

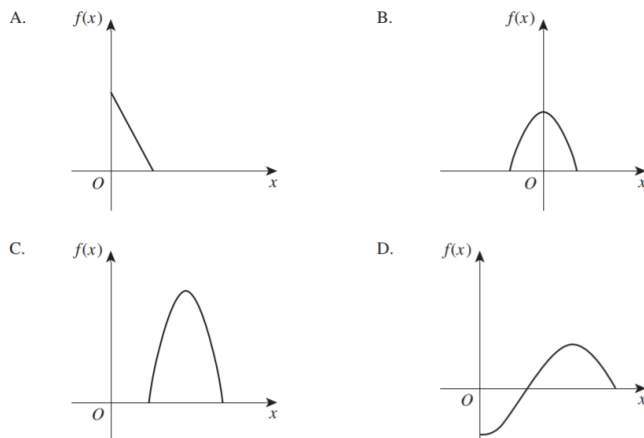


## Multiple Choice

Questions 1 – 10 are multiple choice.

Select the best response and circle your answer on the corresponding answer sheet provided.

1. Which of the following graphs does NOT represent a probability density function?



2. The table shows the future value of a \$1 annuity for varying interest rates over different time periods.

Time period	Interest rate				
	1%	2%	3%	4%	5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

What is the present value of an annuity, correct to the nearest dollar, that would provide a future value of \$52 200 after 4 years at 2% per annum, compounded half-yearly?

- A. \$6082.50                      B. \$6300.01  
 C. \$12292.50                    D. \$12664.50
3. The amount of water that Elaine uses to wash her car is normally distributed with a mean of 20 litres and a standard deviation of 4 litres. On what percentage of occasions would Elaine expect to use between 12 litres and 24 litres of water to wash her car?
- A. 34%                              B. 51%  
 C. 68%                              D. 81.5%

4. If  $P(A) = 0.4$ ,  $P(B) = 0.5$  and  $P(A|B) = 0.2$ , what is the value of  $P(A \cup B)$ ?

- A.** 0.6
- B.** 0.7
- C.** 0.8
- D.** 0.9

5. What is the domain and range of a circle with the equation  $x^2 + 6x + y^2 = 7$ ?

- A.** domain =  $[-3 - \sqrt{7}, -3 + \sqrt{7}]$ ; range =  $[-\sqrt{7}, \sqrt{7}]$
- B.** domain =  $[3 - \sqrt{7}, 3 + \sqrt{7}]$ ; range =  $[-\sqrt{7}, \sqrt{7}]$
- C.** domain =  $[-1, 7]$ ; range =  $[-4, 4]$
- D.** domain =  $[-7, 1]$ ; range =  $[-4, 4]$

6. What is the maximum value of  $y = 3 \sin\left(\frac{x}{2}\right) - 1$  and the value of  $x$  for which this occurs in the domain  $[0, 2\pi]$ ?

- A.** the maximum value of  $y = 3 \sin\left(\frac{x}{2}\right) - 1$  is 2 occurring at  $x = \frac{\pi}{2}$ .
- B.** the maximum value of  $y = 3 \sin\left(\frac{x}{2}\right) - 1$  is 3 occurring at  $x = \frac{\pi}{2}$ .
- C.** the maximum value of  $y = 3 \sin\left(\frac{x}{2}\right) - 1$  is 2 occurring at  $x = \pi$ .
- D.** the maximum value of  $y = 3 \sin\left(\frac{x}{2}\right) - 1$  is 3 occurring at  $x = \pi$ .

7. What is the value of  $a$  if  $\int_a^1 (4-x) dx = 8$ ?

- A.** -2
- B.** -1
- C.** 0
- D.**  $\frac{1}{2}$

8. What is the value of  $\int_0^{\ln 5} -e^{-x} dx$  ?

A.  $-\ln 5$

B.  $e^{-\ln 5}$

C.  $-\frac{4}{5}$

D.  $-e^{-\ln 5}$

9. The function  $y = f(x)$  has a stationary point at  $(2,1)$ .

Which one of the following is definitely a stationary point of  $y = -f\left(\frac{x}{3}\right) + 4$

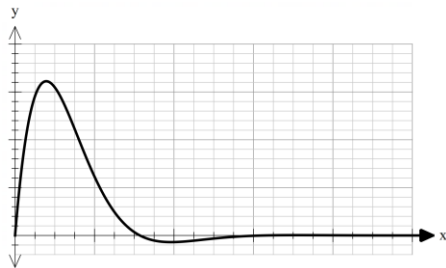
A.  $\left(\frac{2}{3}, -3\right)$

B.  $\left(\frac{2}{3}, 3\right)$

C.  $(6, 3)$

D.  $(6, -3)$

10. The graph of  $y = e^{-x} \sin x$  in the domain  $[0, \infty)$



The  $x$ -coordinate of the minimum turning point is:

A.  $\frac{\pi}{4}$

B.  $\frac{3\pi}{4}$

C.  $\frac{5\pi}{4}$

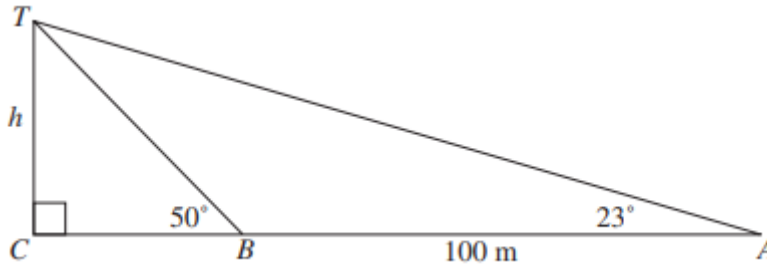
D.  $\frac{7\pi}{4}$

**END OF SECTION A**

**QUESTION 11 (4 marks)**

The elevation of a tower from point  $A$  is  $23^\circ$ .

From a point  $B$ , 100 metres closer to the tower and on the same horizontal level as  $A$  the elevation is  $50^\circ$ .



- (a) Show that  $\angle BTA = 27^\circ$ . (1)

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- (b) Show that  $BT = \frac{100 \sin 23^\circ}{\sin 27^\circ}$  (1)

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- (c) Find the height ( $h$ ) of the tower (2)  
 [Give your answer correct to one decimal place]

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**QUESTION 12 (3 marks)**

The table below shows the marks out of 40 for nine students on two tests.  
 The tests cover similar content, but on one test the students **are allowed** to use a calculator, while on the other test, calculators are **not allowed**.

<i>Student</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>
<i>Calculator allowed (x)</i>	8	10	14	14	16	22	28	32	36
<i>Non – calculator (y)</i>	12	14	6	4	28	16	24	22	18

- (a) Use the marks for students A to I to find the value of the **Pearson correlation coefficient** (2) between the marks  $x$  on the test with the calculator allowed and the marks  $y$  on the non-calculator test, and the **equation of the least squares regression line** of  $y$  on  $x$

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- (b) Student **J** scored 25 on the test with calculators allowed but missed the non-calculator test.

Use the least squares regression line of  $y$  on  $x$  to estimate the mark of student **J** (1) on the non-calculator test.

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**QUESTION 13 (8 marks)**

(a) Find the nature of the stationary points of the curve  $y = 2x^3 - 3x^2 - 12x + 2$  (3)

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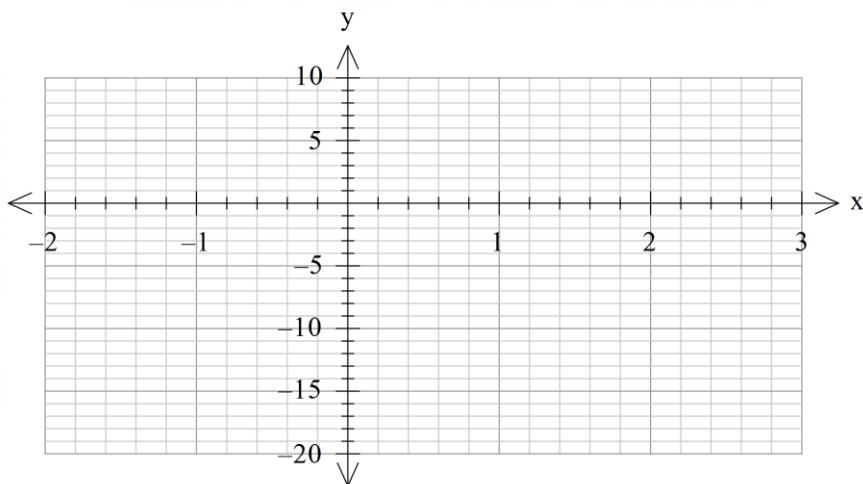
(b) Draw a neat sketch of  $y = 2x^3 - 3x^2 - 12x + 2$  showing the stationary points and the point of inflection for the domain  $-2 \leq x \leq 3$ . (3)

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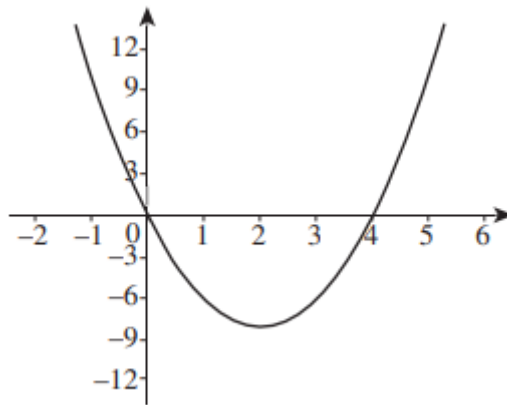
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- (c) Given the graph of  $\frac{dy}{dx}$  below, on the same graph, draw a sketch of a possibility for the graph of  $y$ . (2)



**QUESTION 14 (3 marks)**

- (a) Find the equation of the tangent to the curve  $y = \sqrt{25 - x^2}$  at the point (3, 4). (3)

(Write your answer in general form)

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**QUESTION 15 (4 marks)**

(a) Differentiate  $y = (\ln x)^{-1}$  with respect to  $x$ . (2)

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(b) Differentiate  $y = (x^2 + 4) \tan 3x$  with respect to  $x$ . (2)

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**QUESTION 16 (2 marks)**

Evaluate  $\int_0^{\frac{\pi}{2}} 1 - \cos x \, dx$ . (2)

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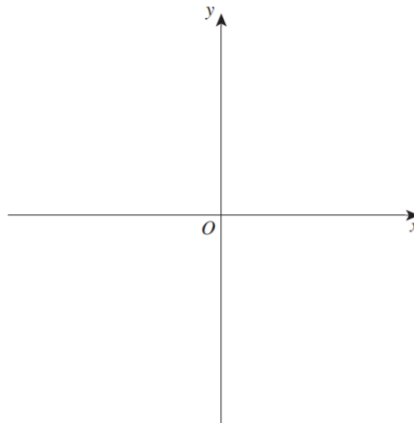
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**QUESTION 17 (2 marks)**

The function  $y = f(x)$  is continuous for all values of  $x$ . (2)  
The following is known of the function's properties.

- When  $x < -1$ ,  $f'(x) > 0$  and  $f''(x) < 0$ .
- When  $x > -1$ ,  $f'(x) > 0$  and  $f''(x) > 0$ .
- $f(-1) = f'(-1) = f''(-1) = 0$

On the axes below, sketch the graph of  $y = f(x)$ , labelling any  $x$ -intercepts.



**QUESTION 18 (3 marks)**

Solve the pair of simultaneous equations: (3)

$\log_2 x + \log_2 y = 3 \quad \dots(1)$

$\log_x y = 2 \quad \dots(2)$

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**QUESTION 19 (2 marks)**

An arithmetic sequence has first term 29 and a common difference of  $-4$ . (2)  
 Find the sum of the first  $n$  terms of this sequence, calculate the least value of  $n$  for which the sum is negative.

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**QUESTION 20 (3 marks)**

Shirley uses her car every day. Sometimes, she needs a few attempts to start her car. Let the discrete random variable  $X$  represent the number of attempts needed. The probability function for the random variable  $X$  is given by:

$$P(X = x) = \begin{cases} a & x = 1 \\ \frac{1}{2}P(X = x - 1) & x = 2, 3, 4, 5 \\ 0 & \text{for all other values of } x \end{cases}$$

(a) Show that  $a = \frac{16}{31}$  (2)

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(b) The probability distribution table for the discrete random variable  $X$  is shown. (1)

$x$	1	2	3	4	5
$P(X = x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

Shirley claims that after an entire year, she needs an average of two attempts to successfully start her car. Calculate the expected value (correct to 2 decimal places).

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**QUESTION 21 (5 marks)**

Complete the table of  $y = 2^x$  .

The table gives the values of  $y = 2^x$  for  $x$ .

$x$	0	2	4	6
$y$	1	4	16	

(a) Complete the table above. **(1)**

(b) Hence, use the trapezoidal rule to find an approximation for  $\int_0^6 2^x dx$ . **(2)**

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(c) Use integration to evaluate  $\int_0^6 2^x dx$  (write your answer correct to 2 decimal places) **(2)**

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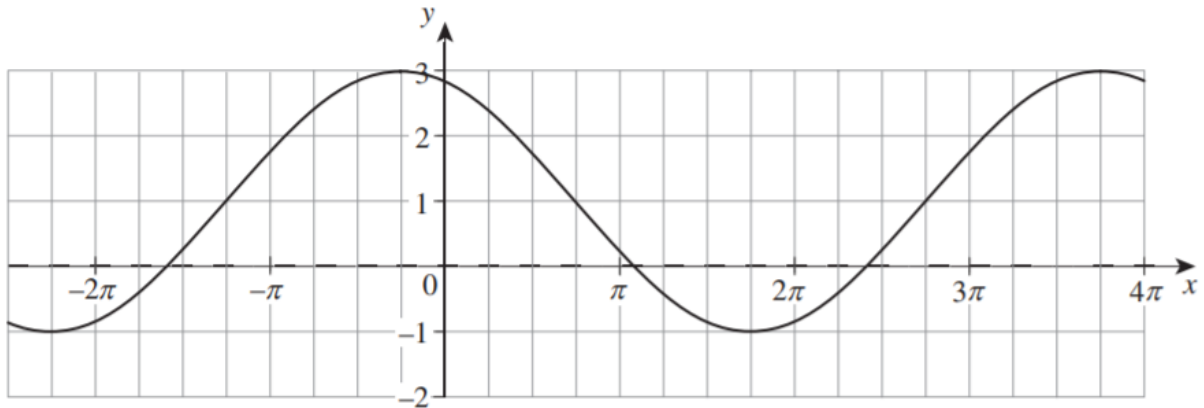
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**QUESTION 22 (3 marks)**

The graph is a function of the form  $y = 2 \cos(a(x+b)) + c$ .



Consider the equation of the function to determine the values of  $a$ ,  $b$  and  $c$  (3)

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**QUESTION 23 (3 marks)**

At time  $t$  hours after 12:00 am, the height  $h$  metres of the deck of a boat (3)

above the level of the jetty is given by  $h = 2 \cos\left(\frac{4\pi}{25}t\right) + 1$ .

Find correct to the nearest minute the first time after 12:00 am the boat **is level** with the jetty.

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**QUESTION 24 (4 marks)**

The students in a school were surveyed on the number of hours of sleep per week.  
The results were normally distributed.  
The survey indicated that 95% of students had between 42 and 54 hours of sleep per week.

- (a) Show that the mean number of hours of sleep per week is 48. **(1)**

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- (b) What was the standard deviation? **(1)**

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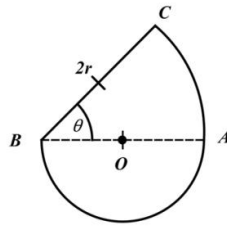
- (c) How many hours of sleep per week would a student have who recorded a  $z$ -score of  $-2.5$ ? **(1)**

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- (d) What percentage of students would have indicated they had between 51 and 57 hours of sleep per week? **(1)**

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**QUESTION 25 (5 marks)**



The diagram shows the cross-section of a cam (a part in a car engine), consisting of a semi-circle diameter AB, centre O and radius  $r$  units; and a sector ABC with centre B, radius  $2r$  units and angle  $\theta$ . The area of the cam ABC is  $\frac{1}{2}$  units<sup>2</sup>.

- (a) Show that  $\theta = \frac{1 - \pi r^2}{4r^2}$  (2)

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- (b) Find the value of  $r$  (correct to 1 decimal place) which gives the least perimeter of the cam. (3)

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**QUESTION 26 (3 marks)**

The continuous random variable, X, has the probability density function

$$f(x) = \frac{1}{2} \sin x \text{ for } 0 \leq x \leq \pi .$$

- (a) Find the cumulative distribution function. **(2)**

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- (b) Find the first quartile of the distribution. **(1)**

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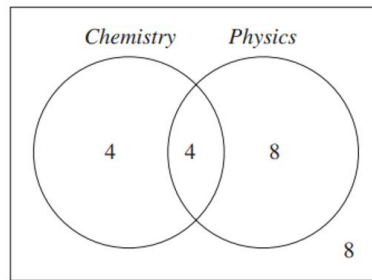
**QUESTION 27 (4 marks)**

A teacher surveyed his class of 24 students about the subjects that they have chosen to study in the following year.

The results indicate that:

- 8 students chose to study neither Physics nor Chemistry
- 8 students chose to study Chemistry
- 12 students chose to study Physics.

A Venn diagram showing this information has been drawn below.



- (a) A student choosing to study Chemistry is event 1. (2)  
A student choosing to study Physics is event 2.  
If a student is chosen at random,  
determine whether these events are independent of each other.

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**QUESTION 27 (continued)**

- (b) The entire year group is given the same survey. (2)

The results indicate that:

- the probability that a student chose to study Chemistry is  $\frac{1}{3}$
- the probability that a student chose to study Physics is  $\frac{2}{5}$
- given that a student chose to study Chemistry, the probability that they chose to study Physics is  $\frac{3}{7}$

Calculate the probability that a student picked at random has chosen to study Chemistry or Physics.

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**QUESTION 28 (6 marks)**

A particle is moving in a straight line with a velocity given by  $\frac{dx}{dt} = -6\sin(2t)$ ,  
where  $x$  is the displacement from the origin in metres and  $t$  is time measured in seconds.

The particle is initially at rest 4 m to the right of the origin.

- (a) Show that the displacement of the particle is given by  $x = 1 + 3\cos(2t)$  (2)

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- (b) Show that the particle comes to rest again when it is 2 m to the **left** of the origin (2)  
and  $t = \frac{\pi}{2}$  seconds.

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- (c) Find a time when the particle reaches its maximum speed. (2)  
What is this maximum speed?

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**QUESTION 29 (4 marks)**

(a) Show that  $\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{1 - \ln x}{x^2}$  (2)

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(b) Use the result in part (a), to evaluate  $\int_e^{e^2} \frac{1 - \ln x}{x \ln x} dx$  (leave your answer in exact form) (2)

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**QUESTION 30 (6 marks)**

A person borrows \$5000 at 1.5% per month compound interest and repays the loan (including interest) by equal monthly payments over 3 years.

(a) Show that the Amount owing after 3 months is:

$$A_3 = 5000(1.015)^3 - M(1 + 1.015 + 1.015^2) \quad \text{where } M \text{ is the monthly payment} \quad (1)$$

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(b) Find the value of each equal monthly payment M. (3)

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(c) How long would the loan have been repaid if the monthly payment was \$200 per month (answer to the nearest month). (2)

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**QUESTION 31 (4 marks)**

At time  $t$  years after it was purchased the value  $\$V$  of a car is given by  $V = 25000e^{-0.5t}$

- (a) Find the loss in value of the car during the third year. (2)

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- (b) Find the year in which the car is losing value at a rate of \$100 per year. (2)

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**QUESTION 32 (3 marks)**

If  $y = \tan^2 x$ , find the values of the constants  $a$  and  $b$  such that  $\frac{d^2y}{dx^2} = ay^2 + by + 2$  (3)

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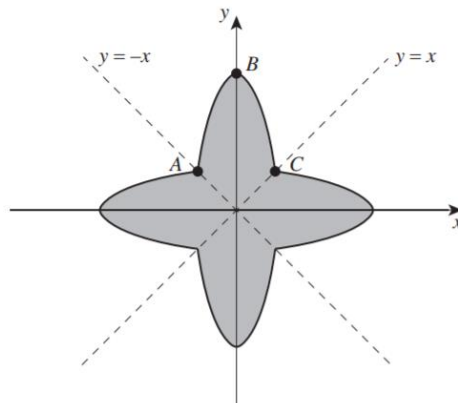
**QUESTION 33 (3 marks)**

A company's logo is designed using the curve ABC, as shown by the shaded region in the diagram.

**(3)**

The curve ABC shows part of the function  $y = 4 - 3x^2$ .

This curve is reflected along the  $x$ -axis and the lines  $y = x$  and  $y = -x$  to form the rest of the logo.



Calculate the area of the company's logo.

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**QUESTION 34 (3 marks)**

The discrete random variable  $X$  has the following probability distribution:

$x$	$-1$	$0$	$1$	$a$	$2a$
$P(X = x)$	$\frac{1}{10}$	$a$	$b$	$b$	$2b$

What is the **range** of possible values for the expected value,  $E(X)$ ? **(3)**

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**End of Paper**