REDDAM HOUSE

2024

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • You may use the *z*-score table for all sections of this paper • In Questions in Section II, show relevant mathematical reasoning and/or calculations • Write your **Student Number** on each writing booklet provided
Total Marks: 70	**Section I – 10 marks** (pages 1 – 4) • Attempt Questions 1 – 10 • Allow about 15 minutes for this section **Section II – 60 marks** (pages 5 – 11) • Attempt Questions 11 – 14 • Allow about 1 hour and 45 minutes for this section

Section I Questions 1 – 10 (1 mark for each question)

Read each question and choose an answer *A*, *B*, *C* or *D*. Write your answer on the Answer Sheet provided. Allow about 15 minutes for this section.

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1. Which of the following is the simplest expression for $\tan\theta + \cot\theta$?

A) $\frac{1}{2}\operatorname{cosec} 2\theta$

B) $2\operatorname{cosec} 2\theta$

C) $2\sec 2\theta$

D) $\frac{1}{2}\sec 2\theta$

2. What is the domain of the function $f(x) = -2\arccos\left(1+\frac{x}{2}\right)$?

A) $[-4, 0]$

B) $[-4, 4]$

C) $[-2, 2]$

D) $[-1, 1]$

3. Which of the following parametric equations represents a parabola with a minimum at $(0, 2)$?

A) $x = t - \frac{1}{t}, \quad y = 2t^2 + \frac{2}{t^2}$

B) $x = 2t + \frac{2}{t}, \quad y = t^2 + \frac{1}{t^2}$

C) $x = 2t - \frac{2}{t}, \quad y = t^2 + \frac{1}{t^2}$

D) $x = 2t - \frac{2}{t}, \quad y = t^2 - \frac{1}{t^2}$

4. What is the smallest number of digits you need to randomly select from $A = \{0,1,2,3,4,5,6,7,8,9\}$ to guarantee a sum of 9?

A) 9

B) 6

C) 10

D) 5

5. When the polynomial $P(x)$ is divided by $(x+1)$ it gives a remainder of n^2, where n is a positive integer. When $P(x)$ is divided by $(x+1)(x+3)$, it gives a remainder $nx+6$. What is the value of n?

A) 2

B) 3

C) 6

D) 1

6. Consider the vector $\underset{\sim}{a} = \sin 3\lambda \underset{\sim}{i} - \cos 3\lambda \underset{\sim}{j}$ where λ is a constant such that $0 \leq \lambda \leq \frac{\pi}{4}$

The length of the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b} = \cos\lambda \underset{\sim}{i} + \sin\lambda \underset{\sim}{j}$ is $\frac{\sqrt{3}}{2}$

Which of the following is $\underset{\sim}{a} = \sin 3\lambda \underset{\sim}{i} - \cos 3\lambda \underset{\sim}{j}$?

A) $\underset{\sim}{a} = 0\underset{\sim}{i} - \underset{\sim}{j}$

B) $\underset{\sim}{a} = \frac{\sqrt{3}}{2}\underset{\sim}{i} - \underset{\sim}{j}$

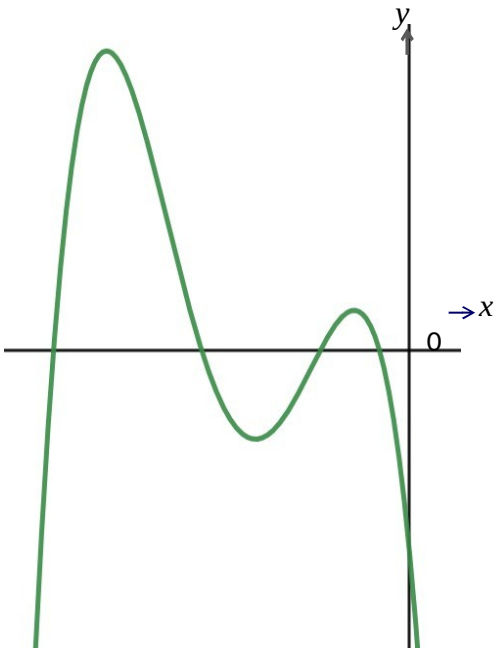
C) $\underset{\sim}{a} = \underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$

D) $\underset{\sim}{a} = \underset{\sim}{i} + 0\underset{\sim}{j}$

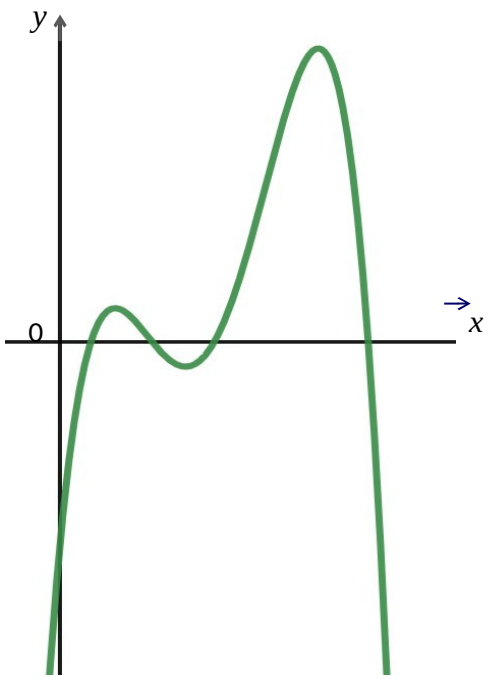
7. Consider the polynomial $f(x) = ax^4 + bx^3 + cx + d$, where the constant a is negative and the constant b is positive.

Which graph could represent $f(x)$?

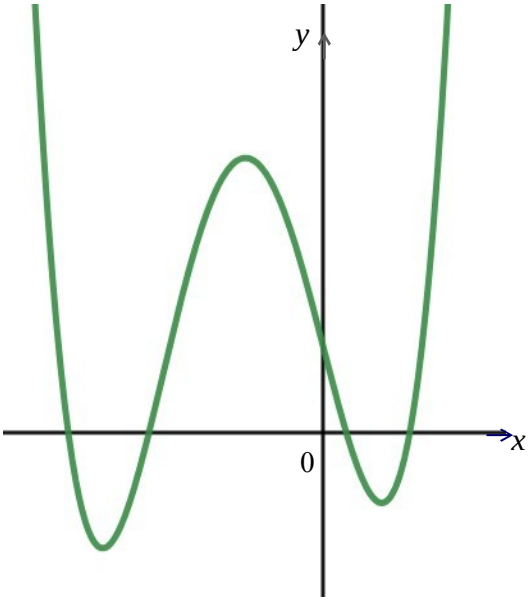
A)



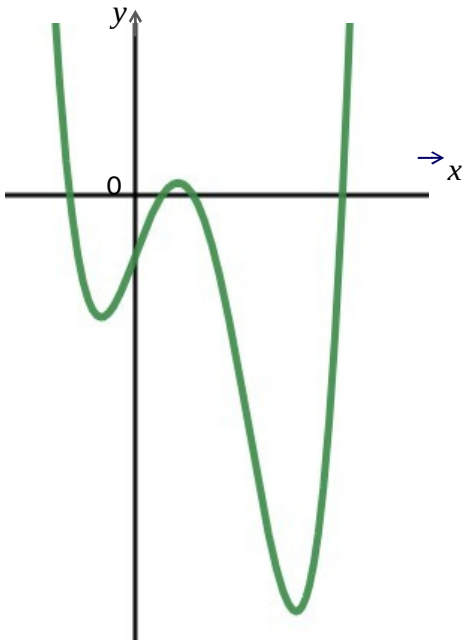
B)



C)



D)



8. The curve with equation $y = \cos x$ undergoes two transformations, after which its equation is: $y = 12\cos x - 5\sin x$. The two possible transformations are:

A) A vertical dilation of a factor 13, then a translation of $\sin^{-1}\left(\frac{5}{13}\right)$ units to the left.

B) A horizontal dilation of a factor 12, then a translation of $\sin^{-1}\left(\frac{5}{12}\right)$ units to the left.

C) A horizontal dilation of a factor 13, then a translation of $\sin^{-1}\left(\frac{5}{13}\right)$ units to the right.

D) A vertical dilation of a factor 169, then a translation of $\sin^{-1}\left(\frac{5}{12}\right)$ units to the right.

9. Consider the one-to-one function $f(x) = -2xe^{x-1}$.
What is the number of solutions to the equation $f(x) = f^{-1}(x)$?

A) 0

B) 1

C) 2

D) 3

10. Jake randomly guesses the answer to 5 multiple choice questions. Each question has four answers of which only one is correct. Having glanced at his responses, a teacher tells Jake that he has at least one wrong answer. Given this information, what is the probability that Jake answered the remaining questions correctly

A) $\frac{1}{1024}$

B) $\frac{1}{341}$

C) $\frac{3}{1024}$

D) $\frac{5}{341}$

End of Section 1

MATHEMATICS EXTENSION 1

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section.

INSTRUCTIONS

- Answer the questions in the appropriate writing booklet.
- Extra writing booklets are available.
- Your responses should include relevant mathematical reasoning and/or calculations.

Section II

Questions 11 – 14 (60 marks)

Allow about 1 hour 45 minutes for this section.

Question 11 (15 marks)

a) Find the coefficient of x^2 in the expansion of $\left(2x+\frac{1}{x}\right)^{10}$ **3**

b) For what values of p are the vectors $\underset{\sim}{a}=\begin{pmatrix}3p+1\\3\end{pmatrix}$ and $\underset{\sim}{b}=\begin{pmatrix}2p\\9\end{pmatrix}$ parallel? **2**

c) Solve the inequality $\frac{1}{x+2}\leq -2$ **3**

d) The probability of tossing a head using a biased coin is 70%.

Layla is to toss this coin 120 times.

Let X be the random variable representing the number of heads she will toss.

This random variable will have a binomial distribution.

i) Find the expected value $E(X)$. **1**

ii) By finding the variance, $\text{Var}(X)$, show that the standard deviation of X is approximately 5. **1**

iii) Use a normal approximation, to find the approximate probability that X is between 74 and 96. **2**

You may use the attached table of values.

e) The area of a regular hexagon is $A=\frac{\sqrt{3}}{24}P^2$ where P is its perimeter. **3**

The area of the hexagon is increasing at the rate of $4\sqrt{3}\ cm^2/s$

At what rate is its perimeter increasing when its perimeter is 40 cm?

Question 12 (15 marks)

a) In how many ways can the letters of the word "MATHEMATICS" be arranged so that the M's are separate? **2**

b) Find $I = \int_0^{\frac{\pi}{6}} \sin 2x \cos x \, dx$ **3**

c) In a fun park there are 6 sliding rides and 8 jumping castles.

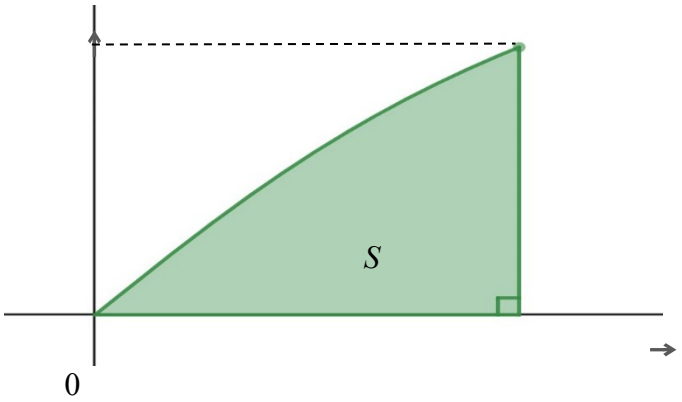
On average, each sliding ride is in use 80% of the time and each jumping castle is in use 70% of the time.

Find the probability that at a certain time

i) exactly 5 of the 6 sliding rides are in use. **1**

ii) exactly 5 of the 6 sliding rides are in use and 7 of the 8 jumping castles are also in use. **2**

d) In the diagram the graphs of the curve $y = \tan^{-1} x$ and the line $y = \frac{\pi}{4}$ intersect at the point $A: \left(1, \frac{\pi}{4}\right)$. The shaded region S is bounded by the curve, the line $x = 1$ and the x-axis as shown.



Find the volume of the solid of revolution when the region S is rotated about the y-axis. **4**

e) Use mathematical induction to prove that, for any integer $n \geq 1$, **3**

$$4 + 7 + 10 + \cdots + (3n + 1) = \frac{n}{2}(3n + 5)$$

Question 13 (15 marks)

a) A recent census found that 40 % of the people living on an island are sailors. **3**

A sample of n randomly selected people living on an island was surveyed.

Let $\hat{p}$ be the sample proportion of surveyed people who are sailors.

A normal distribution has been used and an approximate value of $P(\hat{p} \leq 0.42)$ is calculated to be 0.9251.

Use the standard normal distribution and relevant table to find an approximate value for n which is the number of people surveyed in the sample.

Give your answer correct to the nearest whole number.

b) A model for the population, P, of deer in a forest is $P = \frac{27000}{3 - 27^{-\frac{t}{48}}}$, where t is the time in years from today.

i) When will the population of deer drop to 75% of today's population? **2**

ii) According to the model, what will the eventual population of deer be? **1**

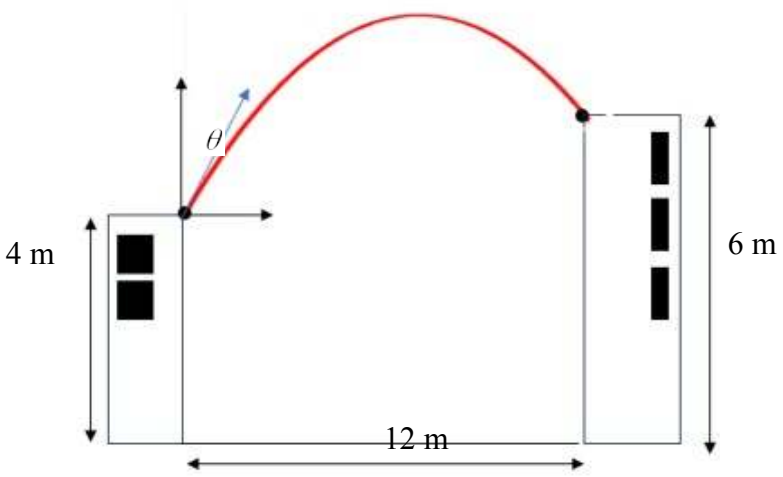
iii) Sketch the graph of the population P. **2**

iv) Show that P satisfies the differential equation $\frac{dP}{dt} = \frac{P \ln 3}{16}\left(1 - \frac{P}{9000}\right)$ **3**

Question 13 continues on page 9

c) A projectile is to be launched from the top of a building of height 4 metres so that it lands at the corner of a building standing opposite, 12 metres away, as shown in the diagram below.

The projectile is launched with an initial velocity of 42 metres/ second at an angle of θ.



Taking the acceleration due to gravity as 10 m/s^2, the position of the particle from the origin at time t, is given by the parametric equation:

$r(t) = \begin{pmatrix} (v\cos\theta)t \\ (v\sin\theta)t - 5t^2 \end{pmatrix}$. Do not prove it.

i) Show that the Cartesian equation of motion is given by $y = x\tan\theta - \frac{5x^2}{v^2}(1+\tan^2\theta)$ **2**

ii) Hence, or otherwise, find the two possible values of θ, the angle of launch.

Present your answer correct to the nearest degree. **2**

End of Question 13

Question 14 (15 marks)

a) The resultant force on a particle, which is moving subject to the forces F_1 and F_2, has magnitude 2 *M* Newtons and its direction is on a bearing of 290°.

The force F_1 has magnitude 90 Newtons and its direction is on a bearing of 320°

The F_2 has magnitude *M* Newtons.

Calculate the magnitude and direction of the force F_2. **3**

b) A particle *P* is moving along the *x*-axis. Initially, it is at the origin.

Its velocity at time *t*, is given by: $v = \dfrac{e^t}{1+e^{2t}}$ where *t* is measured in seconds and *x* in cm.

i) Use integration and the substitution $u = e^t$ to find an expression for the displacement, *x*. **2**

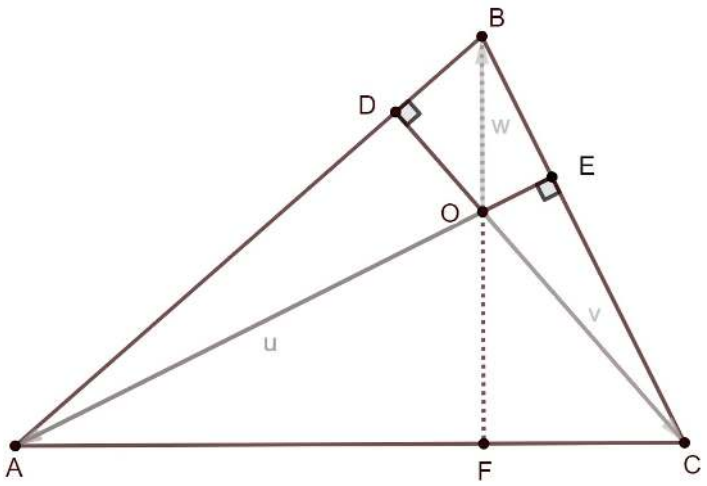
ii) Find the *exact* time when the displacement of the particle is $\arctan\left(\dfrac{1}{2}\right)$ cm. **2**

c) Given the roots of the quadratic equation $8x^2 - 5x + k = 0$ are $\sin\theta$ and $\cos 2\theta$ for some angle θ, find the possible values of *k*. **3**

Question 14 continues on page 11

Question 14 continued

d) In the triangle ABC shown in the diagram, AE and CD are altitudes from A and C to BC and AB respectively. Let the point of intersection of these two altitudes be O.



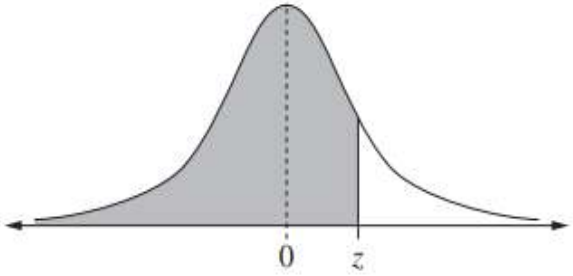
Label $\overrightarrow{OA} = \underset{\sim}{u}$; $\overrightarrow{OC} = \underset{\sim}{v}$; $\overrightarrow{OB} = \underset{\sim}{w}$

i) Show that: $\underset{\sim}{u} \cdot \underset{\sim}{w} - \underset{\sim}{u} \cdot \underset{\sim}{v} = 0$ **2**

ii) Hence, or using alternative vector methods, prove that the segment BF, passing through O, is perpendicular to AC **3**

END OF EXAMINATION

Table of values $P(Z \leq z)$ for the normal distribution $N(0,1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986