
CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2014 Trial Examination

FORM VI

MATHEMATICS EXTENSION 1

Friday 8th August 2014

General Instructions

- Reading time — 5 minutes
- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.

Total — 70 Marks

- All questions may be attempted.

Section I – 10 Marks

- Questions 1–10 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 60 Marks

- Questions 11–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Eleven.
- Write your candidate number on this question paper and submit it with your answers.

Checklist

- SGS booklets — 4 per boy
- Multiple choice answer sheet
- Candidature — 120 boys

Examiner
DS

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

1

Which expression is equivalent to $\cos 2x$?

(A) $\sin^2 x - \cos^2 x$

(B) $2\sin^2 x - 1$

(C) $2\sin^2 x + 1$

(D) $2\cos^2 x - 1$

QUESTION TWO

1

A polynomial of degree four is divided by a polynomial of degree two.
What is the maximum possible degree of the remainder?

(A) 3

(B) 2

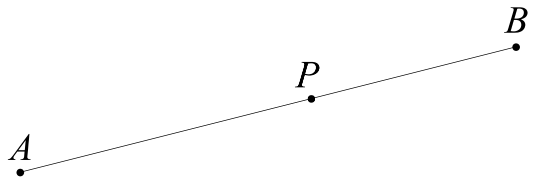
(C) 1

(D) 0

Exam continues next page ...

QUESTION THREE

1



In the diagram above the point P divides the interval AB in the ratio $3:2$. In what ratio does the point A divide the interval BP?

(A) $-5:3$

(B) $-5:2$

(C) $-3:5$

(D) $-2:5$

QUESTION FOUR

1

What is the exact value of $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$?

(A) $-\frac{2\pi}{3}$

(B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

QUESTION FIVE

1

Which function is a primitive of $\dfrac{1}{1+4x^2}$?

(A) $\frac{1}{2}\tan^{-1}\left(\frac{1}{2}x\right)$

(B) $\frac{1}{4}\tan^{-1}\left(\frac{1}{2}x\right)$

(C) $\frac{1}{2}\tan^{-1}(2x)$

(D) $\frac{1}{4}\tan^{-1}(2x)$

Exam continues overleaf ...

QUESTION SIX

1

Which expression is equal to ${}^{n}\mathrm{C}_2$?

(A) $\frac{n}{2}$

(B) $\frac{n^2-n}{2}$

(C) $\frac{n^2+n}{2}$

(D) n

QUESTION SEVEN

1

The velocity v of a particle moving in a straight line is governed by the equation $v = x - 2$, where x is its displacement. The particle started at $x = 5$. What is the displacement function of the particle?

(A) $x = 5e^t$

(B) $x = 2 + \frac{1}{3}e^t$

(C) $x = 2 + e^t$

(D) $x = 2 + 3e^t$

QUESTION EIGHT

1

A particle is moving in simple harmonic motion about the origin according to the equation $x = 2\cos nt$, where x metres is its displacement after t seconds. It passes through the origin with speed $\sqrt{2}$ m/s. What is the value of n?

(A) $\dfrac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

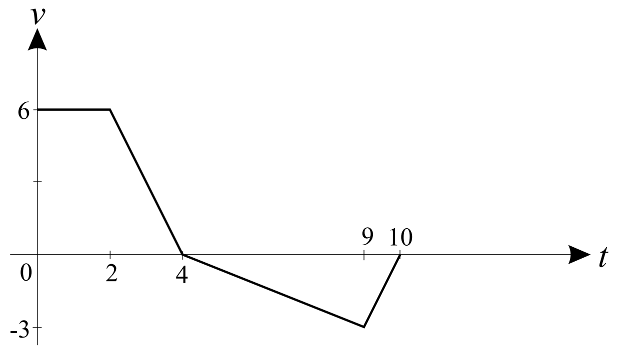
(C) $-\sqrt{2}$

(D) $\dfrac{\pi}{4}$

Exam continues next page ...

QUESTION NINE

1



The diagram above shows the velocity–time graph of an object that moves over a 10 second time interval. For what percentage of the time is the speed of the object decreasing?

(A) 30%

(B) 60%

(C) 70%

(D) It cannot be determined from the graph.

QUESTION TEN

1

How many solutions does the equation $2x + 3\pi \sin x = 0$ have in the domain $0 \leq x \leq 2\pi$?

(A) 1

(B) 2

(C) 3

(D) 4

End of Section I

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) Solve the inequation $\dfrac{2}{x} < 3$. **2**

(b) (i) Sketch the curve $y = \sin^{-1} x$. **1**

(ii) What is the gradient of the curve at $x = 0$? **1**

(c) Solve the equation $\sin 2x = \sin x$ for $-\pi \le x \le \pi$. **3**

(d) A curve is defined parametrically by the equations **2**

$$x = 1 - t$$
$$y = t^2.$$

Find the gradient of the tangent to the curve at the point where $t = -3$.

(e) By using the substitution $u = \sin x$, or otherwise, evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin^5 x \cos x \, dx$. **3**

(f) A spherical balloon, with volume given by the formula $V = \frac{4}{3}\pi r^3$, is being filled with air at the constant rate of $200\,\text{cm}^3/\text{s}$. At what rate is its radius r increasing at the instant when it is 7 cm? Give your answer correct to three significant figures. **3**

Exam continues next page ...

QUESTION TWELVE (15 marks) Use a separate writing booklet. **Marks**

(a) The cubic equation $x^3 + 5x^2 + cx + d = 0$ has three real roots -3, 7 and α.

(i) Use the sum of the roots to find α. **1**

(ii) Find the values of c and d. **2**

(b) Find the coefficient of x^3 in the expansion of $\left(3x^2 - \dfrac{2}{x}\right)^9$. **3**

(c) (i) Write the expression $\sqrt{2}\sin x - \sqrt{6}\cos x$ in the form $A\sin(x - \theta)$, where $A > 0$ and $0 < \theta < \frac{\pi}{2}$. **2**

(ii) Hence write down the maximum value of $\sqrt{2}\sin x - \sqrt{6}\cos x$, and find the smallest positive value of x for which this maximum value occurs. **2**

(d) Let $P(x) = x^3 + 3x - 7$.

(i) Show that the equation $P(x) = 0$ has a root between 1 and 2. **1**

(ii) Use two applications of Newton's method with initial approximation $x_1 = 1$ to approximate this root. Give your answer correct to two decimal places. **2**

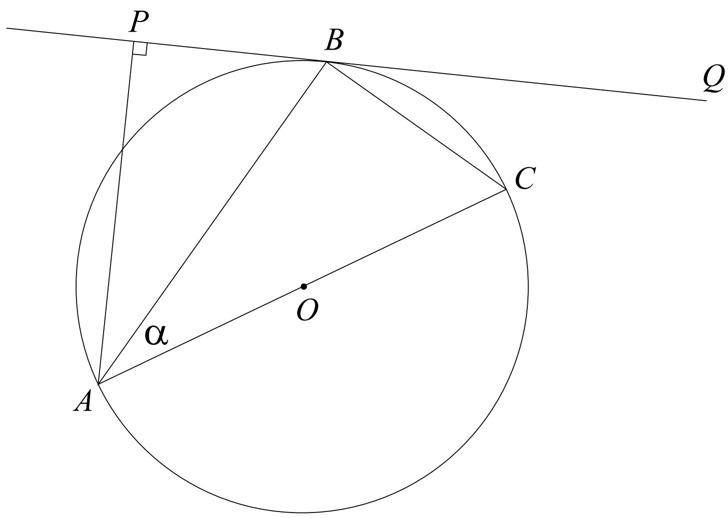
(e) Suppose that θ is the acute angle between the lines $y = kx$ and $(k + 1)y = kx$, where $k + 1 > 0$ and $k \neq 0$.

(i) Find an expression for $\tan\theta$ in simplest form. **1**

(ii) Explain why $\theta < 45°$. **1**

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

(a) **3**



The diagram above shows the points A, B and C lying on a circle, of which AC is a diameter. The line AP is perpendicular to the tangent at B.
Let $\angle BAC = \alpha$.

Prove that BA bisects $\angle PAC$.

(b) A particle is moving in simple harmonic motion. Its acceleration is defined by the equation $\ddot{x} = -9x$. Whenever the particle is 4 cm from the origin its speed is 6 cm/s. Find the amplitude of the motion. **2**

(c) Consider the quadratic polynomial $Q(x) = (x+h)^2 + k$, for some constants h and k. Find the values of h and k given that $x+2$ is a factor of $Q(x)$ and 16 is the remainder when $Q(x)$ is divided by x. **3**

(d) Prove by mathematical induction that for all positive integer values of n, **3**

$$1^2 \times 2 + 2^2 \times 3 + 3^2 \times 4 + \cdots + n^2\,(n+1) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$

QUESTION THIRTEEN (Continued)

(e) A jug of cold water at W° C, where $W > 0$, is taken out of a refrigerator. The air temperature in the room is $2W^\circ$ C. The rate at which the water warms is proportional to the difference between the temperature of the surrounding air and the temperature of the water. Thus $\dfrac{dT}{dt} = k(2W - T)$, where T° C is the temperature of the water after t minutes.

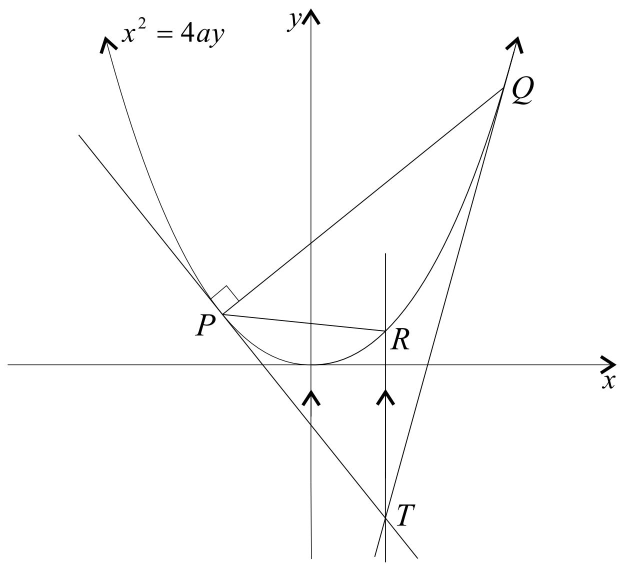
(i) Show that $T = 2W - We^{-kt}$ satisfies the differential equation. **1**

(ii) If the temperature of the water has increased by 50% after 20 minutes, find the value of k. **2**

(iii) Find the percentage increase in the temperature of the water 45 minutes after the water is taken out of the refrigerator. Give your answer correct to the nearest whole percent. **1**

QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

(a)



In the diagram above the normal at $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ meets the parabola again at $Q(2aq, aq^2)$. You may assume that the normal at P has equation $x + py = 2ap + ap^3$.

(i) Show that $p^2 + pq + 2 = 0$. **2**

(ii) Given that the tangents at P and Q intersect at the point $T\,(a(p+q), apq)$, and the line through T parallel to the axis of the parabola meets the parabola at $R(2ar, ar^2)$, prove that PR is a focal chord. (That is, prove that $pr = -1$.) **2**

QUESTION FOURTEEN (Continued)

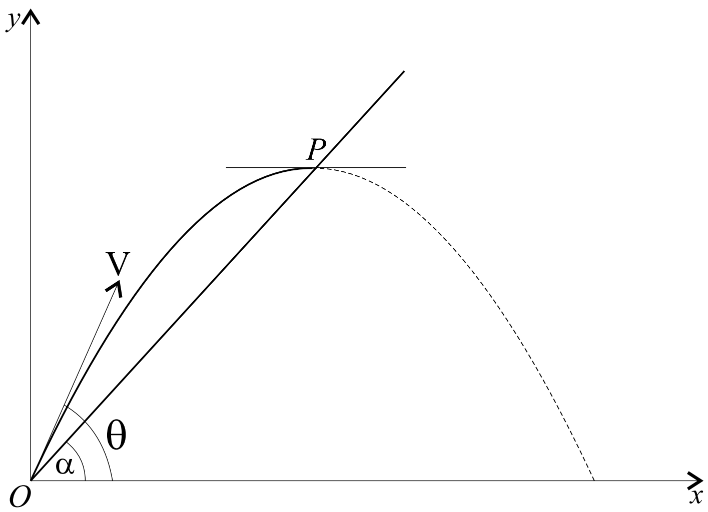
(b) (i) By considering the expansion of $(1+x)^n$, show that **1**

$$\binom{n}{1} + \binom{n}{2}x + \binom{n}{3}x^2 + \cdots + \binom{n}{n}x^{n-1} = \frac{(1+x)^n - 1}{x}.$$

(ii) By applying integration to the identity in part (i), with the substitution $u = 1+x$ on the right-hand-side, show that **3**

$$\binom{n}{1} - \tfrac{1}{2}\binom{n}{2} + \tfrac{1}{3}\binom{n}{3} - \cdots + \tfrac{(-1)^{n-1}}{n}\binom{n}{n} = 1 + \tfrac{1}{2} + \tfrac{1}{3} + \cdots + \tfrac{1}{n}.$$

(c)



In the diagram above the point O is the foot of a plane inclined at an angle α to the horizontal. A particle is projected with speed V from O at an angle of elevation θ to the horizontal, where $\theta > \alpha$. It strikes the inclined plane at P, which is the vertex of the parabolic path of the particle. You may assume that this parabolic path has parametric equations $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$.

(i) Show that $\tan\theta = 2\tan\alpha$. **3**

(ii) Show that the distance OP is given by $\dfrac{2V^2\sec\alpha\tan\alpha}{g(1+4\tan^2\alpha)}$. **4**

End of Section II

END OF EXAMINATION

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The following list of standard integrals may be used:

$$\int x^n \, dx = \frac{1}{n+1} \, x^{n+1}, \;\; n \neq -1; \;\; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \;\; x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} \, e^{ax}, \;\; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \;\; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \;\; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \;\; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \;\; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \;\; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \;\; a > 0, \;\; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \;\; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\text{NOTE}: \;\; \ln x = \log_e x, \;\; x > 0$$

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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

Question Nine

A ◯ B ◯ C ◯ D ◯

Question Ten

A ◯ B ◯ C ◯ D ◯