QUESTION ONE *(12 Marks)* **Marks**

(a) Find $\int \frac{e^{\tan x}}{\cos^2 x} dx$ **1**

(b) Find the exact value of $\cos 2x$ if $\sin x = \sqrt{3} - 1$. **2**

(c) Solve $\frac{5}{2x-1} < 3$. **3**

(d) If α, β and γ are the roots of the equation $x^3 - 3x^2 - 5x + 2 = 0$, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$. **2**

(e) Use the substitution $x = u^2 + 1$ for $u > 0$ to evaluate the integral: **4**

$$\int_1^5 (x+1)\sqrt{x-1}\, dx$$

Question two …

QUESTION TWO *(12 Marks)* Start this question in a new booklet. **Marks**

(a) Use the substitution $t = \tan\frac{x}{2}$ to solve $\sin x - 7\cos x - 5 = 0$ where $0^\circ < x < 360^\circ$. Give your answers correct to the nearest degree. **3**

(b) When the polynomial $P(x)$ is divided by $x^2 - 1$ the remainder is $3x + 1$. What is the remainder when $P(x)$ is divided by $x + 1$? **2**

(c) Nine people are to be seated at a round table.

(i) How many seating arrangements are possible? **1**

(ii) Two people, Amy and Sarah, refuse to sit next to one another. How many seating arrangements are possible now? **2**

(d) (i) Prove that $\cot x + \tan x = 2\operatorname{cosec} 2x$. **2**

(ii) Hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec} 2x \, dx$ **2**

Question Three …

<u>QUESTION THREE</u> *(12 Marks)* Start this question in a new booklet. **Marks**

(a) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $4ay = x^2$ where $a > 0$. Let $S(0, a)$ be the focus of the parabola.

(i) Show that the equation of the tangent at P is $y = px - ap^2$. **2**

(ii) The tangents at P and Q intersect at T. Show that the coordinates of T are $\left(a\left(p+q\right), apq\right)$. **2**

(iii) Show that $SP = a(p^2 + 1)$. **2**

(iv) P and Q move on the parabola in such a way that $SP + SQ = 4a$. Show that the locus of the point T is a parabola and find the coordinates of its vertex and focus. **2**

(b) (i) Explain why the curve $f(x) = x + \log_e x$ is increasing for all values of x in its domain. **1**

(ii) Show that the curve cuts the x-axis between $x = 0 \cdot 5$ and $x = 1$. **1**

(iii) Use Newton's method with a first approximation of $x = 0 \cdot 5$ to find a second approximation to the root of $x + \log_e x = 0$.
Give your answer correct to two decimal places. **2**

Question Four …

QUESTION FOUR *(12 Marks)* Start this question in a new booklet. **Marks**

(a) A particle moving on a horizontal line has a velocity of *v* m/s where $v^2 = 64 - 4x^2 + 24x$.

(i) Prove that the motion is simple harmonic. **2**

(ii) Find the centre of the motion. **1**

(iii) Write down the period and amplitude of the motion. **2**

(iv) Initially the particle is at the centre of motion and is moving to the left. Write down an expression for the displacement of the particle as a function of time. **2**

(b) The rate at which a body cools in air is proportional to the difference between its temperature *T* and the constant temperature $18^{\circ}C$ of the surrounding air. This can be expressed by the equation $\frac{dT}{dt} = -k(T - 18)$. The original temperature of a cup of boiling water was $100^{\circ}\ C$. The water cools to $65^{\circ}\ C$ in 15 minutes.

(i) Show that $T = 18 + Ae^{-kt}$ is a solution of the differential equation. **1**

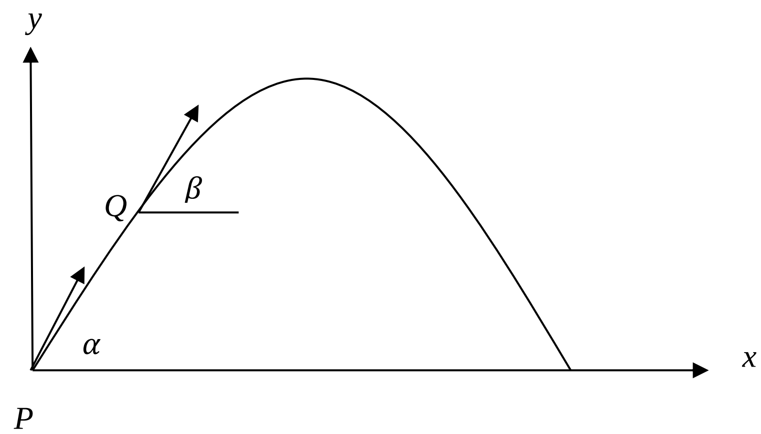
(ii) Find the values of *A* and *k*. **2**

(iii) Find, to the nearest minute, the time for the temperature of the water to reach $50^{\circ}\ C$. **2**

Question Five …

<u>QUESTION FIVE</u> *(12 Marks)* Start this question in a new booklet. **Marks**

(a)



A particle is projected from a point P on horizontal ground with velocity V at an angle of elevation α to the horizontal. It's equations of motion are $\ddot{x} = 0$ and $\ddot{y} = -g$.

(i) Derive equations for the horizontal and vertical displacements of the particle from P after t seconds. **2**

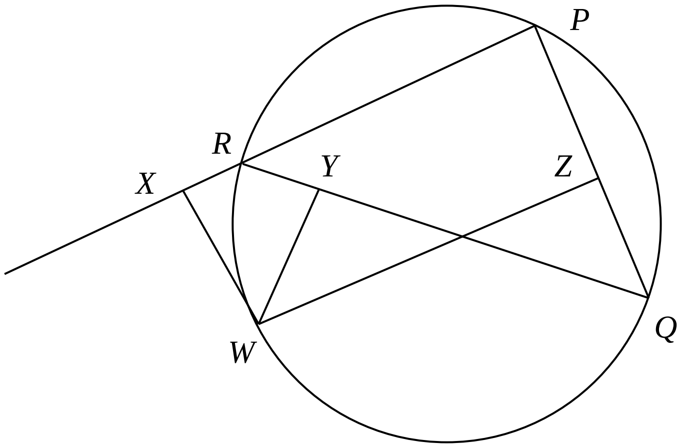
(ii) Determine the time of flight of the particle. **2**

(iii) The particle reaches the point Q where the direction of flight makes an angle β with the horizontal. Prove that the time taken for the particle to travel from P to Q is $\dfrac{V\sin(\alpha-\beta)}{g\cos\beta}$ seconds. **3**

Question Five continued …

Question 5 Continued **Marks**

(b)



PQR is a triangle inscribed in a circle and *W* is a point on the arc *QR*. *WX* is perpendicular to *PR* produced. *WZ* is perpendicular to *PQ* and *WY* is perpendicular to *QR*.

(i) Copy the diagram into your answer booklet.

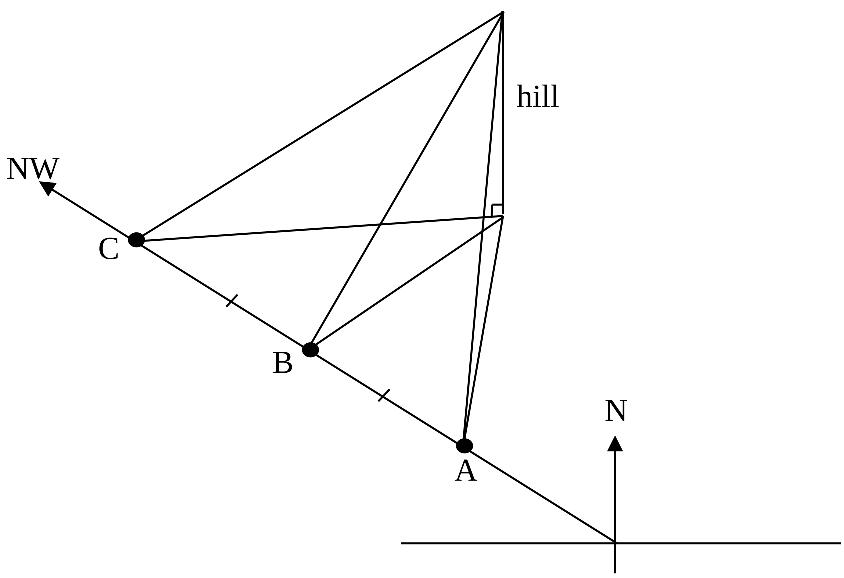
(ii) Explain why *WXRY* and *WYZQ* are cyclic quadrilaterals. **2**

(iii) Show that the points *X*, *Y* and *Z* are collinear. **3**

Question Six …

<u>QUESTION SIX</u> *(12 Marks)* Start this question in a new booklet. **Marks**

(a) A man is travelling along a straight flat road that bears north west. On his journey he passes through three points *A*, *B* and *C* in that order, where $AB = BC = 200$ m. From these three points he observes the angle of elevation of the top of a hill to the right of the road. These angles are 30°, 45° and 45° respectively.



(i) Find the height of the hill. **3**

(ii) Find the bearing of the hill from *A*. **2**

Question six continued …

Question 6 Continued **Marks**

(b) Find the coefficient of x^3 in the expansion of $\left(2x - \frac{1}{x^2}\right)^9$. **3**

(c) Consider the binomial expansion of $(5 + 11x)^{23}$.

(i) Given $T_k = {}^{23}C_{k-1}\left(11x\right)^{k-1}\left(5\right)^{23-(k-1)}$, show that $\dfrac{T_{k+1}}{T_k} = \dfrac{11x\left(24-k\right)}{5k}$. **2**

(ii) Find an expression for the greatest coefficient in the expansion. **2**

Question Seven …

QUESTION SEVEN *(12 Marks)* Start this question in a new booklet. **Marks**

(a) (i) Show that $\tan^{-1}(n+1) - \tan^{-1}(n-1) = \tan^{-1}\left(\frac{2}{n^2}\right)$. **2**

(ii) Hence or otherwise show that: **3**

$$\frac{\pi}{4} + \sum_{r=1}^{n} \tan^{-1}\left(\frac{2}{r^2}\right) = \tan^{-1}\left(\frac{2n+1}{1-n-n^2}\right)$$

(b) (i) Use the binomial theorem to prove that: **2**

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ ...$$

(ii) Use mathematical induction to prove that $\frac{1}{n!} < \frac{1}{2^{n-1}}$ for $n \geq 3$. **3**

(iii) Hence deduce that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = N$ where $2 < N < 3$. **2**

END OF PAPER