



**KNOX  
GRAMMAR  
SCHOOL**

**2023** TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Advanced

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## General Instructions

- Reading time – 10 minutes.
- Working time – 3 hours.
- Write using black pen.
- NESA approved calculators may be used.
- A reference sheet is provided with this paper.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.

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**Total marks:  
100**

### Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.

### Section II – 90 marks (pages 6 – 34)

- Attempt Questions 11 – 18.
- Allow about 2 hours and 45 minutes for this section.

Question	MC	11	12	13	14	15	16	17	18	Total
Mark	/10	/12	/12	/12	/12	/12	/10	/10	/10	

## Teachers:

Charlier C.  
Cheah S.  
Ekanayake P.  
Meli S.  
Menzies D.  
Naidoo V.  
Willcocks A.  
Zaidi S.

**Examiner:** MED

**Number of Students in Course:** 175

## Section I

10 marks

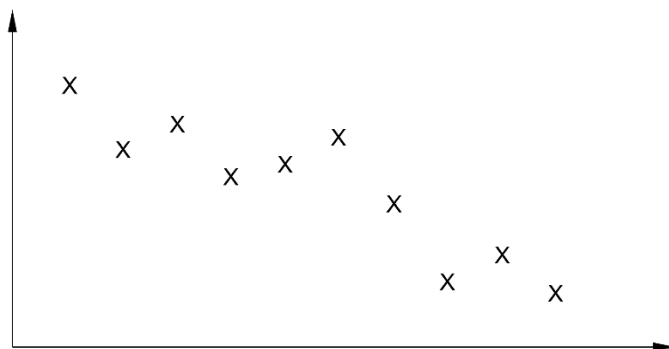
Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1. Consider the bivariate data shown on the scatterplot below.



Which one of the following values is the best estimate for Pearson's correlation coefficient for this data?

- (A)  $-0.9$
- (B)  $-0.2$
- (C)  $0.2$
- (D)  $0.9$
2. The graph of the function  $y = f(x)$  is moved 3 units to the left. Which one of the following represents the new function?
- (A)  $y = f(x+3)$
- (B)  $y = f(x)+3$
- (C)  $y = f(x-3)$
- (D)  $y = f(x)-3$

3. The height of the tide in a harbour can be modelled using the sine function.  
The time,  $t$  in hours, between high tide and low tide is 6 hours.  
Which one of the following could be the function representing the height of the tide?

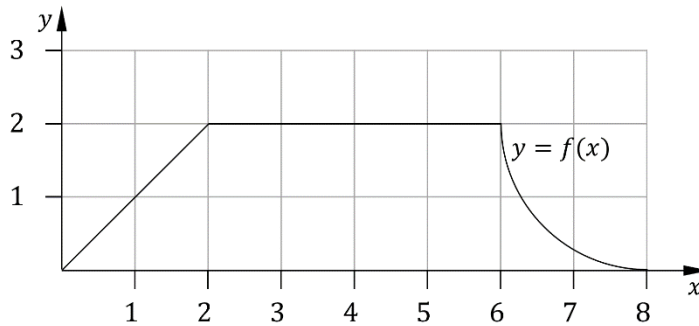
(A)  $h = \sin\left(\frac{\pi t}{3}\right)$

(B)  $h = \sin\left(\frac{\pi t}{6}\right)$

(C)  $h = \sin\left(\frac{\pi t}{12}\right)$

(D)  $h = \sin(2\pi t)$

4. The graph  $y = f(x)$  is shown below:



DRAWN  
TO  
SCALE

What is the exact value of  $\int_0^8 f(x) dx$ ?

- (A)  $10 + \pi$
- (B)  $10 - \pi$
- (C)  $14 + \pi$
- (D)  $14 - \pi$
5. The product of the 1<sup>st</sup> term and 2<sup>nd</sup> term of a geometric sequence is 48 while the product of the 3<sup>rd</sup> term and the 4<sup>th</sup> term of the sequence is 3888.  
What is the product of the 4<sup>th</sup> term and the 5<sup>th</sup> term of the sequence?
- (A) 11 664
- (B) 15 552
- (C) 34 992
- (D) 186 624

6. The displacement of a particle moving along the  $x$  axis at time  $t$  is given by:

$$x = t^3 + \ln(2t^3 + 1) \text{ for } t \geq 0$$

How many times is the particle stationary?

- (A) 0  
(B) 1  
(C) 2  
(D) 3
7.  $\triangle ABC$  is an isosceles triangle such that  $AB = AC = 8$  cm.  
Given that the area of  $\triangle ABC$  is  $16 \text{ cm}^2$ , which one of the following gives the possible values of  $\angle ABC$ ?

- (A)  $15^\circ, 70^\circ$   
(B)  $15^\circ, 75^\circ$   
(C)  $30^\circ, 120^\circ$   
(D)  $30^\circ, 150^\circ$

8. It is given that  $P(A) = \frac{1}{2}$  and  $P(A \cup B) = \frac{3}{5}$

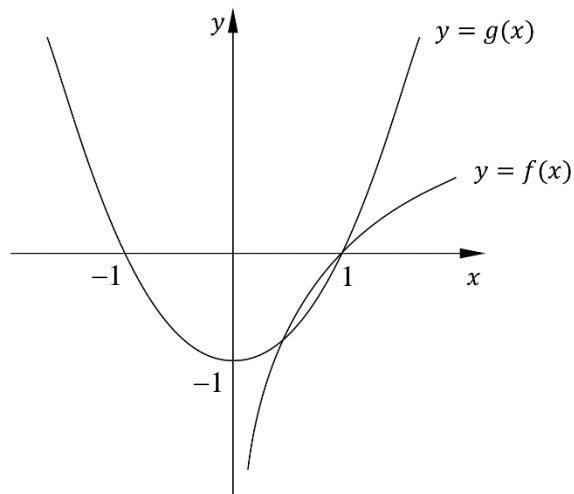
What is the value of  $P(\bar{B} | \bar{A})$ ?

- (A)  $\frac{1}{4}$   
(B)  $\frac{3}{10}$   
(C)  $\frac{3}{4}$   
(D)  $\frac{4}{5}$

9. What is the derivative of  $(\tan^2 x + 1)^2$ ?

- (A)  $4\sec^3 x$
- (B)  $4\sec^4 x$
- (C)  $4\sec^3 x \tan x$
- (D)  $4\sec^4 x \tan x$

10. The graph shows  $y = f(x)$  and  $y = g(x)$ , where  $f(x) = \ln x$  and  $g(x) = x^2 - 1$ .



How many solutions does the equation  $[f(x)]^2 - [g(x)]^2 = 0$  have?

- (A) 0
- (B) 1
- (C) 2
- (D) 3



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Student Number

**2023** TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Advanced

## Section II Answer Booklet

### Section II

90 marks

Attempt Questions 11-18

Allow about 2 hours 45 minutes for this section

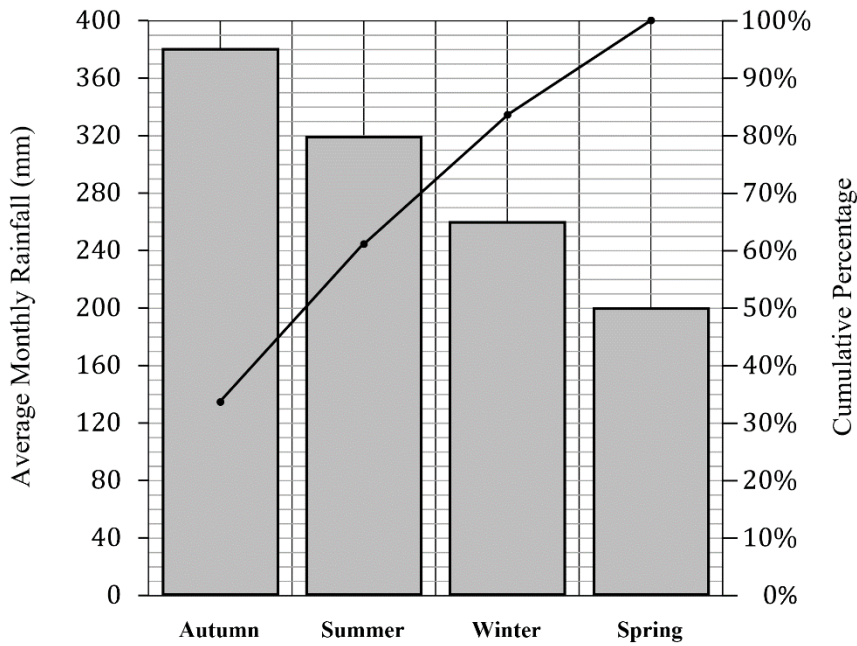
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#### Instructions

- Answer the questions in the spaces provided. These spaces generally provide guidance for the expected length of the response.
  - Your responses should include relevant mathematical reasoning and/or calculations.
  - Extra writing space is provided at the back of this booklet.  
If you use this space, clearly indicate which question you are answering.
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**Question 11 (12 marks)**

(a) Below is a Pareto chart showing the average seasonal rainfall in Sydney.



What percentage of the annual rainfall occurs in the two driest seasons?  
Give your answer as a whole percentage.

**1**

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(b) Find the primitive of  $f(x) = \frac{1}{2x-3}$

**2**

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(c) Calculate  $\int_{\ln 2}^{2\ln 2} e^{2x} dx$

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(d) It is given that  $f''(x) = 6x$  and that  $f(x)$  has a stationary point at  $(-1, 2)$ .  
Find  $f(x)$ .

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(e) It is given that  $1 + 2x - 3x^2 \geq 0$  in the domain  $[0, 1]$ .

Prove that  $f(x) = 1 + 2x - 3x^2$  is a probability density function for  $[0, 1]$ .

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**Question 12 (12 marks)**

- (a) A normally distributed variable,  $X$ , has a mean of 2 and a standard deviation of 0.5. Use the empirical rule to find  $P(0.5 \leq X \leq 3)$ . **2**

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- (b) Find the second derivative of  $y = x(x+1)^3$ , leaving your answer in fully factorised form. **3**

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(c) The circle  $x^2 + y^2 - 6x + 8y - 11 = 0$  is transformed by a horizontal translation to the left by 4 units and a vertical translation up 3 units.

What is the centre and radius of the new circle?

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(d) The table below shows the probability distribution of a discrete random variable  $X$ .

$x$	0	2	4	5	8	9
$P(X = x)$	$k^2$	0.16	0.18	0.3	$k$	0.12

(i) Show that  $k = 0.2$

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(ii) Calculate  $E(X)$ .

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(e) (i) Show that  $\frac{d}{dx}(x \log_e x - x) = \log_e x$

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(ii) Hence find  $\int_1^e \log_e x \, dx$  as an exact value.

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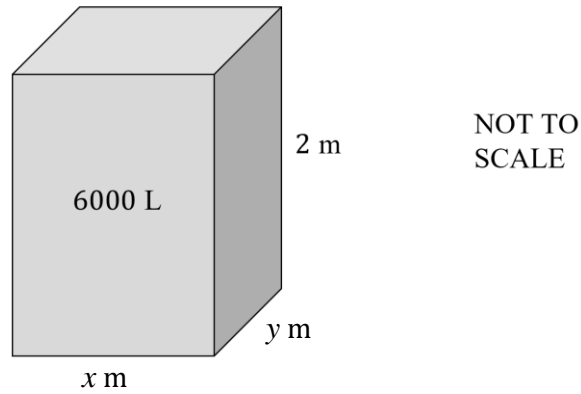
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**Question 13 (12 marks)**

- (a) A rainwater tank manufacturer wants to make a new storage tank that has a standard height, to reduce the obstruction of views. The manufacturer decides that it will be a 2 metre high rectangular prism with all faces made from sheet metal, including the top and bottom, and needs to hold 6000 litres of water, Let the other dimensions be  $x$  metres and  $y$  metres, as shown in the diagram.



For cost reasons, the manufacturer wants to use the least amount of area of sheet metal possible.

- (i) Show that for the given capacity of the tank,  $y = \frac{3}{x}$

**1**

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- (ii) Hence, calculate the exact dimensions of the tank that uses the least amount of area of sheet metal as possible. Show all reasoning.

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**Question 13 (a)(ii) (continued)**




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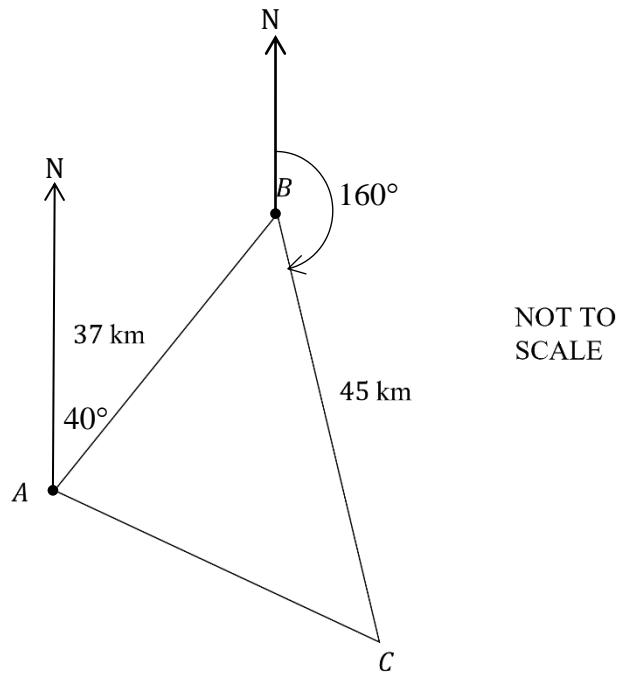
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- (b) There are three towns in a rural area. Town  $B$  is 37 km from Town  $A$  at a bearing of  $040^\circ\text{T}$ . Town  $C$  is 45 km from Town  $B$  at a bearing of  $160^\circ\text{T}$ .



- (i) Find the size of  $\angle ABC$ .

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- (ii) Find the distance from Town  $A$  to Town  $C$ , correct to 1 decimal place.

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**Question 13 (b)** continues on the next page

**Question 13 (b)** (continued)

(iii) Find the bearing of Town A from Town C.

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(c) (i) Show that  $f(x) = x^3 - 5x + \frac{xe^{-x^2}}{1+x^4}$  is an odd function.

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(ii) Hence evaluate  $\int_{-5}^5 \left( x^3 - 5x + \frac{xe^{-x^2}}{1+x^4} \right) dx$ .

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**Question 14 (12 marks)**

- (a) A quantity  $Q$  of the radioactive isotope Carbon-14 decays according to the equation of natural decay  $Q = Q_0 e^{-kt}$  where  $Q_0$  is the initial mass of Carbon-14,  $k$  is a constant and  $t$  is the time in years. The half-life of a radioactive substance is the time it takes to reduce to half its original mass. Carbon-14 has a half-life of 5730 years.

(i) Find the exact value of  $k$ .

**2**

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- (ii) A fossilised leaf contains 9% of its original amount of Carbon-14. How old is the fossilised leaf? Give your answer correct to the nearest year.

**2**

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(b) (i) Simplify  $\frac{\cos^2 \theta}{1 - \sin \theta} - \frac{\cos^2 \theta}{1 + \sin \theta}$

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(ii) Hence, solve  $\frac{\cos^2 \theta}{1 - \sin \theta} - \frac{\cos^2 \theta}{1 + \sin \theta} = 1$  for  $0 \leq \theta \leq \frac{\pi}{2}$

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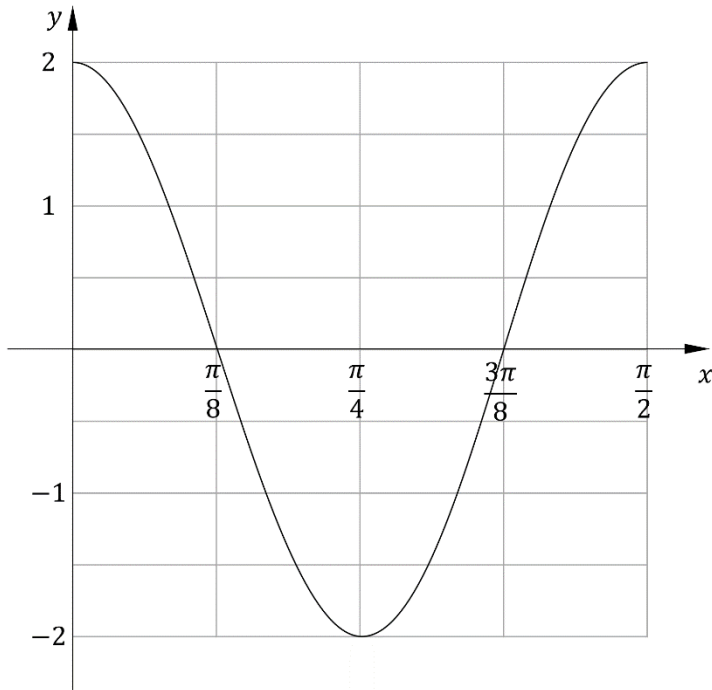
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(c) The graph shown below is  $y = A \cos bx$



(i) Write down the values of  $A$  and  $b$  for this graph.

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(ii) On the axes above, draw the graph of  $y = \sin 2x + 1$  for  $0 \leq x \leq \frac{\pi}{2}$ .

1

(d) Simplify  $\sqrt{x^2 + 2x + 1} - \sqrt{x^2 - 2x + 1}$ .

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**Question 15 (12 marks)**

4

(a) Solve  $\ln(2\sin^2 \theta - \cos \theta) = 0$  for  $0 \leq \theta \leq 2\pi$ .

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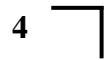
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(b) (i) Find the stationary points and determine their nature, and any points of inflection for  $y = x^4 - 2x^3 + 1$ , showing all reasoning.



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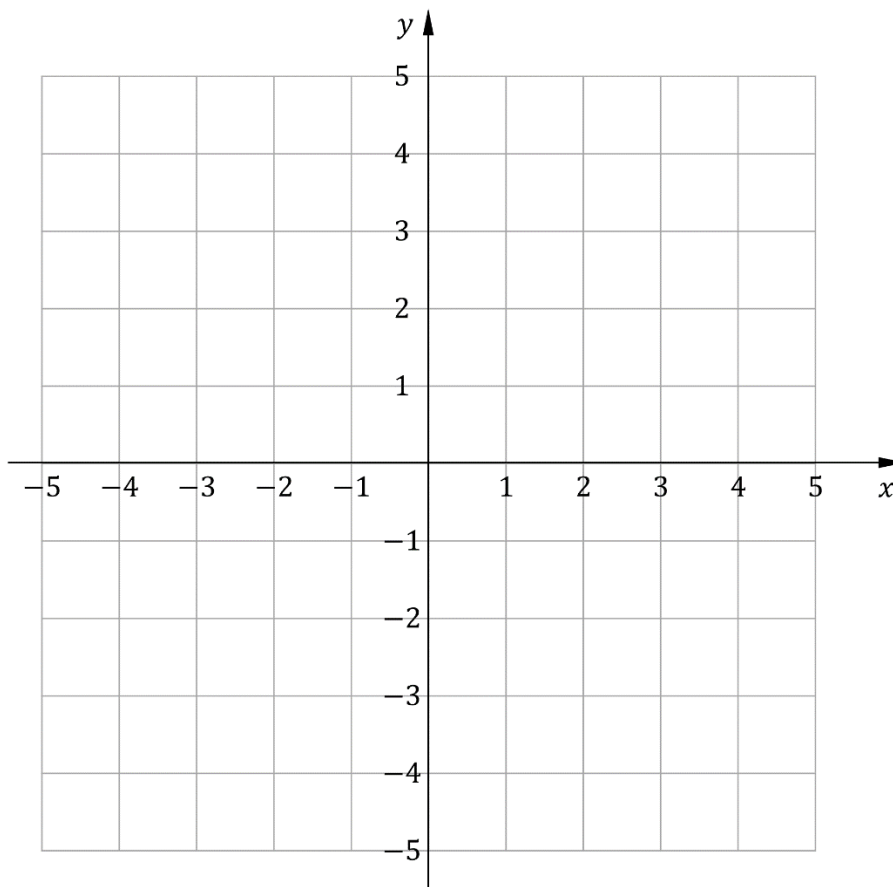
Lined area for student answer.

**Question 15 (b)** continues on next page



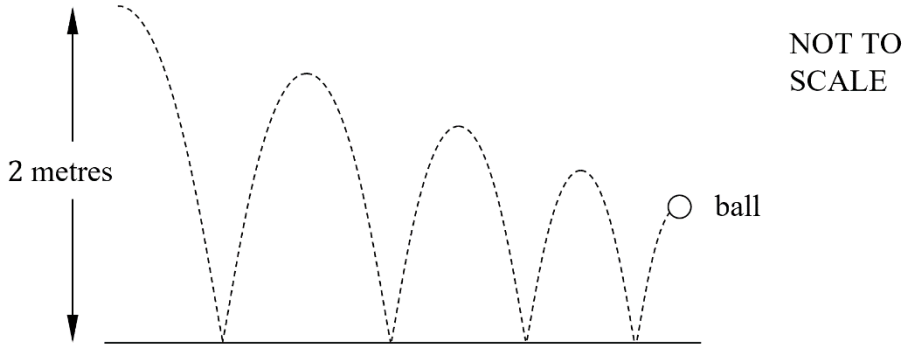
Question 15 (b) (continued)

- (ii) Sketch the graph of  $y = x^4 - 2x^3 + 1$  on the axes below, clearly showing all important points including any turning points and points of inflection. **1**  
It is not necessary to find all  $x$ -intercepts.



(c) The Moon has a lower gravity than Earth, and there is no atmosphere to cause air resistance, so a ball would bounce higher and for much longer on the Moon than on Earth.

When a ball is dropped on the Moon, each bounce is 95% as high as the previous bounce. When an identical ball is dropped on Earth, each bounce is 50% as high as the previous bounce.



Two identical balls are dropped on the Moon and on Earth, each from a height of two metres, and they both eventually stop bouncing.

Calculate the difference in the total vertical distance travelled by these balls.

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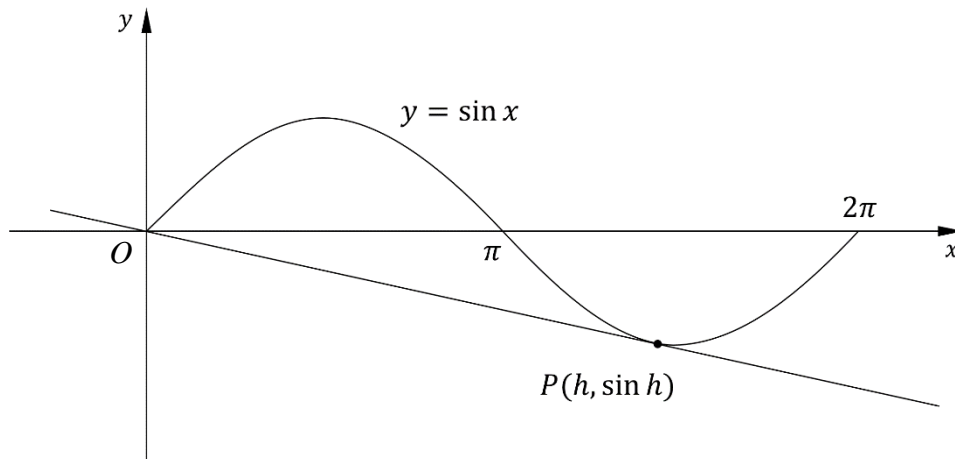


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Question 16 (10 marks)

4

- (a) The graph  $y = \sin x$  only has one tangent in the domain  $[\pi, 2\pi]$  that passes through the origin as shown in the diagram below.  
 Let the point of contact of this tangent be  $P(h, \sin h)$ .



Prove that  $h = \tan h$ .

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- (b) The table below gives the present value interest factors for an annuity of \$1 for various interest rates  $r$  (as a decimal), and the number of periods  $N$ .

Present value interest factors per \$1					
	Interest rate $r$ (as a decimal)				
$N$	<b>0.0025</b>	<b>0.005</b>	<b>0.0075</b>	<b>0.01</b>	<b>0.0125</b>
<b>71</b>	64.98140	59.64121	54.89293	50.66190	46.88363
<b>72</b>	65.81686	60.33951	55.47685	51.15039	47.29247
<b>73</b>	66.65023	61.03434	56.05643	51.63405	47.69627
<b>74</b>	67.48153	61.72571	56.63169	52.11292	48.09508
<b>75</b>	68.31075	62.41365	57.20267	52.58705	48.48897
<b>76</b>	69.13791	63.09815	57.76940	53.05649	48.87800

- (i) Anson plans to invest \$200 each month for 6 years at a rate of 3% p.a. compounded monthly. Use the table of values to calculate the present value of this annuity to the nearest dollar. 2

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- (ii) Monique uses the same table of values to calculate the loan repayments for a business loan for her dance school. She has borrowed \$2 000 000 and she is repaying it in **quarterly** repayments over 19 years, at a rate of 5% p.a. Using the table of values above or otherwise, calculate the amount of each quarterly repayment, correct to the nearest dollar. 2

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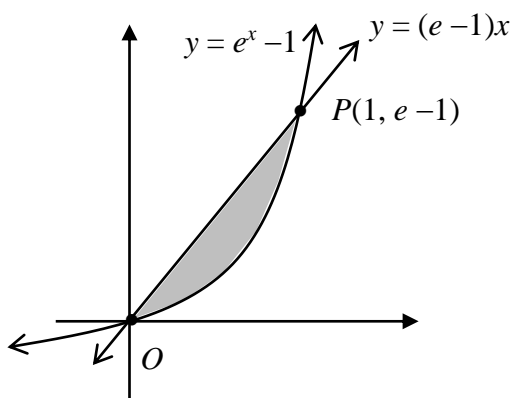
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- (b) The curve  $y = e^x - 1$  meets the line  $y = (e - 1)x$  at the origin and at the point  $P(1, e - 1)$ , as shown in the diagram below.

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Find the exact value of the shaded area.

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(c) The equation  $f(x) = \cos\left(\frac{x}{2}\right)$  is a probability density function for  $\frac{\pi}{3} \leq x \leq \pi$ .

(DO NOT PROVE THIS)

(i) Using the definition of a cumulative distribution function

2

$$F(x) = \int_a^x f(t) dt \text{ for } a \leq x \leq b$$

prove that the cumulative distribution function for the given domain is:

$$F(x) = 2 \sin\left(\frac{x}{2}\right) - 1 \quad \frac{\pi}{3} \leq x \leq \pi$$

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**Question 17 (c)** continues on the next page

(ii) Hence, or otherwise, find the median of this data, correct to one decimal place.

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**Question 18 (10 marks)**

- (a) Sergio decides to buy a top-of-the-line gaming laptop for \$12 500 from the retailer Jay-Bee. Jay-Bee offers the following terms:
- The customer pays for the laptop with equal monthly repayments of \$ $R$  at the end of each month for 5 years.
  - The first six months are interest free. After six months, interest is charged on the balance owing at 9% p.a. compounded monthly.
  - The equal monthly repayment is still made at the end of each month during the first six months.

(i) Let  $A_n$  be the amount owing after  $n$  months.

**3**

Show that after seven months, the total amount Sergio owes is:

$$A_7 = 12500 \times 1.0075 - R(6 \times 1.0075 + 1)$$

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**Question 18 (a)** continues on the next page

Question 18 (a) (continued)

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- (ii) Calculate the monthly repayment  $\$R$  required to pay back the loan, correct to the nearest dollar.

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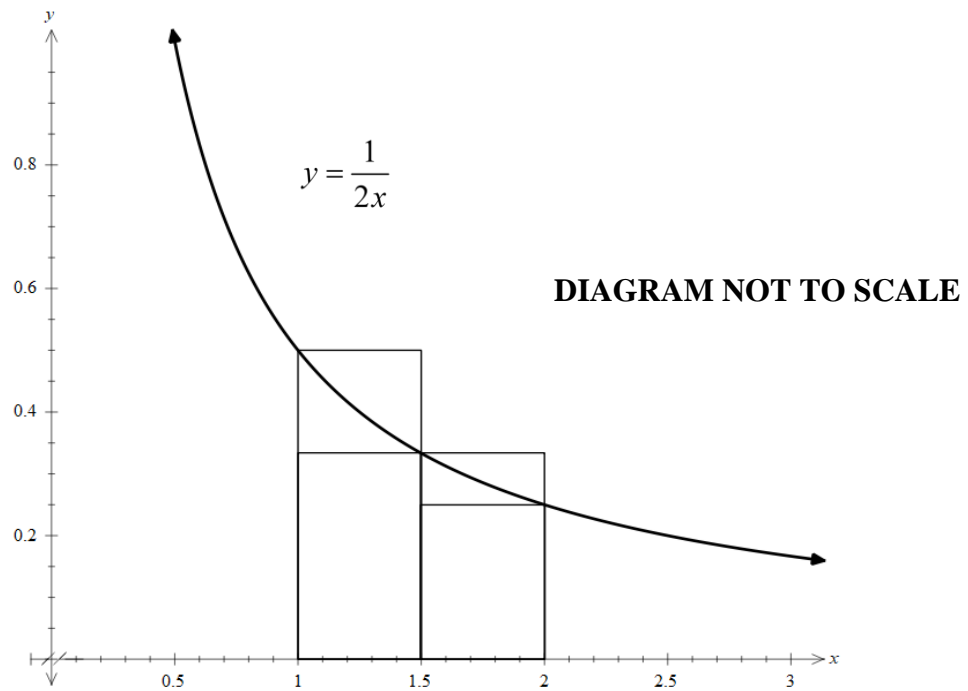
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- (b) The graph of  $y = \frac{1}{2x}$  is shown below.



Upper and lower rectangles of width 0.5 units are drawn from the  $x$ -axis to the curve between  $x = 1$  and  $x = 2$  as shown in the diagram.

By considering the areas of the rectangles, show that  $\frac{7}{24} < \ln \sqrt{2} < \frac{5}{12}$ .

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**Section II extra writing space**

**If you use this space, clearly indicate which question you are answering.**

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