NORTH SYDNEY GIRLS HIGH SCHOOL

2013
TRIAL HSC EXAMINATION

Mathematics Extension 1

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at
- the back of this booklet
- Show all necessary working in questions 11 – 14

Total Marks – 70

Section 1 10 marks

- Attempt Questions 1 -10
- Allow about 15 minutes for this section.

Section 2 60 Marks

- Attempt Questions 11 - 14

NAME:______________________________ **TEACHER:**____________________

NUMBER:___________________________

QUESTION	MARK
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
TOTAL	/70

Section I **Objective response questions**

Total marks − 10
Attempt Questions 1−10

Answer each question on the multiple choice answer sheet provided.

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1. The polynomial $P(x)=x^4-kx^3-2x+33$ has $(x-3)$ as a factor.

What is the value of k ?

(A) -5
(B) -4
(C) 4
(D) 5

2. Which is the correct condition for $y=mx+b$ to be a tangent to $x^2=4ay$?

(A) $am-b=0$
(B) $am^2-b=0$
(C) $am+b=0$
(D) $am^2+b=0$

3. What is the correct expression for $\int \frac{dx}{9+4x^2}$?

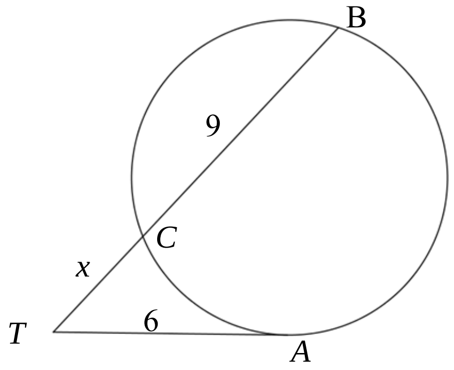
(A) $\frac{1}{4}\tan^{-1}\left(\frac{2x}{3}\right)+c$

(B) $\frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right)+c$

(C) $\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+c$

(D) $\frac{2}{3}\tan^{-1}\left(\frac{2x}{3}\right)+c$

4. Line *TA* is a tangent to the circle at *A*. *TB* is a secant cutting the circle at *B* and *C*.



$TA = 6$ cm

$BC = 9$ cm

$TC = x$ cm

What is the value of x?

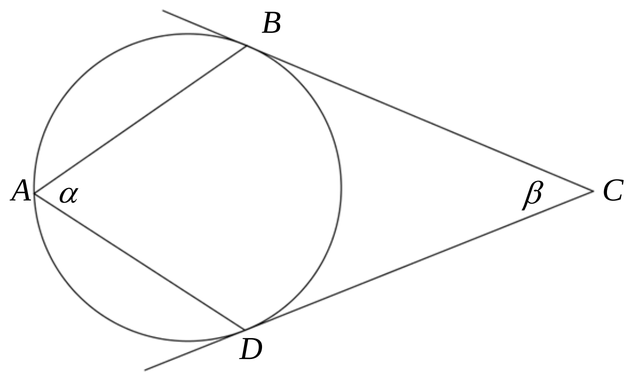
(A) 2

(B) 3

(C) 4

(D) 12

5. In the diagram below, *BC* and *DC* are tangents. Which statement is correct?



(A) $\alpha + \beta = 180^{\circ}$

(B) $2\alpha + \beta = 180^{\circ}$

(C) $\alpha + 2\beta = 180^{\circ}$

(C) $2\alpha - \beta = 180^{\circ}$

6. If $t = \tan\dfrac{\theta}{2}$, which expression is equivalent to $4\sin\theta + 3\cos\theta + 5$?

(A) $\dfrac{2(t+2)^2}{1+t^2}$

(B) $\dfrac{2(t+2)^2}{1-t^2}$

(C) $\dfrac{(t+4)^2}{1-t^2}$

(D) $\dfrac{(t+4)^2}{1+t^2}$

7. A particle moves under simple harmonic motion such that its position x metres after t seconds is given by $x = 8\sin\left(\frac{t}{4} - \frac{\pi}{2}\right)$. Which of the following statements is FALSE?

(A) The maximum speed of the particle is 2 m/s.

(B) The maximum acceleration of the particle is 0.5 m/s^2.

(C) The particle takes 2π seconds to travel between its extremities.

(D) The particle is initially to the left of the origin.

8. Consider the function $f(x) = \sqrt{4-x}$. Which of the following is the correct inverse function, $f^{-1}(x)$?

(A) $f^{-1}(x) = 4 - x^2$

(B) $f^{-1}(x) = 4 - x^2,\ x \geq 0$

(C) $f^{-1}(x) = 4 - x^2,\ x \leq 0$

(D) $f^{-1}(x) = 4 - x^2,\ x \leq 4$

9. Which of the following is an expression for $\int_0^{\frac{1}{2}} x\sqrt{1-2x}\ dx$ using the substitution $u = (1-2x)$?

(A) $\frac{1}{4}\int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}})du$

(B) $\int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}})du$

(C) $\frac{1}{4}\int_0^1 (u^{\frac{3}{2}} - u^{\frac{1}{2}})du$

(D) $\int_0^1 (u^{\frac{3}{2}} - u^{\frac{1}{2}})du$

10. Consider the function f such that $f(x) = a\cos^{-1}(x-b)$ given that f has domain $2 \leq x \leq 4$ and range $0 \leq y \leq 6\pi$. What are the values of a and b?

(A) $a = 6, b = -3$

(B) $a = 12, b = 3$

(C) $a = 12, b = -3$

(D) $a = 6, b = 3$

Section II **Free response questions**
Total marks − 60
Attempt Questions 11−14

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 11. (15 marks) Use a SEPARATE writing booklet.

(a) (i) Express $\cos 2x$ in terms of $\sin^2 x$. **1**

(ii) Hence evaluate $\lim_{x\to 0}\left(\dfrac{\cos 2x - 1}{x\sin x}\right)$ **1**

(b) Find all solutions to the equation $\sin 2\theta + \cos\theta = 0$ for $0 \le \theta \le 2\pi$. **3**

(c) Find the size of the acute angle between the lines $x + y - 2 = 0$ and $2x - y = 0$.

Give your answer to the nearest degree. **2**

(d) Find $\dfrac{d}{dx}\left[\cos^{-1}\left(\dfrac{1}{x}\right)\right]$ **2**

(e) The equation $x^3 - mx + 2 = 0$ has two equal roots.

(i) Write down expressions for the sum of the roots and the product of the roots. **1**

(ii) Hence find the value of m. **2**

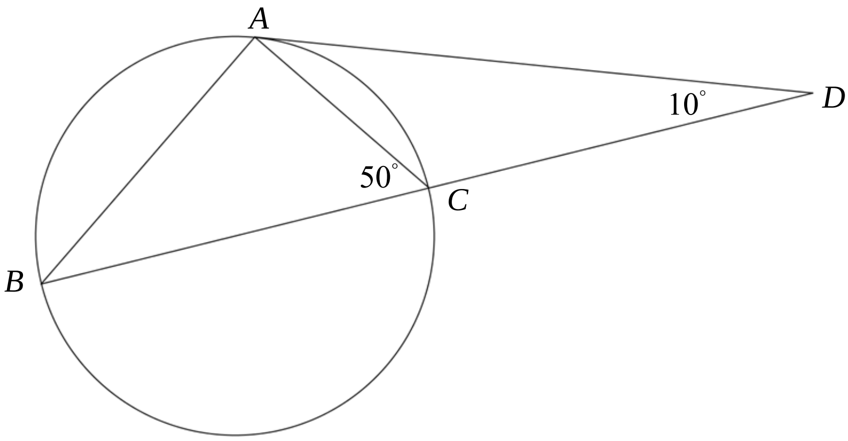
Question 11 continues on page 6

Question 11 (*continued*)

(f) ABC is a triangle inscribed in a circle.

The tangent at A meets BC produced at D.

$\angle ACB = 50^\circ$, $\angle CDA = 10^\circ$



Prove that BC is a diameter of the circle. **3**

Question 12. (15 marks) Use a SEPARATE writing booklet.

(a) At a time t minutes after an oven is switched on, its temperature T° is given by

$$T = 250 - 220e^{-0.1t}.$$

(i) State both the initial and the limiting value of the oven's temperature. **2**

(ii) Find the time taken for the oven's temperature to reach 180°. **2**

(iii) Find the rate at which the temperature is increasing when the temperature reaches 180°. **2**

(b) $T\left(2t, t^2\right)$ is a point on the parabola $x^2 = 4y$ with focus S. The point P divides ST internally in the ratio 1:2.

(i) Write down the coordinates of P in terms of t. **2**

(ii) Hence show that as T moves on the parabola $x^2 = 4y$, the locus of P is the parabola $9x^2 = 12y - 8$. **2**

(c) (i) Show that $\sin(A+B) - \sin(A-B) = 2\cos A \sin B$. **1**

(ii) Hence evaluate $\int \cos 3x \sin x \, dx$ **2**

(d) A particle moves such that its velocity v (metres/second), in terms of its displacement x, is given by $v = 2 + \sin x$.

Find the acceleration of the particle at the origin. **2**

Question 13. (15 marks) Use a SEPARATE writing booklet.

(a) Use the substitution $u = \ln x$ to determine $\int \frac{\sqrt{\ln x} + \ln x}{x} dx$ **3**

(b) Solve the inequality $\frac{2x-5}{x-4} \geq x$. **3**

(c) Use the method of mathematical induction to prove that $5^n - (-1)^n$ is divisible by 6 where *n* is a positive integer. **3**

(d) A particle moves in simple harmonic motion such that its displacement *x* centimetres after *t* seconds is given by $x = 5 + 2\sqrt{3}\cos 3t + 2\sin 3t$.

(i) Express *x* in the form $x = 5 + R\cos(3t - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **2**

(ii) Write down

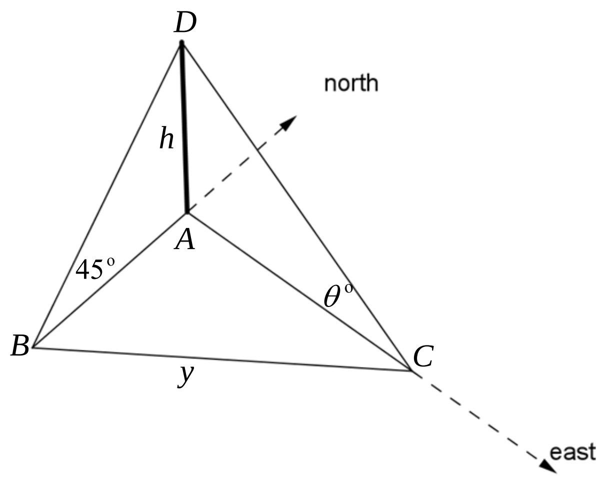
(α) the period of the motion **1**

(β) the range of possible values for *x* **1**

(iii) How much time elapses from when the particle first starts until it is first at the point closest to the origin? **2**

Question 14. (15 marks) Use a SEPARATE writing booklet.

(a)



AD is a tower with a height of *h* metres. *A*, *B* and *C* are 3 points on level ground. *B* is due south of *A* and *C* is due east of *A*. From B, the angle of elevation of D is 45°. From C, the angle of elevation of D is θ°. Let the length $BC = y$.

Show that $h = y\sin\theta$. **3**

(b) Consider the function $f(x) = e^{2x} + e^x$.

(i) Explain why $y = f(x)$ has an inverse function $y = f^{-1}(x)$ for all *x*. **1**

(ii) Draw a neat sketch of $y = f(x)$ and $y = f^{-1}(x)$ showing all intercepts and asymptotes. **2**

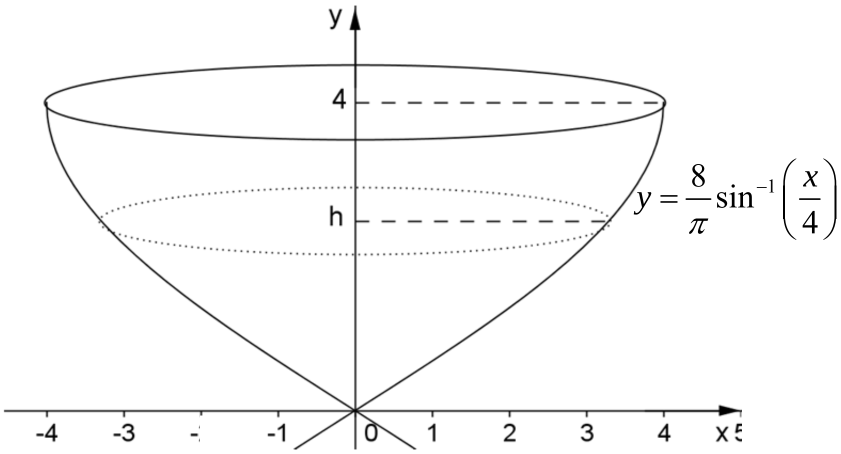
(iii) Find the equation of the inverse function in terms of *x*. **3**

(iv) Hence or otherwise solve $e^{2x} + e^x = 6$. **1**

Question 14 continues on page 10

Question 14 (*continued*)

(c) The area bounded by $y = \frac{8}{\pi}\sin^{-1}\left(\frac{x}{4}\right)$, $y = 4$ and the y – axis is rotated about the y – axis to form a storage tank for oil.



(i) Show that the volume of the oil in cubic metres when the depth is h metres is given by $V = 8\pi h - 32\sin\left(\frac{\pi h}{4}\right)$ **3**

(ii) The tank is being filled at a constant rate of π m^3 / minute. Find the rate the depth is increasing when the depth is 2 m. **2**

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2013 NSGHS EXTENSION 1 TRIAL SOLUTIONS

Section I (answers)

1. **C**
2. **D**
3. **C**
4. **B**
5. **B**
6. **A**
7. **C**
8. **B**
9. **A**
10. **D**

Section I (solutions)

$P(x) = x^4 - kx^3 - 2x + 33$

$P(3) = 3^4 - k \times 3^3 - 2 \times 3 + 33 = 0$

1 $81 - 27k - 6 + 33 = 0$

$27k = 108$

$k = 4$

i.e. **C**

2.

$y = mx + b$

$x^2 = 4ay$

$\therefore x^2 = 4a(mx + b)$

$x^2 - 4amx - 4ab = 0$

$\Delta = 16a^2m^2 + 16ab = 0$

$16a(am^2 + b) = 0$

$\therefore am^2 + b = 0$

i.e. **D**

3.

$\int \frac{dx}{9 + 4x^2}$

$= \frac{1}{4}\int \frac{dx}{\left(\frac{3}{2}\right)^2 + x^2}$

$= \frac{1}{4} \times \frac{2}{3}\tan^{-1}\left(\frac{3x}{2}\right)$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$

i.e. **C**

4.

$x(x+9) = 6^2$

$x^2 + 9x - 36 = 0$

$(x+12)(x-3) = 0$

$x = -12(n.a.), 3$

$\therefore x = 3$

i.e. **B**

5.

$\angle BOD = 180 - \beta$

(cyclic quad $BODC$)

$\angle BOD = 2 \times \angle BAD$

(angle at centre)

$180 - \beta = 2\alpha$

$2\alpha + \beta = 180$

i.e. **B**

6.

$4\sin\theta + 3\cos\theta + 5$

$= 4 \times \frac{2t}{1+t^2} + 3 \times \frac{1-t^2}{1+t^2} + 5$

$= \frac{8t + 3 - 3t^2 + 5 + 5t^2}{1+t^2}$

$= \frac{2t^2 + 8t + 8}{1+t^2}$

$= \frac{2(t+2)^2}{1+t^2}$

i.e. **A**

7.

$x = 8\sin\left(\frac{t}{4} - \frac{\pi}{2}\right)$

$\dot{x} = 2\cos\left(\frac{t}{4} - \frac{\pi}{2}\right)$

A true

$\ddot{x} = -0.5\sin\left(\frac{t}{4} - \frac{\pi}{2}\right)$

B true

$t = 0,\ x = 8\sin\left(-\frac{\pi}{2}\right) < 0$

D true

period $= 2\pi \div \frac{1}{4} = 8\pi$

time $= 4\pi$

C false

i.e. **C**

8.

$f(x) = \sqrt{4 - x},$

$x = \sqrt{4 - y}$, range $y \ge 0$

$x^2 = 4 - y$

$y = 4 - x^2$, domain $x \ge 0$

i.e. **B**

9.

$I = \int_0^{\frac{1}{2}} x\sqrt{1 - 2x}\ dx$

$u = 1 - 2x$

$\therefore du = -2dx$

$x = 0 \to u = 1$

$x = \frac{1}{2} \to u = 0$

$\therefore I = \int_1^0 \frac{1-u}{2}\sqrt{u} \times -\frac{du}{2}$

$= \frac{1}{4}\int_0^1 (1-u)\sqrt{u}\ du$

$= \frac{1}{4}\int_0^1 (u^{\frac{1}{2}} - u^{\frac{3}{2}})du$

i.e. **A**

10.

$f(x) = a\cos^{-1}(x - b)$

domain $-1 \le x - b \le 1$

$b - 1 \le x \le 1 + b$

$b - 1 = 2$

$\therefore b = 3$

$0 \le \cos^{-1}(x - b) \le \pi$

$0 \le a\cos^{-1}(x - b) \le 6\pi$

$\therefore a = 6$

i.e. **D**

SECTION II

Question 11

(a)

(i) $\cos 2x = 1 - 2\sin^2 x$

(ii)

$$\lim_{x\to 0}\left(\frac{\cos 2x - 1}{x\sin x}\right)$$
$$= \lim_{x\to 0}\left(\frac{1-2\sin^2 x - 1}{x\sin x}\right)$$
$$= \lim_{x\to 0}\left(\frac{-2\sin^2 x}{x\sin x}\right)$$
$$= -2\times\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$$
$$= -2$$

(b)

$$\sin 2\theta + \cos\theta = 0$$
$$2\sin\theta\cos\theta + \cos\theta = 0$$
$$\cos\theta(2\sin\theta + 1) = 0$$
$$\cos\theta = 0,\ \sin\theta = -\tfrac{1}{2}$$
$$\therefore \theta = \tfrac{\pi}{2}, \tfrac{3\pi}{2}, \tfrac{7\pi}{6}, \tfrac{11\pi}{6}$$

(c)

$x + y - 2 = 0,\ m_1 = -1$

$2x - y = 0,\quad m_2 = 2$

$$\tan\theta = \left|\frac{-1-2}{1+(-1)\times 2}\right|$$
$$= \left|\frac{-3}{-1}\right|$$
$$= 3$$
$$\therefore \theta = 71.56^\circ$$
$$= 72^\circ \text{ (nearest degree)}$$

(d)

$$\frac{d}{dx}\left[\cos^{-1}\left(\frac{1}{x}\right)\right]$$
$$= \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\times -x^{-2}$$
$$= \frac{1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}}$$
$$= \frac{1}{\sqrt{x^4 - x^2}} = \frac{1}{x\sqrt{x^2-1}}$$

(e)

(i)

$$x^3 - mx + 2 = 0$$
$\therefore$ roots α, α, β
$$\therefore 2\alpha + \beta = 0,\ \alpha^2\beta = -2$$

(ii)

$$\beta = -2\alpha$$
$$\therefore \alpha^2 \times -2\alpha = -2$$
$$\therefore \alpha^3 = 1$$
$$\therefore \alpha = 1,\ \beta = -2$$
$$-m = \alpha^2 + 2\alpha\beta$$
$$\therefore m = -(1^2 + 2\times 1\times -2)$$
$$i.e.\ m = 3$$

(f)

$\angle ACD = 180 - 50 = 130$
(straight angle)
$\angle CAD = 180 - 130 - 10 = 40$
(angle sum ΔCAD)
$\angle ABC = \angle CAD = 40$
(angle in alternate segment)
$\angle BAC = 180 - 40 - 50 = 90$
(angle sum ΔABC)
$\therefore BC$ a diameter
(angle in semicircle)

Question 12

(a)

$$T = 250 - 220e^{-0.1t}$$

(i)

Initially, $\quad T = 250 - 220e^0$
$= 30^\circ$
as $t \to \infty,\quad T \to 250 - \dfrac{220}{e^\infty}$
$\therefore T \to 250^\circ$

(ii)

$$180 = 250 - 220e^{-0.1t}$$
$$220e^{-0.1t} = 70$$
$$-0.1t = \ln\left(\frac{70}{220}\right)$$
$$t = 10\ln\left(\frac{22}{7}\right)$$
$= 11.45$ minutes

(iii)

$$T = 250 - 220e^{-0.1t}$$
$$\frac{dT}{dt} = -220\times -0.1e^{-0.1t}$$
$$= 0.1\times 220e^{-0.1t}$$
$$= 0.1(250 - T)$$
when $T = 180$
$$\frac{dT}{dt} = 0.1(250 - 180)$$
$= 7^\circ$ / minute

(b) (i)

$S(0,1)\ \ T\left(2t, t^2\right)$

$$P\left(\frac{1\times 2t + 0}{3}, \frac{1\times t^2 + 2\times 1}{3}\right)$$
$$P\left(\frac{2t}{3}, \frac{t^2+2}{3}\right)$$

(ii)

$x = \frac{2t}{3}$

$\therefore t = \frac{3x}{2}$

$y = \frac{t^2 + 2}{3}$

$3y = \left(\frac{3x}{2}\right)^2 + 2$

$3y - 2 = \frac{9x^2}{4}$

$9x^2 = 12y - 8$

(c) (i)

$\sin(A+B) - \sin(A-B)$

$= \sin A\cos B + \cos A\sin B$

$-(\sin A\cos B - \cos A\sin B)$

$= 2\cos A\sin B$

(ii)

$\int \cos 3x \sin x \, dx$

$= \frac{1}{2}\int \sin(3x+x) - \sin(3x-x)\, dx$

$= \frac{1}{2}\int \sin(4x) - \sin(2x)\, dx$

$= \frac{1}{2}\left(-\frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x\right) + c$

$= \frac{1}{4}\cos 2x - \frac{1}{8}\cos 4x + c$

(d)

$v = 2 + \sin x$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{1}{2} \times \frac{d}{dx}(2+\sin x)^2$

$= \frac{1}{2} \times 2 \times (2+\sin x) \times \cos x$

$= \frac{1}{2} \times 2 \times (2+\sin 0) \times \cos 0$ initially

$= \frac{1}{2} \times 2 \times 2 \times 1$

$= 2$

Question 13.

(a)

$\int \frac{\sqrt{\ln x} + \ln x}{x} dx$

$u = lnx$

$\frac{du}{dx} = \frac{1}{x}$

$\therefore \int \frac{\sqrt{\ln x} + \ln x}{x} dx$

$\int (\sqrt{u} + u)\frac{dx}{x}$

$= \int (\sqrt{u} + u) du$

$= \frac{2}{3}u^{\frac{3}{2}} + \frac{1}{2}u^2 + c$

$= \frac{2}{3}(\ln x)^{\frac{3}{2}} + \frac{1}{2}(\ln x)^2 + c$

(b)

$\frac{2x-5}{x-4} \ge x$

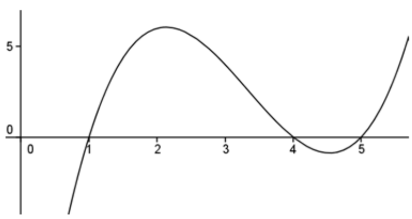
$\frac{(2x-5)(x-4)^2}{(x-4)} \ge x(x-4)^2$

$(2x-5)(x-4) \ge x(x-4)^2$

$x(x-4)^2 - (2x-5)(x-4) \le 0$

$(x-4)(x^2 - 4x - 2x + 5) \le 0$

$(x-4)(x-1)(x-5) \le 0$



$x \le 1$ or $4 < x \le 5$

(c)

to prove $5^n - (-1)^n$ divisible by 6

$n = 1.\quad 5^n - (-1)^n = 5^1 - (-1)^1$

$= 5 + 1 = 6$

$\therefore$ statement true for $n = 1$

$\therefore$ can assume statement true for $n = k$

i.e. $5^k - (-1)^k = 6M$, M integer

$\{5^k = 6M + (-1)^k\}$

to show true for $n = k+1$

i.e. $5^{k+1} - (-1)^{k+1}$ is divisible by 6

$5^{k+1} - (-1)^{k+1} = 5 \times 5^k - (-1)(-1)^k$

$= 5 \times 5^k + (-1)^k$

$= 5[6M + (-1)^k] + (-1)^k$

$= 30M + 5(-1)^k + (-1)^k$

$= 30M + 6(-1)^k$

$= 6[5M + (-1)^k]$

i.e. factor of 6

$\therefore$ true for next term

$\therefore$ true for all $n \ge 1$ by induction

(d) (i)

$x = 5 + 2\sqrt{3}\cos 3t + 2\sin 3t$

$\equiv 5 + R\cos(3t - \alpha)$

i.e. $2\sqrt{3}\cos 3t + 2\sin 3t \equiv R\cos 3t\cos\alpha + R\cos 3t\sin\alpha$

$R\cos\alpha \equiv 2\sqrt{3}$

$R\sin\alpha \equiv 2$

$\therefore \tan\alpha = \dfrac{1}{\sqrt{3}}$

$\therefore \alpha = \dfrac{\pi}{6}$

$R^2(\cos^2\alpha + \sin^2\alpha) = \sqrt{12}^2 + 2^2$

$\therefore R = 4$

$x = 5 + 4\cos\left(3t - \dfrac{\pi}{6}\right)$

(ii)

(α) period $= \dfrac{2\pi}{n} = \dfrac{2\pi}{3}$

(β) $1 \le x \le 9$

(iii)

$x = 1$

$1 = 5 + 4\cos\left(3t - \dfrac{\pi}{6}\right)$

$\cos\left(3t - \dfrac{\pi}{6}\right) = -1$

$3t - \dfrac{\pi}{6} = \pi$

$3t = \pi + \dfrac{\pi}{6} = \dfrac{7\pi}{6}$

$\therefore t = \dfrac{7\pi}{18}$ seconds

Question 14.

(a)

$\dfrac{h}{AB} = \tan 45$

$\therefore AB = h$

$\dfrac{h}{AC} = tan\theta$

$\therefore AC = h\cot\theta$

$y^2 = AB^2 + AC^2$ (Pythag.)

$= h^2 + h^2\cot^2\theta$

$= h^2(1 + \cot^2\theta)$

$= h^2\cos ec^2\theta$

$\therefore y = h\cos ec\theta$

$y = \dfrac{h}{\sin\theta}$

$\therefore h = y\sin\theta$

(b) (i)

$f(x) = e^{2x} + e^x$

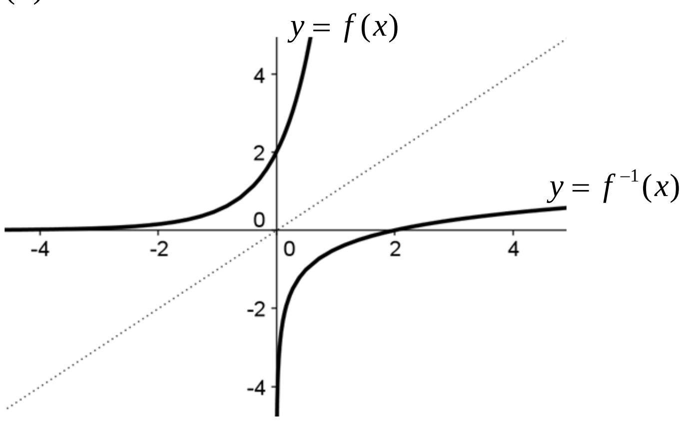
$f'(x) = 2e^{2x} + e^x$

$f''(x) = 4e^{2x} + e^x$

Always increasing as $f'(x)$ always positive

$\therefore$ inverse for all x

(ii)



(iii)

let $y = e^{2x} + e^{x}$

for inverse,

$x = e^{2y} + e^{y}$

$e^{2y} + e^{y} + \frac{1}{4} = x + \frac{1}{4}$

$\left(e^{y} + \frac{1}{2}\right)^{2} = \frac{4x+1}{4}$

$e^{y} + \frac{1}{2} = \pm\frac{\sqrt{4x+1}}{2}$

$e^{y} = \frac{-1 \pm \sqrt{4x+1}}{2}$

$y = \ln\left(\frac{-1 \pm \sqrt{4x+1}}{2}\right)$

but $-1 \pm \sqrt{4x+1}$ must be positive

$\therefore\ f^{-1}(x) = \ln\left(\frac{\sqrt{4x+1}-1}{2}\right)$

(iv)

$e^{2x} + e^{x} = 6$

$f(x) = 6$

$\therefore x = f^{-1}(6)$

$= \ln\left(\frac{\sqrt{4\times 6+1}-1}{2}\right)$

$= \ln 2$

(c) (i)

$y = \frac{8}{\pi}\sin^{-1}\left(\frac{x}{4}\right)$

$\sin^{-1}\left(\frac{x}{4}\right) = \frac{\pi y}{8}$

$\frac{x}{4} = \sin\left(\frac{\pi y}{8}\right)$

$x = 4\sin\left(\frac{\pi y}{8}\right)$

$V = \pi\int_{0}^{h} x^{2}dy$

$= \pi\int_{0}^{h} \left(4\sin\left(\frac{\pi y}{8}\right)\right)^{2} dy$

$= 16\pi\int_{0}^{h} \sin^{2}\left(\frac{\pi y}{8}\right)dy$

$= 16\pi\int_{0}^{h} \frac{1}{2}(1-\cos\left(\frac{\pi y}{4}\right))dy$

$= 8\pi\left[y - \frac{4}{\pi}\sin\left(\frac{\pi y}{4}\right)\right]_{0}^{h}$

$= 8\pi\left[h - \frac{4}{\pi}\sin\left(\frac{\pi h}{4}\right)\right]$

$= 8\pi h - 32\sin\left(\frac{\pi h}{4}\right)$

(ii)

$\frac{dV}{dt} = \pi$

$V = 8\pi h - 32\sin\left(\frac{\pi h}{4}\right)$

$\frac{dV}{dh} = 8\pi - 8\pi\cos\left(\frac{\pi h}{4}\right)$

$\frac{dV}{dt} = \frac{dV}{dh}\times\frac{dh}{dt}$

$\pi = \left(8\pi - 32\cos\left(\frac{\pi h}{4}\right)\right)\times\frac{dh}{dt}$

$\pi = \left(8\pi - 32\cos\left(\frac{\pi\times 2}{4}\right)\right)\times\frac{dh}{dt}$

$\pi = 8\pi\times\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1}{8}$ cm / minute