1}{2\sin\theta} + \frac{1}{2\sin\theta} + \frac{3\tan\theta - 2}{4\tan\theta}$

$= \frac{1}{\sin\theta} + \frac{3}{4} - \frac{\cos\theta}{2\sin\theta}$

$= \left(\frac{3}{4} + \frac{2 - \cos\theta}{2\sin\theta}\right) \times 60$ minutes

$= 45 + \frac{60 - 30\cos\theta}{\sin\theta}$

$t = 15\left(3 + \frac{4 - 2\cos\theta}{\sin\theta}\right)$ QED!

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iii) $t' = 15\left(\frac{2 - 4\cos\theta}{\sin^2\theta}\right) = 0$ for min time.

$t' = 2 - 4\cos\theta = 0$ $\therefore \theta = 60°$

$2 = 4\cos\theta$

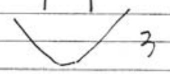
$\therefore \cos\theta = \frac{1}{2}$

$\therefore t = 15\left(3 + \frac{4 - 2\cos 60}{\sin 60}\right)$

$t = 96.96$

$\therefore t \doteqdot 97$ minutes

| 50° | 60° | 70° |
|---|---|---|
| −14.6 | 0 | 28.16 |



3

$\therefore$ min when $t = 97$