



Newington College

Name: _____

Teacher (please circle):

DMH AMM GVA KST

Student number

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2024

Trial Higher School Certificate Examination

Mathematics Advanced

General Instructions

- Reading time — 10 minutes
- Working time — 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

**Total marks:
100**

Section I - 10 marks (pages 2—6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 8—33)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

	Section I	Section II	Total
Functions & further graphs	<i>/3</i>	<i>/21</i>	<i>/24</i>
Trigonometry	<i>/1</i>	<i>/12</i>	<i>/13</i>
Calculus	<i>/1</i>	<i>/29</i>	<i>/30</i>
Statistics and Probability	<i>/3</i>	<i>/15</i>	<i>/18</i>
Modelling financial situations	<i>/2</i>	<i>/13</i>	<i>/15</i>
Total	<i>/10</i>	<i>/90</i>	<i>/100</i>

Section I

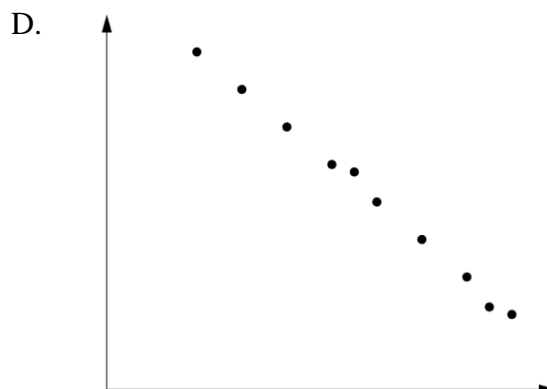
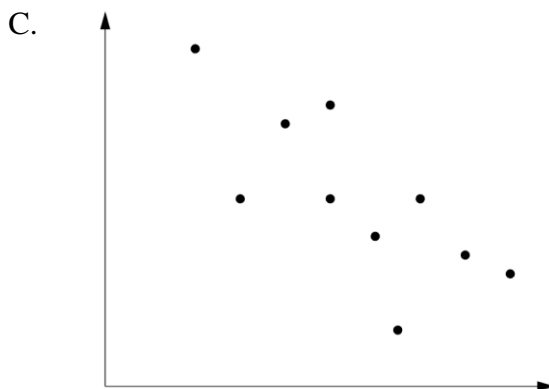
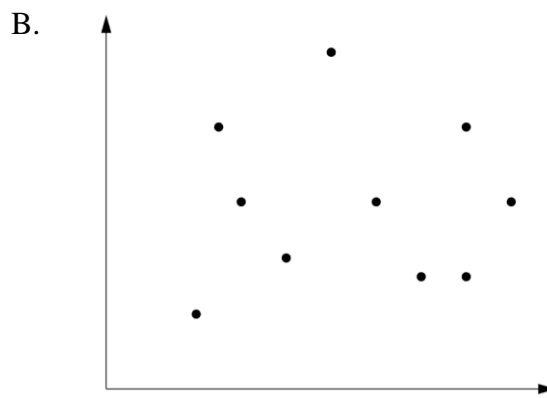
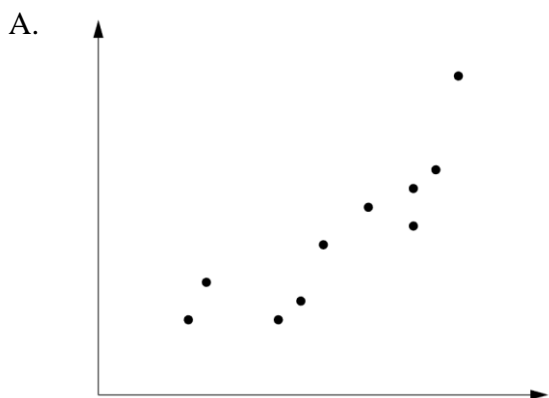
10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- 1 The probability that it rains on any particular day in Sydney is 25%.
 If three consecutive days of the year are chosen, what is the probability that it rains on all three of those days?
- A. 1.56%
 B. 25%
 C. 42.19%
 D. 75%
- 2 Which of the following scatterplots best represents a data set with a value for Pearson's correlation coefficient of $r = -0.75$?



3 What is the value of $\int_0^4 \sqrt{16 - x^2} dx$?

- A. 2π
- B. 4π
- C. 6π
- D. 8π

4 The n^{th} term in a sequence is denoted by T_n and the sum of the first n terms in a sequence is denoted as S_n . For a particular sequence, $S_8 = 50$ and $S_9 = 92$.

What is the value of T_9 ?

- A. 25
- B. 42
- C. 76
- D. 93

- 5 The table below shows the future value of an annuity of \$1.

Periods	Interest rate (per period)				
	2%	3%	4%	5%	6%
2	2.020	2.030	2.040	2.050	2.060
3	3.060	3.091	3.122	3.153	3.184
4	4.122	4.184	4.246	4.310	4.375
5	5.204	5.309	5.416	5.526	5.637
6	6.308	6.468	6.633	6.802	6.975
7	7.434	7.663	7.898	8.142	8.394
8	8.583	8.892	9.214	9.549	9.898

Angela deposits \$5500 into an annuity account every six months for 3 years. The annuity earns 4% per annum, compounded every six months.

What is the value of Angela's investment at the end of 3 years?

- A. \$11 220
 B. \$16 830
 C. \$34 694
 D. \$36 482
- 6 The table below shows the values of two functions, $f(x)$ and $g(x)$, for various values of x .

It is known that $f(x)$ is an even function and $g(x)$ is an odd function.

x	1	2	4	6	7
$f(x)$	3	5	6	-1	-4
$g(x)$	2	0	1	4	7

What is the value of $f(g(-4))$?

- A. 2
 B. 3
 C. 4
 D. 5

7 The point $A(4,2)$ lies on the graph of $y = f(x)$.

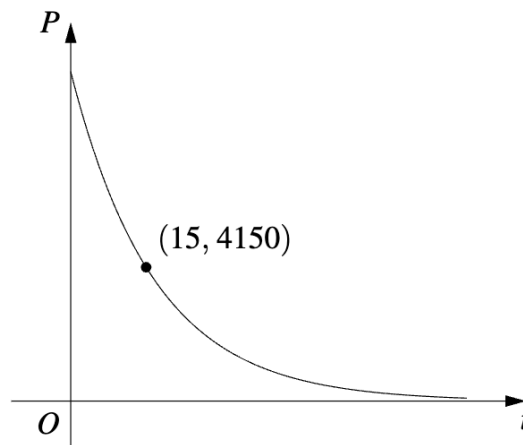
What are the coordinates of the point A' , the image point of A , on the graph of $y = g(x)$ if $g(x) = -f\left(\frac{x}{2}\right)$?

- A. $(-4, -2)$
- B. $(-4, 2)$
- C. $(8, 2)$
- D. $(8, -2)$

8 The population, P , of an agricultural pest can be modelled by the equation

$$P = P_0 e^{-0.32t}$$

where t is the time, in days, since the use of a new insecticide is introduced.



At what rate is the population of pests decreasing after 15 days?

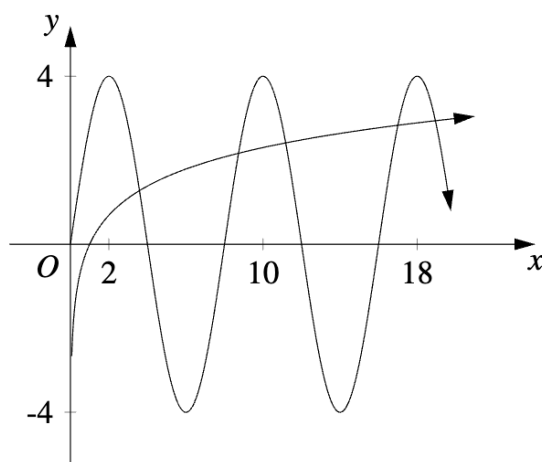
- A. 277 pests/day
- B. 1328 pests/day
- C. 1500 pests/day
- D. 12 696 pests/day

- 9 It is known that $P(R) = \frac{2}{5}$, $P(R | S) = \frac{1}{3}$ and $P(S | R) = \frac{1}{2}$.

What is the value of $P(R \cup S)$?

- A. $\frac{1}{5}$
- B. $\frac{1}{3}$
- C. $\frac{3}{5}$
- D. $\frac{4}{5}$

- 10 The graphs of $y = 4\sin\left(\frac{\pi}{4}x\right)$ and $y = \ln x$ are shown on the number plane below.



How many solutions exist to the equation $4\sin\left(\frac{\pi}{4}x\right) = \ln x$ in the domain $0 \leq x \leq 50$?

- A. 10
- B. 11
- C. 12
- D. 13

End of Section I

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Section II

90 marks

Attempt Questions 11–31

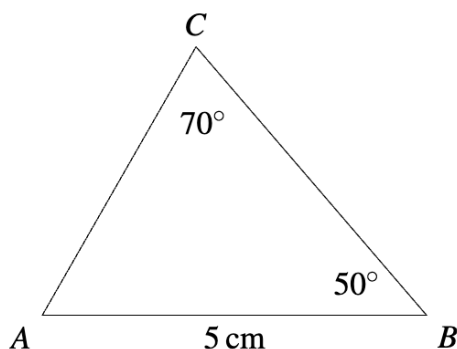
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra exam writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (4 marks)

- (a) The diagram shows triangle ABC . It is known that $AB = 5$ cm and $\angle ABC = 50^\circ$ and $\angle BCA = 70^\circ$. **2**



Find the length of AC , correct to two decimal places.

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- (b) Hence, find the area of triangle ABC , correct to the nearest square centimetre. **2**

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Question 12 (4 marks)

The volume, V , of a puddle, measured in milliliters (ml) is proportional to the square of the length, L , of the puddle, measured in centimetres (cm).

When the puddle is $8cm$ long the volume is $29.44ml$.

- (a) Find the equation relating V and L .

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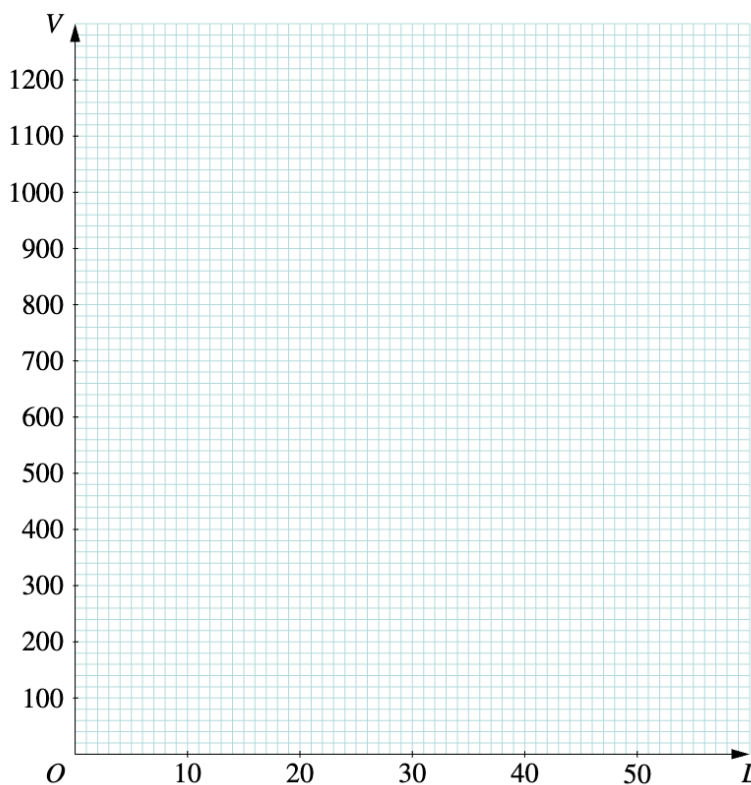
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- (b) By first completing the table of values, graph the relationship between the length of the puddle and the volume of the puddle from $L = 10cm$ to $L = 50cm$.

2

L	10	20	30	40	50
V					



Question 13 (2 marks)

Evaluate $17 + 14 + 11 + \dots - 106$.

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Question 14 (1 mark)

State the domain of $f(x) = \frac{1}{\sqrt{x-2}}$

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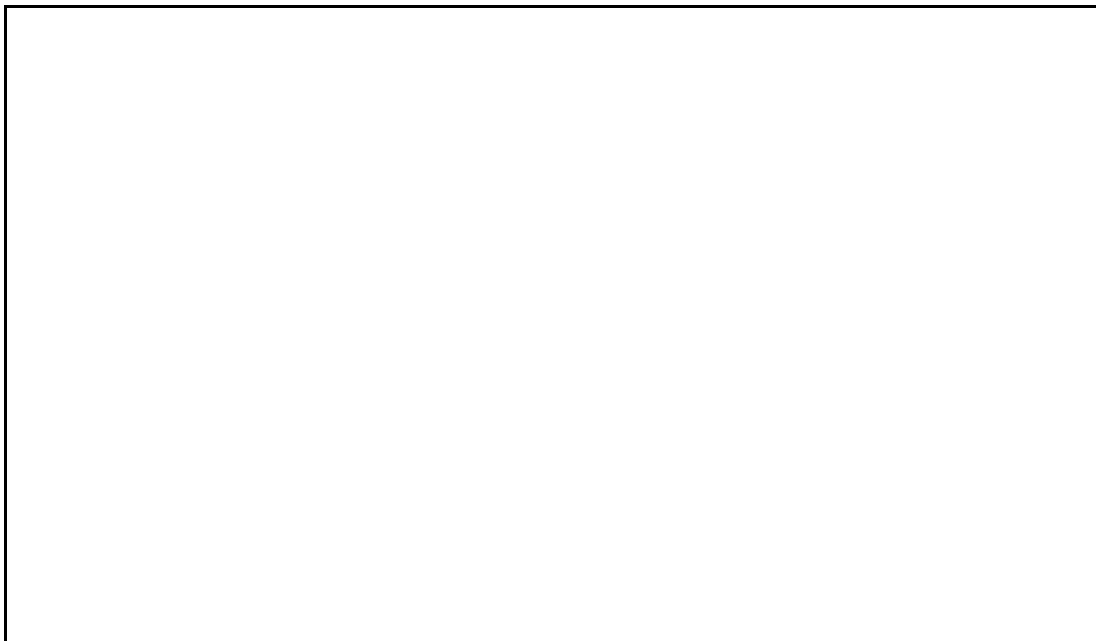
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Question 15 (4 marks)

- (a) In the space below, sketch the graph of $y = \ln(x - 3)$, showing the vertical asymptote, x -intercept and one other point on the curve.

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- (b) Using the trapezoidal rule with 4 sub-intervals, approximate the area enclosed by the curve, the x -axis and the line $x = 6$, correct to one decimal place.

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Question 16 (2 marks)

Evaluate $\int_1^4 \sqrt{3x-2} \, dx$, correct to two decimal places.

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Question 17 (3 marks)

Find the exact value(s) of x such that $9\log_5 x = 25\log_x 5$.

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Question 18 (3 marks)

Let $f(x) = \sin\left(x - \frac{\pi}{3}\right)$.

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It is known that $\int_{\frac{\pi}{3}}^k f(x) \, dx = \int_{\frac{\pi}{3}}^0 f(x) \, dx$ for some $k \in \mathbb{R}$.

Find the smallest value of k in the domain $[0, 2\pi]$.

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Please turn over

Question 19 (8 marks)

Let $f(x) = \frac{(x-4)^2(x+1)}{3}$ for $-2 \leq x \leq 6$.

- (a) Find the coordinates of the stationary points on the graph of $y = f(x)$ and determine their nature. **3**

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- (b) Find the coordinates of the point of inflexion on the graph of $y = f(x)$. **2**

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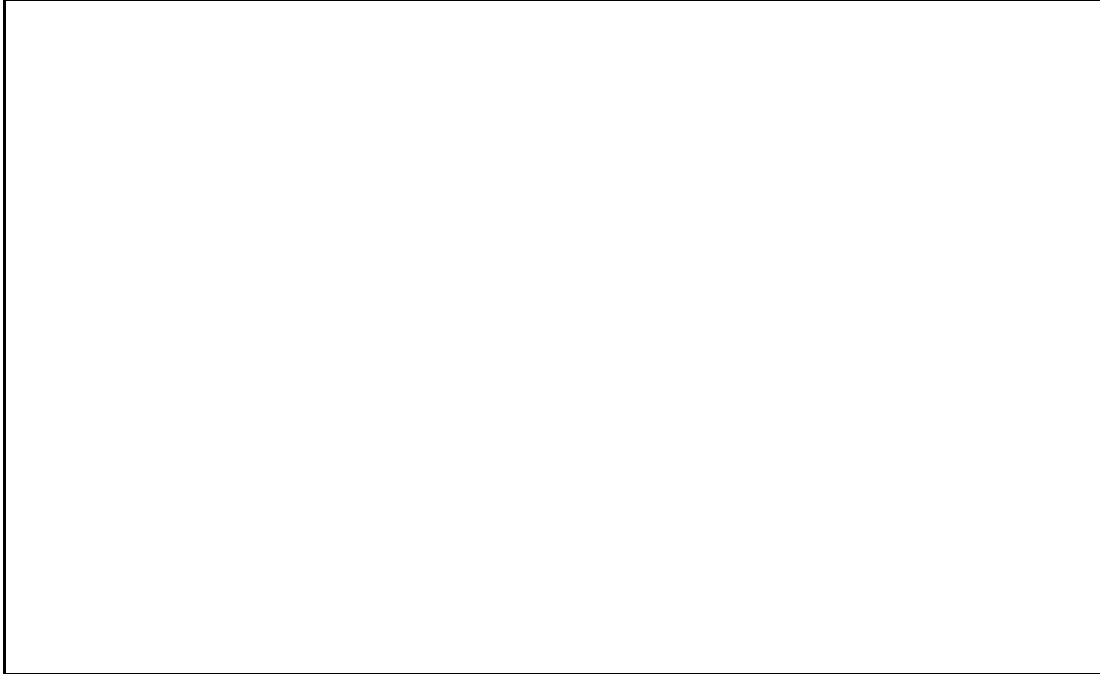
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Question 19 continues on next page

Question 19 (continued)

(c) In the space below, sketch the graph of $y = f(x)$.

2



(d) Hence, state the maximum value of $f(x)$.

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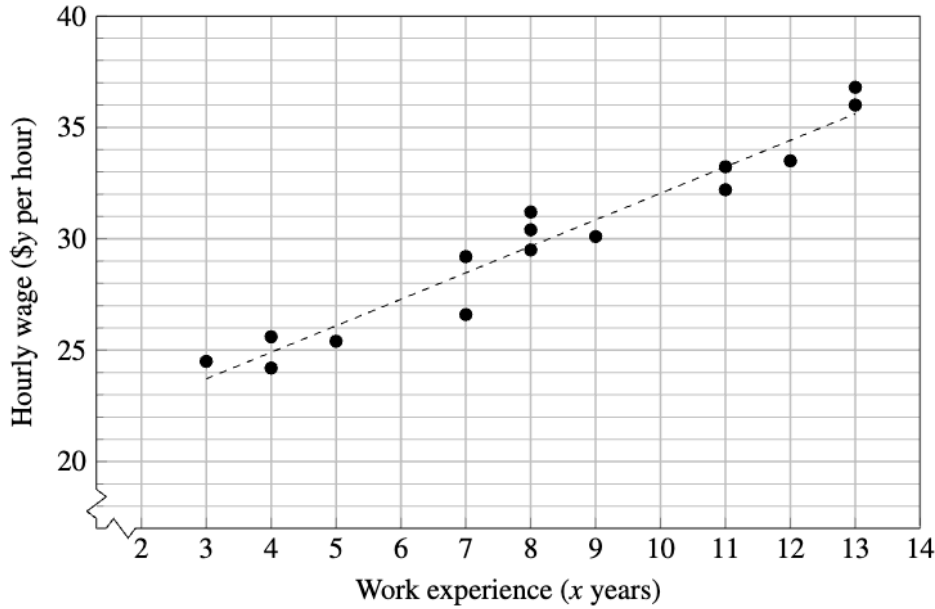
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End of Question 19

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Question 20 (4 marks)

A survey of 15 employees was conducted. Their work experience, x years, and their hourly wage, $\$y$ per hour, were recorded. The graph shows the data as well as the least-squares regression line.



The equation of the least-squares regression line is $y = 1.189x + c$.

The value of Pearson's correlation coefficient is $r = 0.921$.

- (a) Interpret the gradient of the least-squares regression line in the given context. **1**

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- (b) It is known that one of the respondents with 11 of years experience earns \$33.20 per hour, and that this point lies on the least-squares regression line. **2**

Predict the hourly wage of an employee who has 10 years of experience.

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Question 20 continues on the next page

Question 20 (continued)

- (c) Another group of 15 employees was surveyed. Their work experience, in years, and their hourly wage, in dollars per hour, were recorded. The value of Pearson's correlation coefficient for this bivariate dataset was found to be $r = 0.850$.

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Which dataset would be better to use to predict the hourly wage of employees based on work experience? Give a reason for your answer.

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End of Question 20

Please turn over

Question 21 (3 marks)

(a) Show that $\frac{\sec^2 x}{\tan^2 x} = \operatorname{cosec}^2 x$.

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(b) Hence, or otherwise evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec}^2 x \, dx$. Leave your answer in exact form.

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Question 22 (6 marks)

Let X represent the non-negative difference between the numbers on their uppermost faces when two dice are tossed.

- (a) Find the probability that $X = 3$. **2**

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- (b) Find $E(X)$ correct to two decimal places. **2**

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- (c) Calculate the standard deviation of X , correct to two decimal places. **2**

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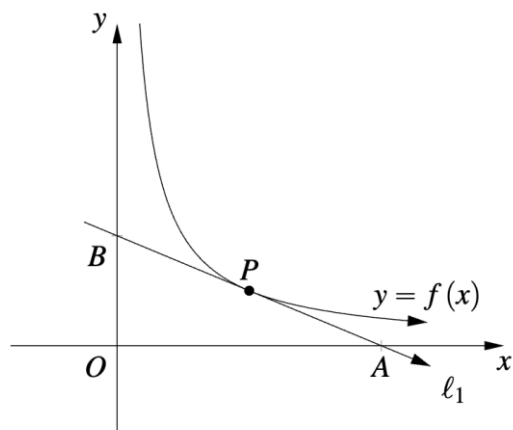
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Question 23 (4 marks)

Let $f(x) = \frac{k}{x}$, where $x > 0$ and $k > 0$.

Point $P\left(p, \frac{k}{p}\right)$ lies on the curve $y = f(x)$. The line ℓ_1 is a tangent to the graph of $y = f(x)$ at the point P , meets the x -axis at A and y -axis at the point B , as shown.



- (a) Show that the equation of ℓ_1 is $kx + p^2y = 2pk$.

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- (b) Find the area of triangle AOB in terms of k .

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Question 24 (4 marks)

- a) Describe in correct order a dilation and a translation that can be used to transform the graph of the curve $y=f(x)$ into the graph of the curve $y=f(2x-1)$. **2**

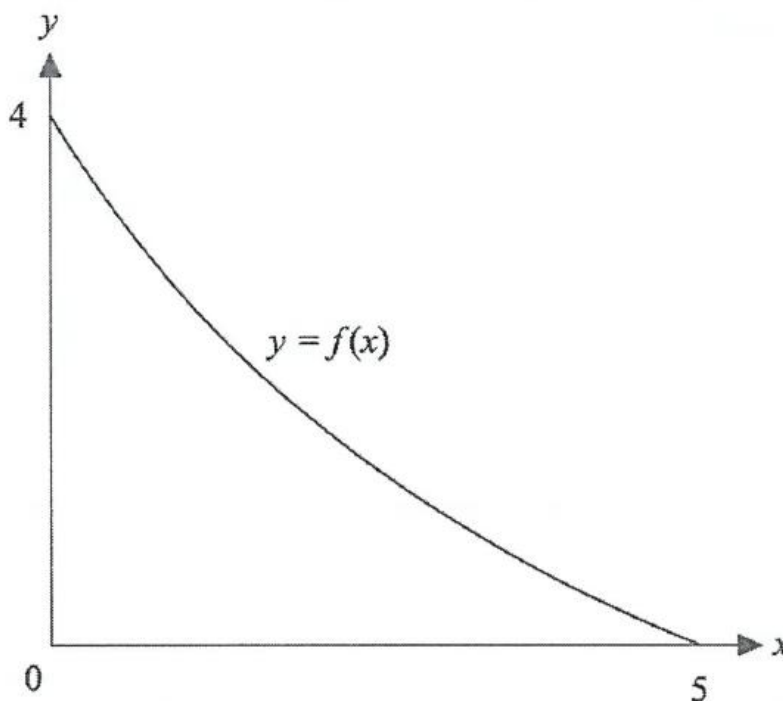
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- b) **2**



The diagram shows the graph of the curve $y=f(x)$ with endpoints $(0, 4)$ and $(5, 0)$
On the diagram above, sketch the curve $y=f(2x-1)$, clearly showing the co-ordinates of the endpoints.

End of Question 24

Please turn over

Question 25 (7 marks)

Noah purchases a home and takes out a reducing-balance loan of \$350 000 to be repaid over 25 years. The interest rate charged is 6% per annum, compounding monthly.

- (a) The table below shows the present value interest factors for a loan of \$1 for various interest rates (r) and number of periods (N).

2

Table of present value interest factors					
$N \backslash r$	Interest rate per period (as a decimal)				
	0.002	0.003	0.004	0.005	0.006
100	90.5529	86.2830	82.2862	78.5426	75.0339
150	129.4799	120.6474	112.6328	105.3500	98.7226
200	164.7061	150.2317	137.4884	126.2406	116.2873
250	196.5832	175.7009	157.8465	142.5203	129.3113
300	225.4297	197.6274	174.5210	155.2069	138.9683
350	251.5337	216.5039	188.1784	165.0933	146.1288

How much should the monthly payment be in order to repay the loan in equal monthly repayments?
Give your answer to the nearest cent.

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- (b) The interest charged and the balance owing for the first three months of the loan are shown in the spreadsheet below.

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Month	Starting Balance	Interest Charged	Repayment	End Balance
1	350000	A	2255.05	349494.95
2	39494.95	1747.47	2255.05	348987.36
3	348987.36	1744.94	2255.05	348477.25

What is the value of A?

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Question 25 continues on the next page

Question 25 (continued)

(c) How much interest did Noah pay over the course of the 25 years?

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(d) After 15 years, Noah decides to make a lump sum payment of \$50 000 and continues to make the same monthly repayments. The rest of the loan was then repaid over a further 86 months.

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How much interest did Noah save by making the lump sum payment?

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End of Question 25

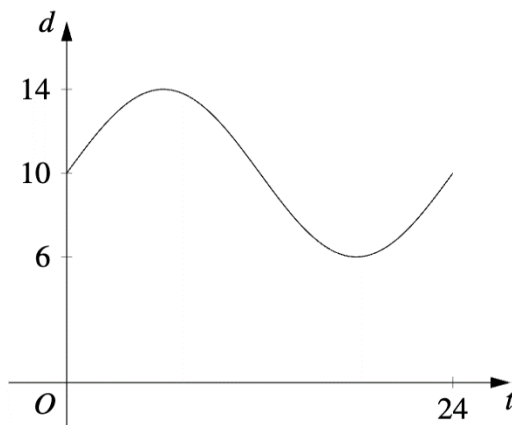
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Question 26 (5 marks)

The depth, d metres, of water in a reservoir is shown below. The depth is modelled using the function

$$d(t) = c + k\sin(bt)$$

where $0 \leq t \leq 24$ is the time in days.



- (a) Find the values of c and k . 2

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- (b) Find the value of b . 1

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- (c) Find the duration when the depth of water in the reservoir is 11 metres or more, correct to one decimal place. 2

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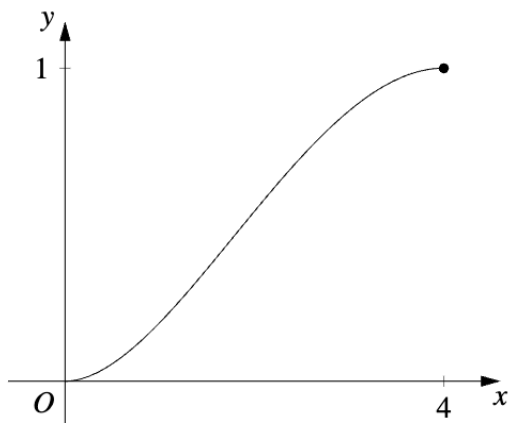
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Question 27 (5 marks)

A continuous random variable, X , has a cumulative distribution function given by:

$$F(x) = \frac{1}{16}x^2(5 - 2\sqrt{x}) \text{ for } 0 \leq x \leq 4$$

The graph of $y = F(x)$ is shown below.



- (a) Find $P(1 \leq X \leq 3)$ correct to two decimal places.

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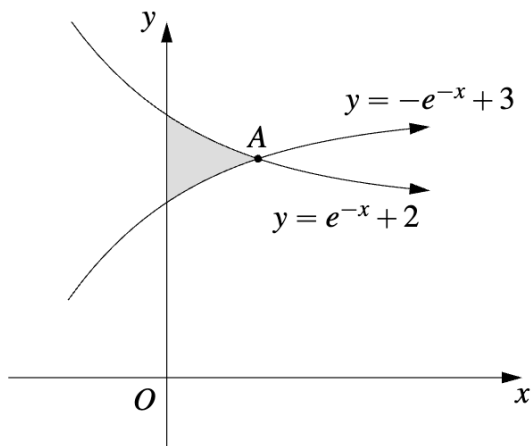
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Question 28 (4 marks)

The region enclosed by the curves $y = e^{-x} + 2$, $y = -e^{-x} + 3$ and the y -axis is shaded in the diagram below. The curves intersect at A .

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Find the area of the shaded region, leaving your answer in simplest exact form.

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Question 29 (4 marks)

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A herb farmer produces herbs with an initial volume of 10 kg. The herbs immediately increase in volume at a rate of 15% per week. She then harvests 500 grams of the herbs each week to sell at the local farmers' market.

A supermarket chain then orders 50 kg of herbs.

Find the minimum number of weeks the farmer can continue to sell at the local farmers' market and also be able to fulfil the order from the supermarket chain.

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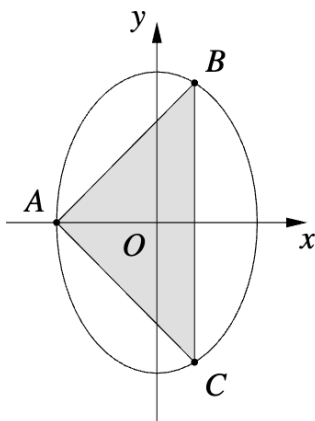
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Question 30 (5 marks)

The diagram below shows the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. The point A has coordinates $(-2,0)$. Points B and C both lie on the ellipse such that the interval BC is vertical. Triangle ABC is shaded.



- (a) Show that for points in the top half of the ellipse, $y = \sqrt{9 - \frac{9}{4}x^2}$.

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Question 31 (8 marks)

A motorcycle stunt rider performs a stunt in which they must travel in a straight line. The motorcycle is initially travelling at a velocity of 70 km/h. The stunt requires the rider to accelerate to a final velocity of 275 km/h in two stages.

In the first stage of the stunt, the motorcycle's acceleration, \ddot{x} km/h², at time, t hours, is given by

$$\ddot{x} = 120\sin(2t)$$

for $0 \leq t \leq k$ where k is the first time the rider reaches a velocity of 160 km/h.

- (a) Find an expression for the velocity of the motorcycle during the first stage. **2**

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- (b) Find the distance travelled by the motorcycle after k seconds. Answer correct to two decimal places. **3**

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