2023 Mathematics Advanced Trial Solutions and feedback

1 A stone is thrown straight up, and the height in metres after t seconds is given by the formula

$$h = 9.8t - 4.9t^2 \ .$$

What is the stone's height when $t = 1$?

A. 4.9 metres

B. 5.25 metres

C. 6.75 metres

D. 9.8 metres

2 The graph of the function $y = f(x)$ is moved 3 units to the left.

Which of the following is the new function?

A. $y = f(x+3)$

B. $y = f(x-3)$

C. $y = f(x) + 3$

D. $y = f(x) - 3$

3 The height of the tide in a harbour can be modelled using the sine function. The time, t in hours, for the tide to complete one full cycle from high tide to low tide and back to high tide is 12 hours.

Which of the following could be the function representing the height of the tide?

A. $h = \sin\left(\frac{\pi t}{3}\right)$

B. $h = \sin\left(\frac{\pi t}{6}\right)$

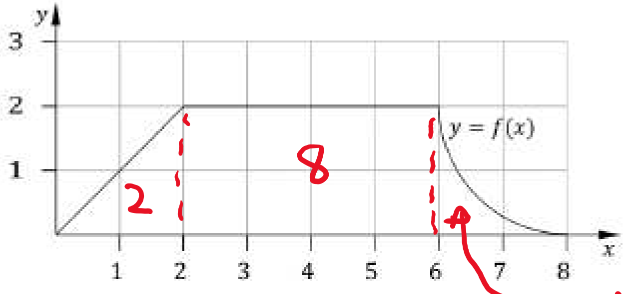
C. $h = \sin\left(\frac{\pi t}{12}\right)$

D. $h = \sin\left(\frac{\pi t}{18}\right)$

$T = \frac{2\pi}{n} = 12$

$n = \frac{\pi}{6}$

4 The graph of $y = f(x)$ is shown.



DRAWN TO SCALE

$4 - \frac{\pi \times 2^2}{4}$

What is the exact value of

$$\int_0^8 f(x)\,dx\,?$$

A. $10 + \pi$ units^2

B. $10 - \pi$ units^2

C. $14 + \pi$ units^2

D. $14 - \pi$ units^2

5 Which of the following is an arithmetic series?

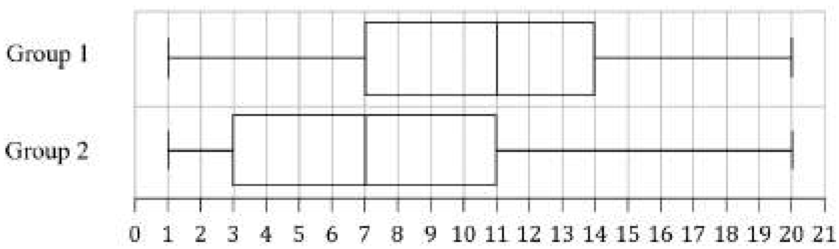
A. 2, 4, 6, 8, ...

B. $2 + 4 + 6 + 8 + \ldots$

C. 2, 4, 8, 16, ...

D. $2 + 4 + 8 + 16 + \ldots$

6 Consider the parallel box plots below.



Which of the following statements is CORRECT?

A. Group 1 is positively skewed.

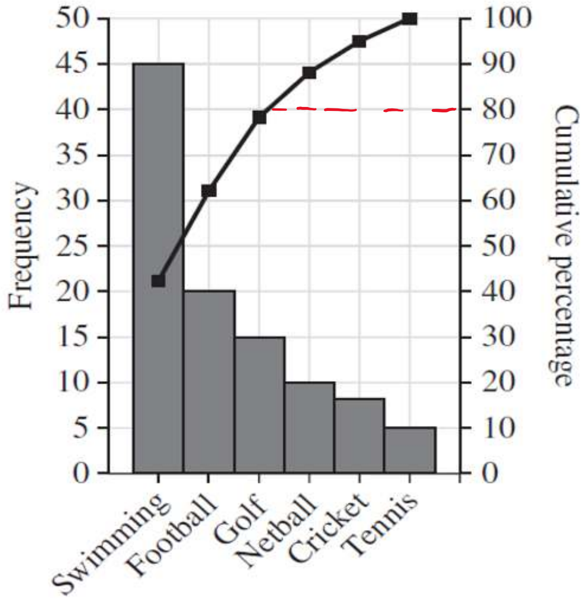
B. Group 2 is negatively skewed.

C. The difference between the median and Q_1 of Group 1 is the same as the difference between the median and Q_3 of Group 2.

D. The range and IQR are equal for both sets of data.

7 Using the Pareto Chart, which statement is true?

Complaints about spectator misbehaviour at a sports venue



A. Using the 80/20 rule, the complaints that should be resolved are golf, netball, cricket and tennis.

B. Using the 80/20 rule, the complaints that should be resolved are netball, cricket and tennis.

C. Using the 80/20 rule, the complaints that should be resolved are swimming, football and golf.

D. Using the 80/20 rule, the complaints that should be resolved are swimming, football, golf and netball.

8 It is given that $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$.

What is the value of $P(B|A)$?

A. $\frac{3}{4}$

B. $\frac{6}{5}$

C. $\frac{3}{2}$

D. $\frac{3}{5}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{1}{2} + \frac{2}{5} - \frac{3}{5}$

$= \frac{3}{10}$

$\therefore P(B|A) = \frac{3/10}{1/2}$

$= 3/5$

9 What is the derivative of $y = \left(\tan^2 x + 1\right)^2$?

$y' = 2(\tan^2 x + 1) \times 2\tan x \times \sec^2 x$

$= 4(\sec^2 x) \times \tan x \times \sec^2 x$

$= 4\sec^4 x \tan x$

NB: $\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\therefore \tan^2 x + 1 = \sec^2 x$

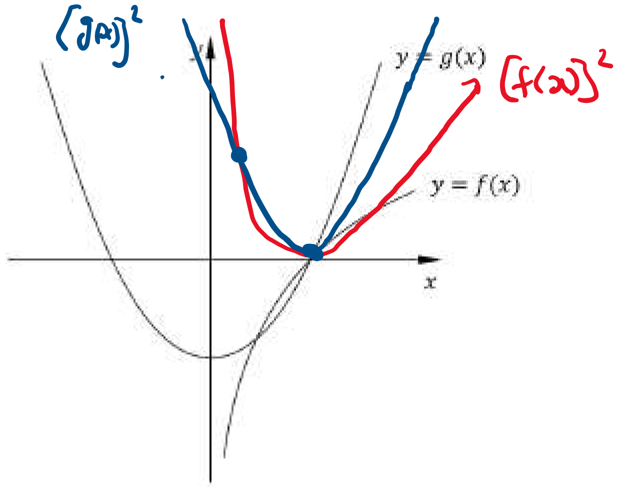
A. $4\sec^3 x$

B. $4\sec^4 x$

C. $4\sec^3 x \tan x$

D. $4\sec^4 x \tan x$

10 The graph shows $y = f(x)$ and $y = g(x)$, where $f(x) = \ln x$ and $g(x) = x^2 - 1$.



How many solutions does the equation $[f(x)]^2 - [g(x)]^2 = 0$ have?

A. 0

B. 1

C. 2

D. 3

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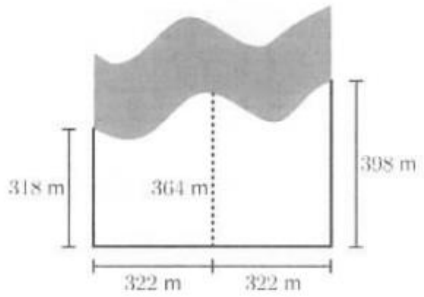
Question 11 (3 marks)

The following piece of land has straight boundaries on the east, west and south borders and follows a creek at the north. The land has been divided into two sections so we can use the trapezoidal rule to approximate the area.



(a) Find the approximate area of the piece of land by using two applications of the trapezoidal rule. Give your answer in square metres. **2**

$A = \frac{322}{2}(318 + 364) + \frac{322}{2}(364 + 398)$

$= 232484 \ m^2$

</td>
<td>Poorly done. Many used the wrong formula or used and as the height or vice versa. Some showed no working. If the wrong value of h was used, and the rest was correct – awarded 1 mark.</td>
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(b) 35.2 mm of rain fell during a heavy storm. Find the volume of water that lands on this property in cubic metres. Round your answer to the nearest cubic metre. **1**

$232484 \times 0.0352 = 8183.4368$

$= 8183 \ m^3$

correct application for 1 strip ✓
using height 644

</td>
<td>Some forgot the formula for the volume. Some did not convert 35.2 mm to m.</td>
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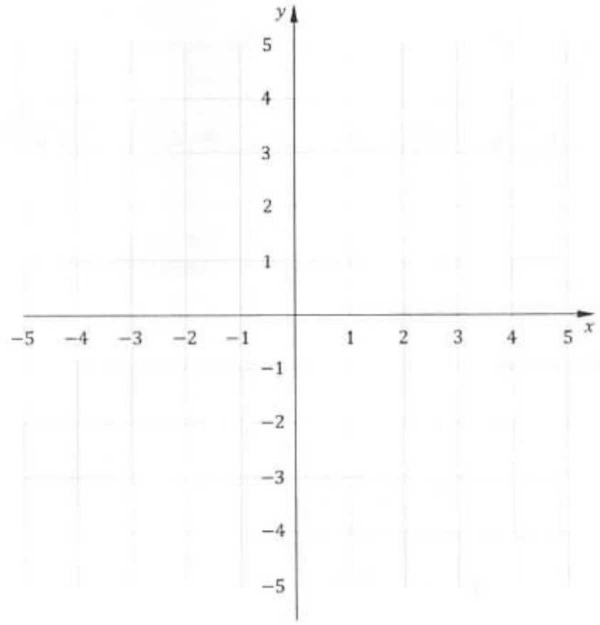
Question 12 (3 marks)

A line passes through $(-3,4)$ and $(3,0)$. **3**

By sketching the line, or otherwise, find the equation of the line.



$m = \frac{4-0}{-3-3} = \frac{4}{-6} = \frac{-2}{3}$ ✓

$(3, 0)$

$y = -\frac{2}{3}(x-3)$ ✓

$y = \frac{-2}{3}x + 2$ or $2x + 3y - 6 = 0$ ✓

or using the graph

Well done. Some who used the drawing did not realise the gradient is negative. A few did not know how to draw the graph.

Question 13 (2 marks)

Find the anti-derivative of **2**

$$f(x) = \frac{1}{2x-3}.$$

$$\frac{1}{2}\ln|(2x-3)| + c$$

1 mark off - no absolute value sign
1 mark off - no 'c'

Well done. Some did not read the question and found the derivative instead. A few left out the 'c' - lost 1 mark. A few left out the absolute value sign – 1 mark.

Question 14 (3 marks)

Calculate **3**

$$\int_{\ln 2}^{2\ln 2} e^{2x}\,dx.$$

$$= \frac{1}{2}\left[e^{2x}\right]_{\ln 2}^{2\ln 2} \checkmark$$

$$= \frac{1}{2}\left[e^{4\ln 2} - e^{2\ln 2}\right] \checkmark$$

$$= \frac{1}{2}\left[e^{\ln 16} - e^{\ln 4}\right]$$

$$= \frac{1}{2}(16-4) = 6 \checkmark$$

Well done. A few are writing the integral sign even though they have worked out the integral. Some students are writing or for the integration. There are issues with not knowing how to evaluate .

Question 15 (3 marks) It is given that $f''(x) = 6x$ and that $f(x)$ has a stationary point at $(-1,2)$. **3** Find $f(x)$. $f'(x) = \frac{6x^2}{2} + c_1$ $= 3x^2 + c_1$ ✓ $f'(x) = 0$ at $x = -1$ $0 = 3(-1)^2 + c_1$ $c_1 = -3$ ✓ $f'(x) = 3x^2 - 3$ $f(x) = x^3 - 3x + c_2$ at $x = -1$, $y = 2$ $2 = (-1)^3 - 3(-1) + c_2$ $2 = -1 + 3 + c_2$ $c_2 = 0$ $\therefore f(x) = x^3 - 3x$ ✓	Generally well done. Some students were using = 2 which was in fact the y value to determine the first constant. Some were unable to find the second constant.

Question 16 (3 marks) (a) The fifth term of an arithmetic sequence is 5 and the nineteenth term is 47. **2** Find the first term and the common difference. $T_5 = a + 4d = 5 \quad \ldots (1)$ $T_{19} = a + 18d = 47 \ldots (2)$ $(2) - (1) \qquad 14d = 42$ $d = 3$ ✓ $a = -7$ ✓ (b) Find the sum of the first 25 terms of the above sequence. **1** $S_{25} = \frac{25}{2}[2(-7) + 24 \times 3]$ $= 725$ ✓	Excellent.

Question 17 (4 marks)

Consider the functions $y = -x^3 - 1$ and $y = -x - 1$.

(a) Find the x coordinates of the points of intersection given $x \geq 0$. **1**

$-x^3 - 1 = -x - 1$

$x - x^3 = 0$

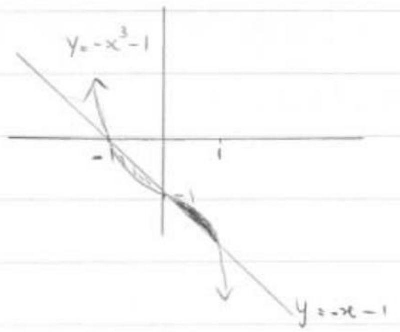
$x(1 - x^2) = 0$

$x = 0, \pm 1$ but $x \geq 0$

$\therefore x = 0$ or 1 ✓

Some students are not reading the condition that and lost the mark.

(b) Find the area between the two graphs given $x \geq 0$. **3**



$A = \int_0^1 [(-x^3 - 1) - (-x - 1)]\,dx$ ✓

$= \int_0^1 (-x^3 + x)\,dx$

$= \left[-\frac{x^4}{4} + \frac{x^2}{2}\right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\ u^2$ ✓ ✓

If they calculated for both areas $\frac{2}{3}$

1 mark ← Wrong order ✗ and no acknoledgment of absolute sign $\int_{-1}^{0} f - g + \int_0^1 g - f = \frac{1}{2}$ ✓✓

DRAW the graph so you can see which is the top graph and which is the bottom. Some decided to put in the absolute value sign at the end as they realised it was negative area. Some added the two areas together. Setting out is poor. Some are using as the limit even though the question specifically stated .

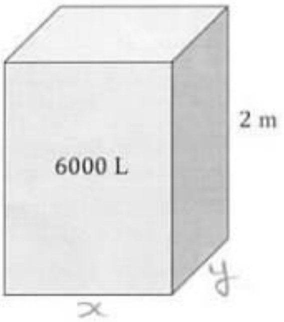
Question 18 (4 marks) Find and fully factorise the first and second derivatives of $y = x(x+1)^3$. **4** $y' = 1(x+1)^3 + 3x(x+1)^2 \times 1$ $= (x+1)^2[x+1+3x]$ $= (x+1)^2[4x+1]$ $y'' = 2(x+1)(4x+1) + 4(x+1)^2$ $= 2(x+1)(4x+1+2(x+1))$ $= 2(x+1)(6x+3)$ $= 2(x+1)3(2x+1) \leftarrow$ accepted $= 6(x+1)(2x+1)$	Generally well done, however, a handful of candidates did not fully factorise the answers costing them marks. A few did not know how to differentiate using the product rule. Careless errors in differentiations, hence, need to check the work on algebra. A few expanded instead of factorising.
Question 19 (3 marks) The circle $x^2 + y^2 - 6x + 8y - 11 = 0$ is transformed by a horizontal translation to the left by 4 units and a vertical translation up 3 units. **3** What is the centre and radius of the new circle? $x^2 - 6x + 9 + y^2 + 8y + 16 = 11 + 9 + 16$ $(x-3)^2 + (y+4)^2 = 36$ $C(3, -4)$ New centre $(3-4, -4+3)$ $(-1, -1)$ r: 6 units.	A lot of issues with completing the square method. Errors in transformations. Directions must be reversed for both x and y coordinates.

Question 20 (3 marks) (a) Prove that **1** $$\frac{d}{dx}\left(xe^x\right) = xe^x + e^x.$$ $1 \times e^x + x \cdot e^x$ $= xe^x + e^x$ ✓	Need to show evidence in differentiation and not rewriting the question. Need to see the product rule being applied.
(b) Hence find **2** $$\int xe^x \, dx.$$ $\int (xe^x + e^x)\,dx = \int \frac{d}{dx}(xe^x)\,dx$ ✓ $\int xe^x\,dx = xe^x - \int e^x\,dx$ $= xe^x - e^x + c$	Very poor setting out. Not correctly using the symbols for integrals.

Question 21 (4 marks)

A rainwater tank manufacturer wants to make a new tank that is lower than the standard home fence height, to reduce the disturbance of views from neighbouring houses. **4**

The manufacturer decides that the tank will be a 2 metre high rectangular prism, including a roof, and needs to hold 6000 litres of water. ($1\text{m}^3 = 1$ kL)



NOT TO SCALE

For environmental and commercial reasons the manufacturer wants to use the least amount of material possible.

Calculate the exact dimensions of the tank that uses the smallest amount of material possible.

This means smallest surface Area

$6000\text{L} = 6\text{kL}.$

$\therefore 2xy = 6$

$xy = 3 \quad — ①$

$SA = 2xy + 2 \times 2y + 2 \times 2x$

$= 2xy + 4y + 4x \qquad$ but $y = \frac{3}{x}$ from 1.

From ① $\quad SA = 6 + 4x + \frac{12}{x}$

Still not well done.

Only a few students recognised the need to change L to kL.
(6000 L = 6kL)

Very few students wrote a correct expression for surface area.

Only some students wrote surface area as a function of only 1 variable.

Being a 'minimum SA' question, students needed to differentiate SA and solve. This found a value for *x*.

This value needed to be tested to confirm minimum or maximum.

Finally, answer the question by writing the three dimensions.

Question 21 (continued)

$SA = 6 + 4x + \cancel{\frac{12}{x}}\ 12x^{-1}$ $\quad \left[SA = 6 + 4x + \frac{12}{x} \right]$

$\frac{d\,SA}{dx} = 4 - 12x^{-2}$

$0 = 4 - 12x^{-2}$

$4x^2 = 12$

$x = \sqrt{3} \quad x > 0$

$SA'' = 24x^{-3}$

> 0 for $x = \sqrt{3}$ $\therefore$ minimum.

$xy = 3 \quad \therefore y = \sqrt{3}$

$\therefore$ Dimensions are $\sqrt{3}, \sqrt{3}, 2$

Comments

1. Nearly all students failed to recognise it was calculus min/max question.
2. Many failed to see the question was SA
3. Large number failed to recognise units required was metres $\Rightarrow 1m^3 = 1KL$
$\therefore 6000L = 6KL = 6m^3$

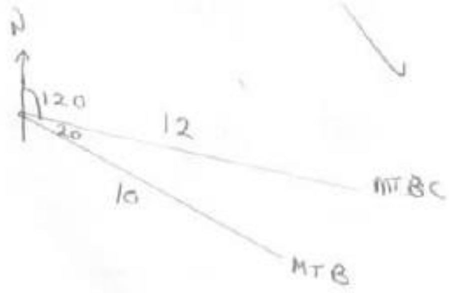
End of Question 21

Question 22 (4 marks)

From the top of Mount Adee a bushwalker sees two other mountains.

- Mount Bailey is 10 kilometres away at a bearing of 140°.
- Mount Charlee is 12 kilometres away at a bearing of 120°.

(a) Draw a diagram showing all relevant information. **1**



Very well done

Diagram was very well done.

(b) Find the distance from Mount Bailey to Mount Charlee. Answer to the nearest hundred metres. **3**

$x^2 = 12^2 + 10^2 - 2 \times 10 \times 12 \times \cos 20$

$x \doteqdot 4.298\ldots$ km

$\doteqdot 4.300$ km

$\doteqdot 4300$ m

(Correct rounding)

Full correct according to diagram

Very well done.

This was very well done.

Some did not note the rounding off requirements.

Correct according to the diagram 3 marks unless the diagram simplified the problem.

Question 23 (3 marks)

Luke needs a twelve-month loan for $8000. He has the following loan options: 3

- A payday lender at 0.92% per week, interest compounded weekly.
- A bank at 11.99% p.a., interest compounded monthly.

He will repay the loan with a single lump sum at the end of the twelve months.

How much more will it cost him if he takes the loan from the payday lender rather than the bank?

Pay day $8000 \times (1\cdot0092)^{52} \doteq \$12\,879.72$

Bank $8000 \times \left(1 + \frac{0\cdot1199}{12}\right)^{12} = \$\cancel{9007.46}$
$\$9013.71$

$\therefore$ Extra $= \cancel{\$3872.26}$ $\$3866.01$

$[12879.72 - \overset{9013.71}{\cancel{3866.0}}]$

rounding too early ($\frac{0.1199}{12}$ rounded to early)

Well done.

* Some did not use correct weeks in one year.

* Some answers said bank was more X (told that payday was more)

Well done.

Problems – some used 48 weeks in a year.

Some did not note that the question says payday costs more than the bank and then stated that the bank costs more. This needs to alert students that they have done something wrong.

Question 24 (3 marks)

(a) Simplify **2**

$$\frac{\cos^2\theta}{1-\sin\theta}-\frac{\cos^2\theta}{1+\sin\theta}.$$

$$\frac{\cos^2\theta(1+\sin\theta)-\cos^2\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$=\frac{\cos^2\theta+\cos^2\theta\sin\theta-\cos^2\theta+\cos^2\theta\sin\theta}{\cos^2\theta}$$

$$=2\sin\theta$$

(b) Hence, solve **1**

$$\frac{\cos^2\theta}{1-\sin\theta}-\frac{\cos^2\theta}{1+\sin\theta}=1 \quad \text{for } 0\le\theta\le\frac{\pi}{2}.$$

$2\sin\theta = 1$

$\sin\theta = 1/2$

$\theta = \pi/6$

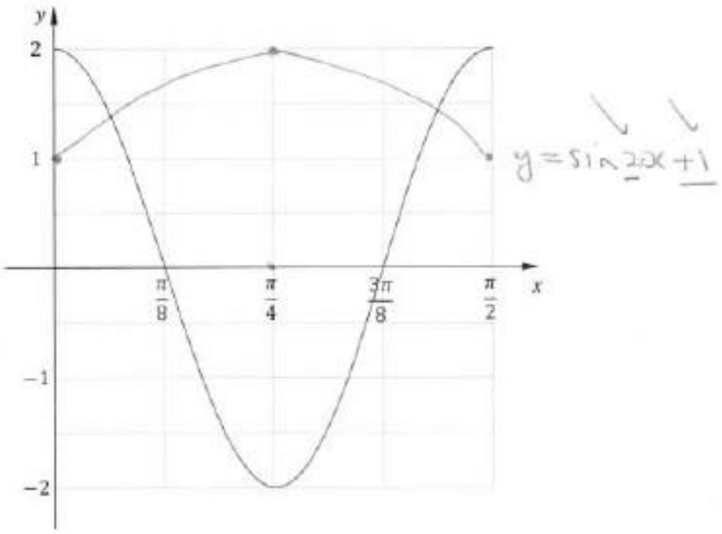
Quite well done.

There were two methods used – the first as done here. The second was to convert $cos^2\theta$ to $1 - sin^2\theta$ and then factorised using difference of 2 squares. Finally cancelling and simplifying.

Those who completed (a) correctly, were able to complete (b) with the only issue being not applying the domain correctly.

Question 25 (4 marks)

The graph shown is $y = A\cos bx$.



Well done

(a) Explain why $A = 2$. **1**

A = amplitude and amplitude = 2

oR ⇒ $x = 0$ $\quad 2 = A\cos 0$ $\quad \therefore A = 2$

$= A \times 1$

Well done

Well done

(b) Explain why $b = 4$. **1**

Period $= \frac{2\pi}{n}$ $\quad \therefore \frac{\pi}{2} = \frac{2\pi}{n}$ $\quad \therefore b = 4$

Well done

Well done

(c) On the axes above, draw the graph of $y = \sin 2x + 1$. **2**

Well done but some issues in properly identifying the shift up by one and the appropriate period.

Question 26 (6 marks)

The height in metres, h, of eight competitors at the school shot put, and the furthest distance they achieved in metres, d, are shown in the form (h, d) below.

$(1.6, 5.5)$ $(1.2, 4.7)$ $(1.8, 6.2)$ $(1.5, 5.4)$ $(1.7, 6.1)$ $(1.4, 5.3)$

(a) Determine the equation of the least-squares line of best fit in the form $d = mh + c$. Show m and c correct to 1 decimal place. **2**

$d = 2.5h + 1.7$ 1 mark if not 1 d.p.

This was well done – use of the computer was very good.

(b) Find the average increase in the distance students achieved for each extra 10 cm in height. Justify your answer. **2**

$2.5 \text{ m/m} = \frac{2.5}{10} \text{ m} / 10 \text{ cm}$

$\Rightarrow 25 \text{ cm} / 10 \text{ cm}$ in height

$\therefore$ Average increase is 25 cm

$d = 2.5h + 1.7$
$h = 0$
$d = 1.7$
$h = 0.1$
$d = 1.95$
$\therefore 1.95 - 1.7$
$= 0.25$

Check only a few students knew to do / understood this. Could use equation and sub in $h = 0$, 0.1 metres (for example).

Poorly done. Few students recognised that the gradient in (a) was the average rate as 2.5 metres per metre.

(c) Find the correlation coefficient and describe the strength of the correlation. **2**

$r = 0.975...$

strength is strong positive

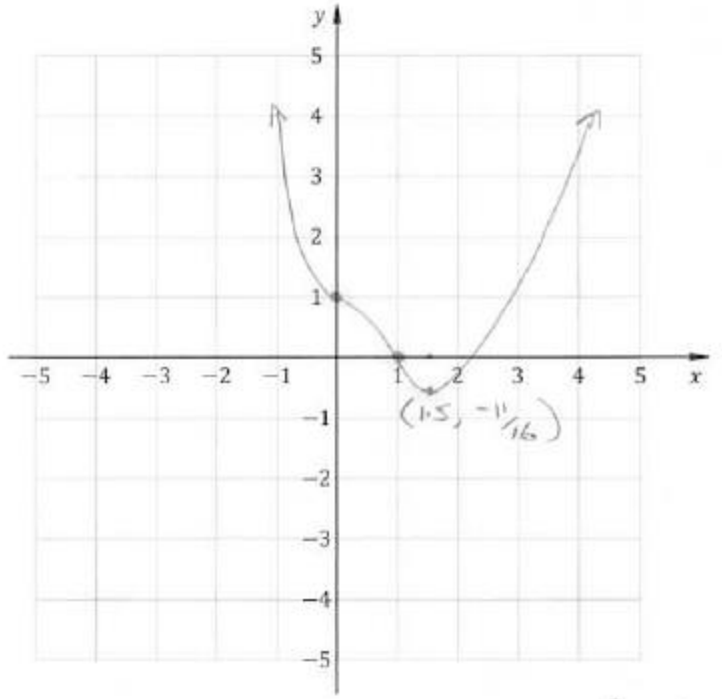
strong only

Well done

Question 27 (4 marks) (a) Find the turning points and points of inflection on $y = x^4 - 2x^3 + 1$. **3** $y' = 4x^3 - 6x^2$ $= x^2(4x-6)$ $0 = x^2(4x-6)$ $\therefore x = 0,\ 1.5$ T.P. $x=0 \quad y=1 \qquad (0,1)$ $x=1.5 \quad y=-0.6875 \qquad (1.5, -^{11}/_{16})$ min. $y'' = 12x^2 - 12x$ $= 12x(x-1)$ max/min At $x=0$ $y''=0$ possible POI. At $x=1.5$ $y''>0$ $\therefore$ min. $y''=0$ at $x=1$ POI x: -1, 0, 0.5, 1, 2 y'': >0, 0, <0, 0, >0 $\therefore x=0, x=1$ change in concavity. $\therefore (0,1)$ and $(1,0)$ points of inflection. Poorly done – process does not seem to be understood. **Question 27 continues on page 25** Must check for all POI.	Poorly done. There are not many students who understand the required process of finding tuning points, points of inflections and the required testing to show max/min and POI.

Question 27 (continued)

(b) Sketch the graph $y = x^4 - 2x^3 + 1$ on the axes below, clearly showing the turning points and points of inflection. It is not necessary to find all x-intercepts. **1**



Require correct curve shape and the 3 points highlighted.

End of Question 27

Some did not recognise $x^4 \rightarrow$

Many problems with drawing "smooth" curve particularly joining (0,1) (1,0) (1.5, -11/16).

Not well done as part (a) was required and part (a) was poorly done.

Curves are often not draw 'smoothly'.

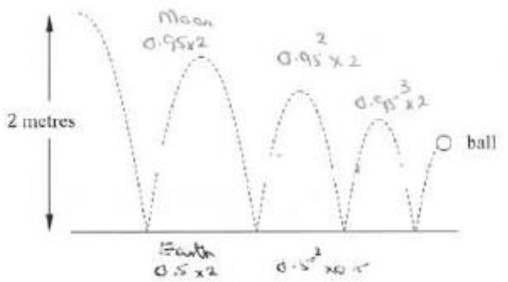
Some students were not able to use the data from (a) to correctly draw the curve.

(0, 1), (1,0) and (1.5, -11/16) needed to be identified and correctly drawn as POIs and minimum turning point.

Question 28 (3 marks)

The Moon has a lower gravity than Earth, and there is no atmosphere to cause air resistance, so a ball would bounce higher and for much longer on the Moon than on Earth. **3**

When a ball is dropped on the Moon each bounce is 95% as high as the previous bounce. When an identical ball is dropped on Earth each bounce is 50% as high as the previous bounce.



NOT TO SCALE

$a = 2 \Rightarrow 80 - 2 = 78$

$a = 2 \Rightarrow 8 - 2 = 6$

[80+2 and 8+2 1 error]

Two identical balls are dropped on the Moon and on Earth, each from a height of two metres.

$S_\infty = \frac{a}{1-r}$

Calculate the difference in the total vertical distance travelled by these balls.

Moon $2 + 2 \times \left(\frac{0.95 \times 2}{0.05}\right) = 78$ $r = 0.95$ $a = 0.95 \times 2$

Earth $2 + 2 \times \frac{0.5 \times 2}{0.5} = 6$ $r = 0.5$ $a = 0.5 \times 2$

$\therefore$ Different height = 72 m

Moon $= 2 + 2 \times 0.95 \times 2 + 2 \times (0.95 \times 2) \times 0.95 + \ldots$ $a = 0.95 \times 2$, $r = 0.95$

$\Rightarrow 2 + \frac{2 \times (0.95 \times 2)}{0.05}$

Earth $= 2 + 2 \times 0.5 \times 2 + 2 \times (0.5 \times 2) \times 0.5 + \ldots$ $a = 0.5 \times 2$, $r = 0.5$

$\Rightarrow 2 + \frac{2 \times (0.5 \times 2)}{0.05}$

Correctly applies S_∞ for at least one – 1 mark. [=36 1 mark]

Many not using that every bounce equals is $\times 2$.

1 mark – correctly identifies $\times 2$

Poorly done.

Many did not recognise that every bounce consisted of an up and a down motion (other than the initial drop).

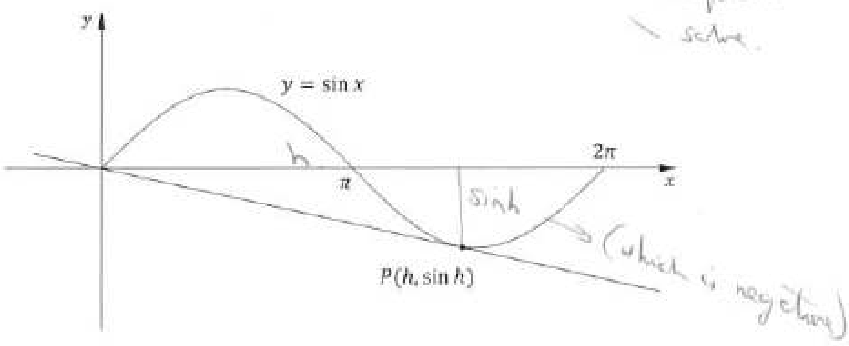
Most who attempted the question recognised sum to infinity, however, a number did not identify 'a' and/or 'r' correctly.

Question 29 (4 marks)

The graph $y = \sin x$ only has one tangent in the domain $[\pi, 2\pi]$ that passes through the origin. **4**

gradient using pts
gradient using y'
equate
solve

Let the point of contact of this tangent be $P(h, \sin h)$.



Prove that $h = \tan h$.

$m = \frac{\sin h}{h}$

$y = \sin x$

$y' = \cos x \quad \therefore$ at $x = h$

$m = \cos h.$

$\therefore \cos h = \frac{\sin h}{h}.$ (equate gradients)

$h = \frac{\sin h}{\cos h}$

$h = \tan h.$

Poorly done.

Very few recognised that there were 2 ways to find the gradient of the tangent – one using the two points and one using differentiation.

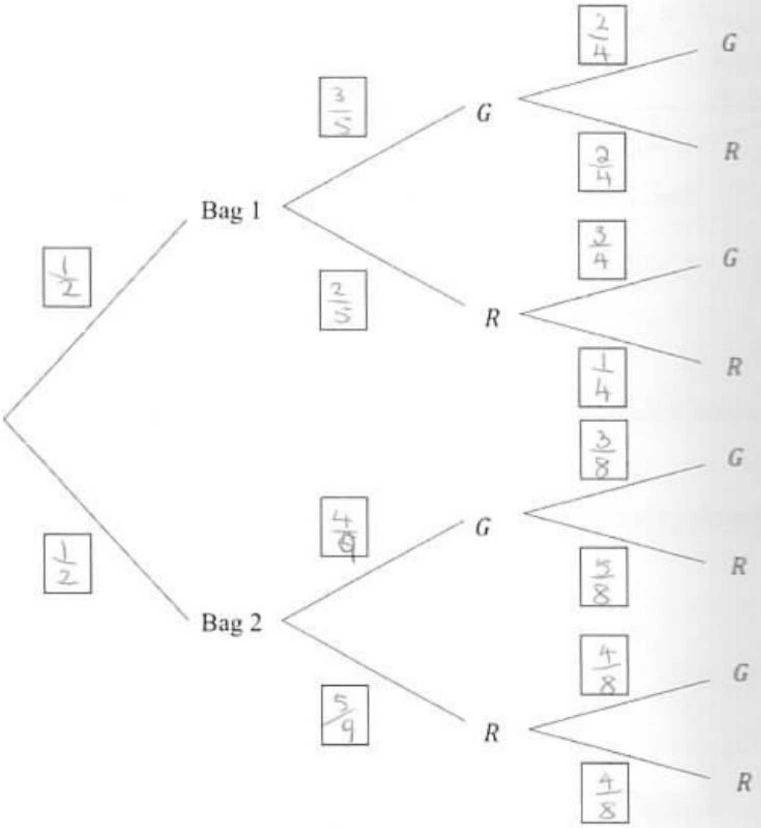
The key was the two gradients and then equating them as they were equal.

Question 30 (5 marks)

There are two bags of lollies. One bag has three green frogs and two red frogs, while the second bag has four green frogs and five red frogs.

One bag is chosen at random, and two frogs are picked from the bag, without replacement.

(a) Complete the below tree diagram below by writing probabilities in the boxes. **2**



✓

✓ reduced by 1

Question 30 (continued) (b) Calculate the probability of picking two red frogs. **2** $\frac{1}{2}\left(\frac{2}{5} \times \frac{1}{4} + \frac{5}{9} \times \frac{4}{8}\right)$ $= \frac{17}{90}$ (c) Calculate the probability of picking at least one green frog. **1** P(at least 1 green) = 1 − P(no green) $= 1 - \frac{17}{90}$ $= \frac{73}{90}$	Many didn't know what probabilities to put in for the two bags, however, the other probabilities were completed much better. Some did not consider that there was no replacement.
Question 31 (2 marks) Find **2** $\frac{d}{dx}\left[\log_3\left(x^2\right)\right].$ $\frac{d}{dx}\left[\frac{\ln x^2}{\ln 3}\right] = \frac{1}{\ln 3} \times \frac{2x}{x^2}$ $= \frac{2}{x \ln 3}$	b) Many did not multiply by ½. c) CFE, not bad as they understood the use of complementary event to part b).

Question 32 (5 marks) The volume of a solid is directly proportional to the cube of its radius ($V = kr^3$). When the radius is 7 cm, the volume is 1372 cm^3. (a) Find the value of k, and hence write the equation for the volume. **3** $V = kr^3$ $1372 = k \times 7^3$ ✓ $k = \frac{1372}{7^3}$ $= 4$ ✓ $V = 4r^3$ ✓ (b) Find the volume given the radius is 11 cm. **1** $V = 4 \times 11^3$ $= 5324\ cm^3$ ✓	Need to use log rule. Mostly well done, a few did not simplify, hence lost a mark.
Question 32 (continued) (c) Find the radius if the volume is 42 592 cm^3. **1** $42592 = 4r^3$ $r^3 = \frac{42592}{4}$ $r = \sqrt[3]{\frac{42592}{4}}$ $= 22\ cm$ ✓	a) -c) Excellent.

Question 33 (7 marks)

Declan decides to buy a top-of-the-line gaming laptop for $12 499.

The retailer offers the following terms:

- The customer pays for the laptop with equal monthly instalments of $R at the end of each month for 5 years.
- Interest is charged on the balance owing at 9% p.a. compounded monthly.
- The first six months are interest free. The regular monthly instalment is still made at the end of each month during the first six months.

(a) Show that after seven months the total amount owing is **3**

$$A_7 = 12\,499 \times 1.0075 - R \times (6 \times 1.0075 + 1).$$

$A_1 = 12499\left(\cancel{1+\frac{0.09}{12}}\right) - R$

$A_2 = 12499 - 2R$

$\therefore A_6 = 12499 - 6R$

with correct r

$A_7 = (12499 - 6R)\left(1+\frac{0.09}{12}\right) - R$

$= 12499(1.0075) - R(6 \times 1.0075 + 1)$

$[0.09 \div 12 = 0.0075]$

$= 12499(1.0075) - R(7.045)$

1 mark A_6

1 mark $r = 0.0075$

1 mark correct development of A_7

Not well done.
Students must go through the process of developing each monthly amount.
They did not develop the pattern for the first 6 months and then the new pattern for the next months.

Question 33 (continued)

(b) Calculate the regular monthly repayment, $R, required to pay back the loan. **4**

$A_7 = 12499(1.0075) - R(6 \times 1.0075 + 1)$ given in part a

$A_8 = 12499(1.0075)^2 - R(6 \times 1.0075^2 + 1.0075 + 1)$

$\therefore A_{60} = 12499(1.0075)^{54} - R(6 \times 1.0075^{54} + 1.0075^{53} + \cdots 1.0075 + 1)$

Paid off $\therefore A_{60} = 0$

$S_{54} = \dfrac{1(1.0075^{54} - 1)}{0.0075}$ $n = 54$

$\therefore 12499(1.0075)^{54} = R\left[6 \times 1.0075^{54} + \dfrac{1.0075^{54} - 1}{0.0075}\right]$

$R \doteqdot \$248.64$

Those who developed the monthly totals and therefor saw the pattern where able to obtain a correct answer.

Question 34 (5 marks)

A decibel is a measurement of sound intensity based on a logarithmic scale.

Zero decibels (0 dB), the quietest sound the average person can hear, has an intensity of approximately 1 picowatt per square metre. The formula to find the decibels from the intensity of a sound is

$$D = 10\log_{10}\left(\frac{I}{I_0}\right),$$

where D is the number of decibels, I is the sound intensity and $I_0 = 1 \times 10^{-12}\ \text{W/m}^2$.

(a) Calculate the decibels of a set of headphones which has a sound intensity of $1 \times 10^{-2}\text{W/m}^2$. **1**

$D = 10\log_{10}\left[\frac{1 \times 10^{-2}}{1 \times 10^{-12}}\right]$

$= 10\log_{10} 10^{10}$

$= 100\ \text{dB}$ ✓

(b) A jet engine has a decibel value of 150 dB. **2**

Calculate the magnitude of the sound intensity, I, in terms of I_0.

$150 = 10\log_{10}\left[\frac{I}{1 \times 10^{-12}}\right]$

$15 = \log_{10}\left[\frac{I}{1 \times 10^{-12}}\right]$ ✓

$10^{15} = \frac{I}{1 \times 10^{-12}}$

$I = 1000$ ✓

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Question 34 (continued)

(c) Sounds above 85 dB can damage hearing over time. 2

A student is mowing the lawn with a damaged mower which is running at 102 dB.

Compared with the safe noise level of 85 dB, how many times more intense is the noise from the damaged mower? Answer to the nearest integer.

$85 = 10\log_{10}\left[\frac{I}{1\times10^{-12}}\right]$ $102 = 10\log_{10}\left[\frac{I}{1\times10^{-12}}\right]$
$8.5 = \log_{10}\left[\frac{I}{1\times10^{-12}}\right]$ $10.2 = \log_{10}\left[\frac{I}{1\times10^{-12}}\right]$
(✓) $10^{8.5} = \frac{I}{1\times10^{-12}}$ $10^{10.2} = \frac{I}{1\times10^{-12}}$
$I = 10^{8.5}\times1\times10^{-12}$ $I = 10^{10.2}\times1\times10^{-12}$
$= 0.000316\ldots$ $= 0.0158\ldots$
Or simply $\frac{10^{10.2}}{10^{8.5}} = 50.1187\ldots$ ✓

End of paper

50 times ✓
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a) Calculator work after correct substitution was good.

Many received a mark to show , however, did not know what to do next. Need revision of log rules.
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<td>Same problem as part b). To find the intensity, a lot found the difference instead of dividing.</td>
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