Student's name **Student's number** **Teacher's name**

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PRESBYTERIAN LADIES' COLLEGE SYDNEY 1888

2014

TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using blue or black pen Black is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 100

Section I: Pages 3-6
10 marks

- Attempt questions 1-10, using the answer sheet on page 17.
- Allow about 15 minutes for this section

Section II: Pages 7-14
90 marks

- Attempt questions 11-16, using the lined paper provided.
- Allow about 2 hours 45 minutes for this section

Multiple Choice	11	12	13	14	15	16	Total
							%

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Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

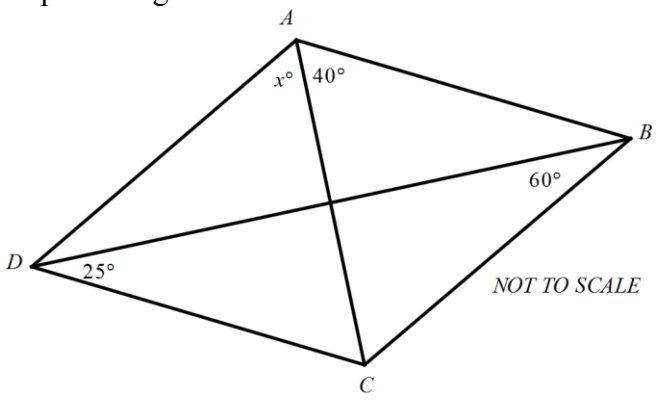
1. What is the equation of the straight line through $(4,5)$ and parallel to $2x-y+1=0$?

(A) $2x-y-13=0$
(B) $2x-y-3=0$
(C) $x+2y-10=0$
(D) $x+2y-7=0$

2. What are the nature of the roots of the quadratic equation $x^2-5x-6=0$?

(A) Real, rational and unequal
(B) Real, irrational and unequal
(C) Unreal, rational and unequal
(D) Unreal, irrational and unequal

3. *ABCD* is a parallelogram.



What is the value of *x*?

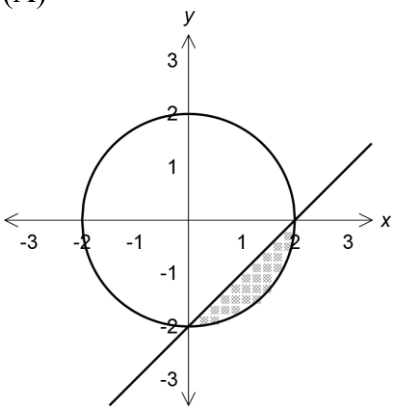
(A) 40
(B) 45
(C) 55
(D) 60

4. What is the equation of the directrix of the parabola $y^2 = -16(x-2)$?

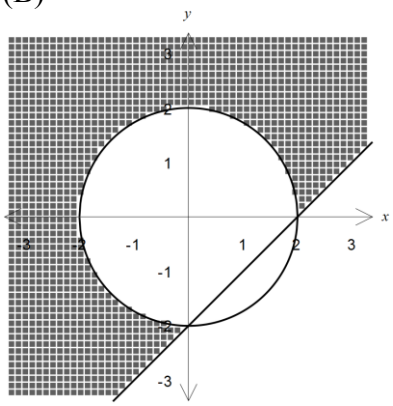
(A) $x = -2$
(B) $x = 6$
(C) $y = -2$
(D) $y = 6$

5. Which of the following diagrams show where $x^2 + y^2 \geq 4$ and $y \leq x - 2$ hold simultaneously?

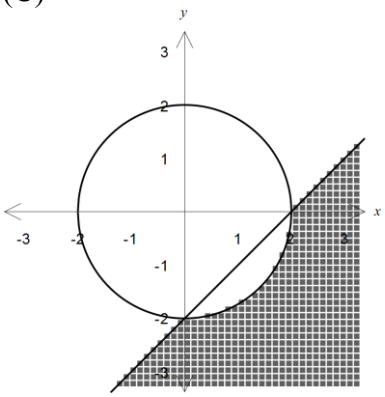
(A)



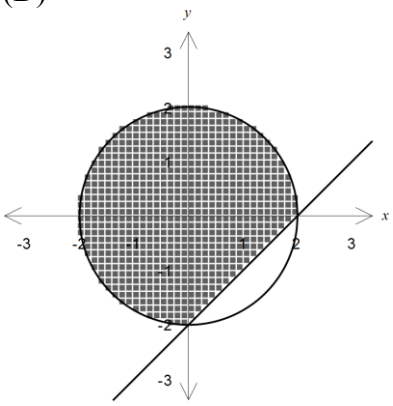
(B)



(C)



(D)



6. If $log_a 3 = -1.585$ and $log_a 5 = -2.322$, what is the value of $log_a\left(\frac{27a}{5}\right)$?

(A) -10.943
(B) -1.433
(C) $2.048a$
(D) $6.143a$

7. What is the equation of the curve that passes through $(4, 5)$ if the gradient function is $\sqrt{2x+1}$?

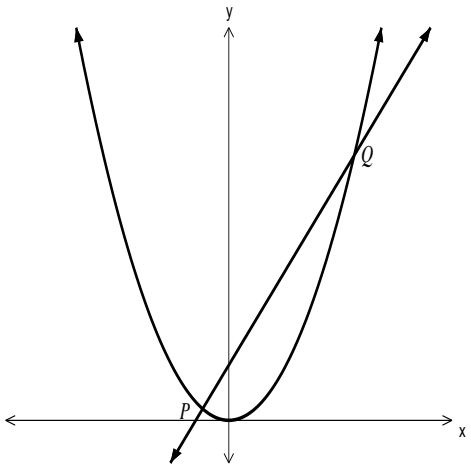
(A) $y = \frac{1}{3}(2x+1)^{\frac{3}{2}} - 4$

(B) $x - 3y + 11 = 0$

(C) $3x - y - 7 = 0$

(D) $y = \frac{2}{3}(2x+1)^{\frac{3}{2}} - 13$

8. The x values of P and Q are the solutions to which quadratic equation?



(A) $x^2 - 3x - 3 = 0$

(B) $x^2 - 3x + 3 = 0$

(C) $x^2 + 3x - 3 = 0$

(D) $x^2 + 3x + 3 = 0$

9. If $\sin x = -\frac{1}{5}$ and $\pi \le x \le \frac{3\pi}{2}$, then $\cot x$ equals

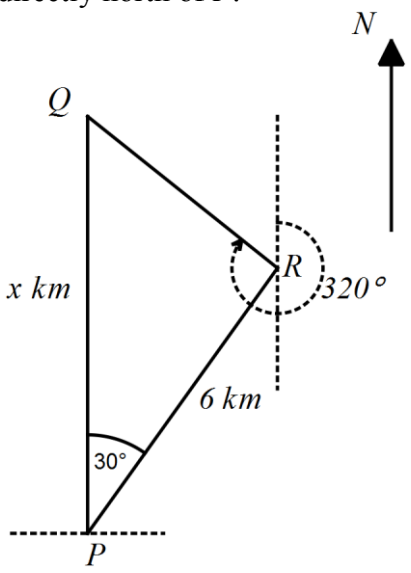
(A) $-\frac{1}{2\sqrt{6}}$

(B) $-2\sqrt{6}$

(C) $\frac{1}{2\sqrt{6}}$

(D) $2\sqrt{6}$

10. A ship leaves a port, P, and sails 6 km on a bearing of 030° to position R. It then heads on a bearing of 320° until it reaches a port, Q, which is directly north of P.



Which of the following will give the value for x?

(A) $\dfrac{x}{\sin 30^\circ} = \dfrac{6}{\sin 110^\circ}$

(B) $\dfrac{x}{\sin 40^\circ} = \dfrac{6}{\sin 110^\circ}$

(C) $\dfrac{x}{\sin 110^\circ} = \dfrac{6}{\sin 30^\circ}$

(D) $\dfrac{x}{\sin 110^\circ} = \dfrac{6}{\sin 40^\circ}$

Section II

90 marks

Attempt Questions 11–16

Allow about 2 hours 45 minutes for this section

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Solve for x: **2**

$$|3-2x| = 2x$$

b) Evaluate a and b if $\dfrac{2-\sqrt{5}}{2+\sqrt{5}} = a+\sqrt{b}$. **2**

c) Find $\lim\limits_{x\to 0}\left[\dfrac{x^2-9x}{5x}\right]$ **1**

d) Find the domain of $y=\sqrt{3-2x-x^2}$. **2**

e) Differentiate the following with respect to x.

(i) $\dfrac{e^{x^3}}{5x}$ **2**

(ii) $log_e\left(x^2+1\right)^{\frac{1}{2}}$ **2**

f) Find $\int \dfrac{3-x^2}{x}\,dx$ **2**

g) Show that $\int_0^{log_e 2} \dfrac{2e^{2x}}{e^{2x}+1}\,dx = log_e\left(\dfrac{5}{2}\right)$. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Find the shortest distance between the lines $3x-5y=8$ and $3x-5y=-1$. **2**

b) If $f(x)=x^3-x+4$, find the value of $f(f(-1))$. **2**

c) (i) Use Simpson's Rule with 3 function values to find an approximation to the area under the curve $y=\frac{1}{x}$ between $x=a$ and $x=3a$ where a is positive. **2**

(ii) Hence show that $log_e 3 \div \frac{10}{9}$. **1**

d) Sketch a continuous smooth curve for $x \geq 0$, where: **2**

$f(0)=1$,

$f'(x)<0$ and $f''(x)>0$ for $0<x<2$,

$f'(2)=0$,

$f(2)=-2$

$f'(x)>0$ and $f''(x)>0$ for $x>2$.

e) Prove $sec\theta - tan\theta - \frac{1}{sec\theta - tan\theta} = -2tan\theta$. **3**

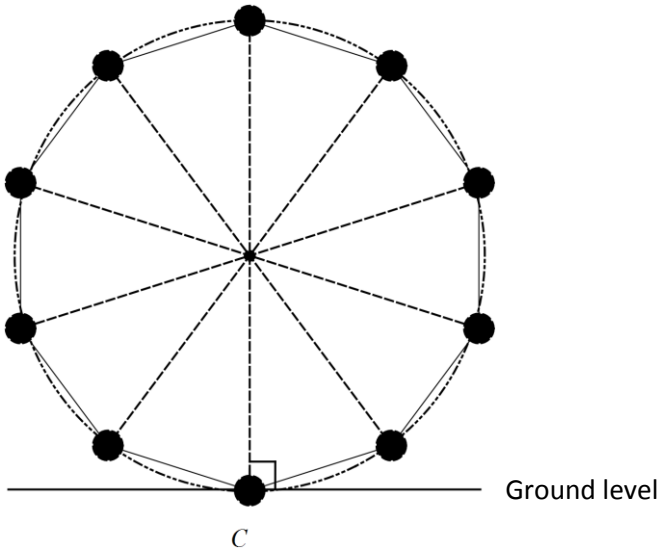
f) Find the equation of the locus of the point $P(x,y)$ such that the distance from P to the point $A(2, 3)$ is twice the distance from P to the point $B(-1, 4)$. Write your answer in simplest form. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) Find the value(s) of p such that $x^2+(p-1)x-(2p+1)>0$ for all values of x. **2**

b) A Ferris Wheel has a radius of 40 metres and 10 cages. A particular cage, C, starts at ground level and travels on a circular path.



(i) If the Ferris Wheel suddenly stops after cage C has moved 100 metres to a point D, through what angle has the wheel rotated? **1**

(ii) What is the area of the sector COD where O is the centre of the Ferris Wheel, C is the starting point of cage C and D is the point where cage C stopped? **1**

(iii) How far is cage C when it stops, in a straight line, from its starting point at ground level? Write your answer correct to 1 decimal place. **2**

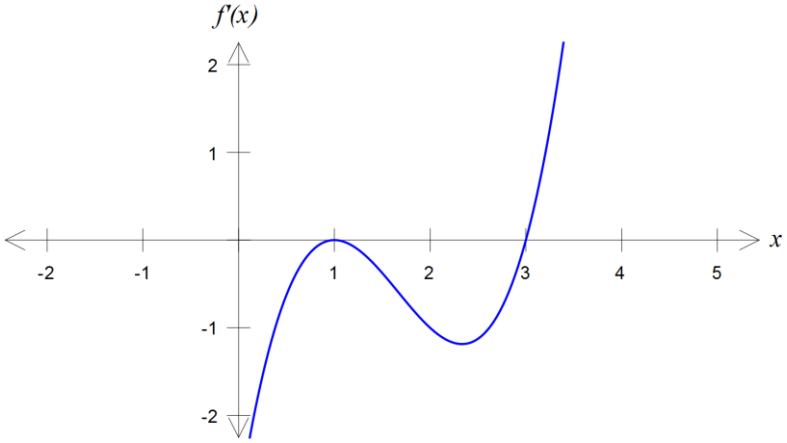
(iv) What is its height, to the nearest metre, above the ground now? **2**

c) The region between $y=\sec x$ and the x-axis, bounded by $x=\frac{\pi}{6}$ and $x=\frac{\pi}{3}$ is rotated about the x-axis. Find the exact volume of the solid of revolution formed. **3**

Question 13 continues on page 10

d) If the roots of the equation $x^2 + ax + k = 0$ differ by $3a$, show that $k = -2a^2$ **2**

e) The graph of $y = f'(x)$ is shown below. **2**



In your answer booklet, draw $y = f(x)$, clearly showing any stationary points.

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) A particle moving in a straight line with its velocity, v *m/s* at time t *seconds* is given by $v = 3cos\left(2t - \frac{\pi}{2}\right)$.

(i) Show that the particle is initially at rest? **1**

(ii) What is the maximum speed of the particle? **1**

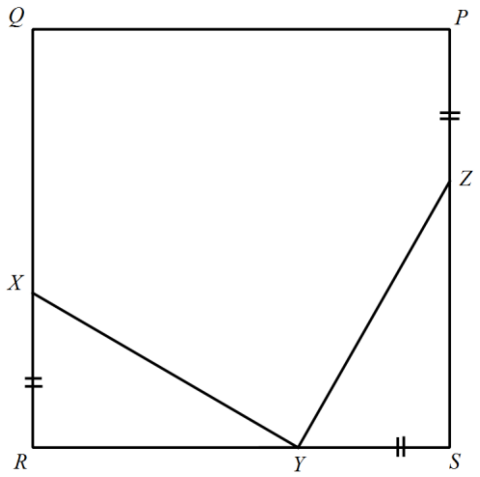
(iii) Find the first time the velocity of the particle reaches $\frac{3\sqrt{3}}{2} m/s$. **2**

(iv) Find the acceleration of the particle at time t. **1**

(v) Sketch the acceleration of the particle as a function of time for $0 \le t \le \pi$. **2**

(vi) If the particle is initially at the origin, find its distance travelled in the first π seconds. **3**

b) In the diagram PQRS is a square.



(i) Prove that $\Delta XYR \equiv \Delta YZS$. **3**

(ii) Prove that $\angle XYZ = 90^o$. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) The population of feral pigs is growing at a rate proportional to the current population. The population of pigs, P, at time t years is given by, $P = P_0 e^{kt}$, where P_0 and k are constants. In 2010, the feral pig population was first recorded. The population in 2012 was 350 and in 2014 it was 410.

(i) Find the exact value of P_0 and k. **2**

(ii) What is the expected population of the pigs in 2020? **1**

(iii) In what year will the feral pig population reach 3000? **2**

b) ABC is an isosceles triangle in which $a = b = 1cm. \angle C$ is obtuse. The perpendicular from B to AC produced, meets AC in D so that $BD = \frac{1}{2} AD$. Let $\angle BCD = \theta$. Show that $\sin\theta = \frac{1+\cos\theta}{2}$ **2**

c) For $y = 2x^2 e^x$

(i) Find the x and y intercepts, if any. **1**

(ii) What happens to the function as $x \to -\infty$? **1**

(iii) Show that stationary points exist at $(0, 0)$ and $\left(-2, \frac{8}{e^2}\right)$ **2**

(iv) Determine the nature of the stationary points. **2**

(v) It is known that 2 points of inflexion exist on this curve at $x = -2 \pm \sqrt{2}$. Sketch the curve. **2**

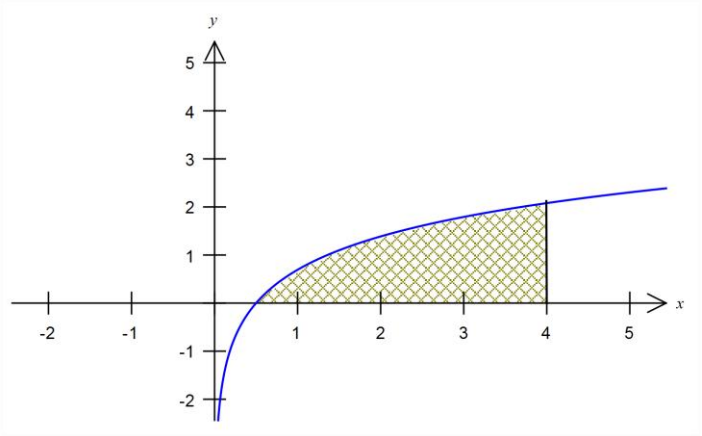
End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

a) (i) Show that the equation of the normal to the parabola $x^2 = 16y$ at the point where $x = 4$ is $2x + y - 9 = 0$. **2**

(ii) A line parallel to this normal is a tangent to the parabola. Find its equation and the co-ordinates of the point of contact. **3**

b) Find the area under the curve $y = log_e 2x$, bounded by $x = 4$ and the x-axis. **3**

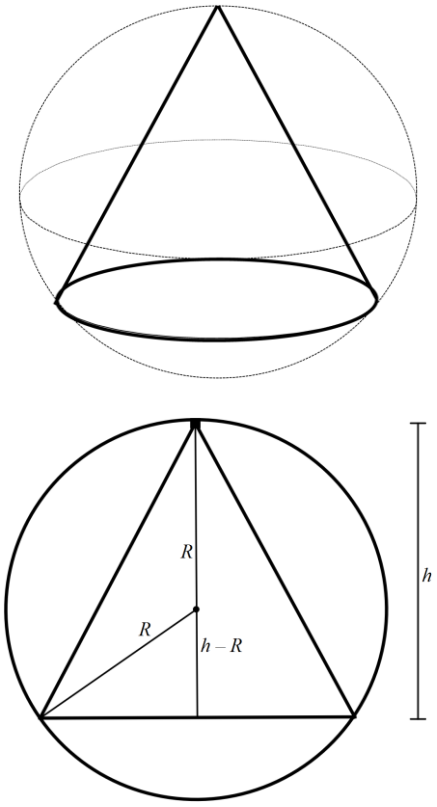


c) A tap is slowly turned on such that the volume flow rate of water, R, varies with time according to the relation $R = kt$, where k is a constant and $t > 0$. **2**
Calculate the total volume of water that flows from the tap in the first 10 seconds if $k = 1.3 m^3 / s^2$.

Question 16 continues on page 14

Question 16 continued

d) A right circular cone is inscribed in a sphere of radius R.



(i) Show that the volume of the cone can be found by **2**

$$V = \frac{1}{3}\pi\left(2Rh - h^2\right)h$$

(ii) Calculate the volume of the largest right circular cone inscribed in a sphere of radius R. Write your answer in terms of R. **3**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \ x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \qquad x > 0$

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Mathematics: Multiple Choice Answer Sheet

Student Number_____

Completely fill the response oval representing the most correct answer.

1.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
2.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
3.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
4.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
5.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
6.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
7.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
8.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
9.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭
10.	**A** ⬭	**B** ⬭	**C** ⬭	**D** ⬭

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