SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2012

MATHEMATICS

Directions to Students

• Reading Time : 5 minutes	• Total Marks **100**
• Working Time : 3 hours	
• Write using blue or black pen. (sketches in pencil).	• This paper contains two sections. • Section 1 contains ten objective response questions. • Section 2 contains six free response questions. • All questions may be attempted.
• Board approved calculators may be used	• Section 1 Q1-10 Multiple Choice 1 mark each • Section 2 Q11-16 15 marks each
• A table of standard integrals is provided at the back of this paper.	
• All necessary working should be shown in every question.	
• **Answer each question in the booklets provided and clearly label your name and teacher's name.**	

Section 1 10 Marks Answer on sheet provided.

1. What is the exact value of $\operatorname{cosec}\frac{4\pi}{3}$?

(A) 2

(B) -2

(C) $\frac{2}{\sqrt{3}}$

(D) $-\frac{2}{\sqrt{3}}$

---

2. Which of the following quadratic equations have two distinct real roots?

(A) $y = x^2 - 4x + 4$

(B) $y = x^2 + 4x + 4$

(C) $y = x^2 - 4x - 4$

(D) $y = x^2 + 4$

---

3. What is the value of $\sum_{r=1}^{3} r\,2^r$?

(A) 384 (B) 34 (C) 2 (D) 24

---

4. A rubber ball is dropped from the top of a building, which is 170 metres high. Suppose each time it hits the ground it rebounds $\frac{2}{3}$ of the distance of the preceding fall. What total distance does it travel before it comes to rest?

(A) $113\frac{1}{3}\,m$ (B) $255\,m$ (C) $510\,m$ (D) $850\,m$

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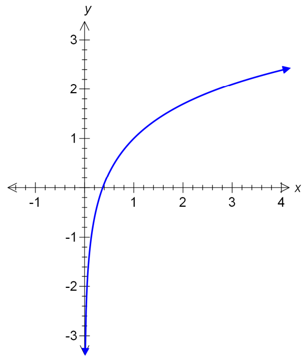
5. What is the equation of the graph below?



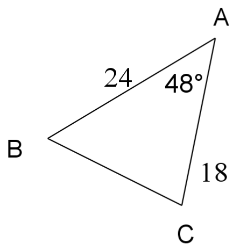
(A) $y = \ln x$

(B) $y = 1 + \ln x$

(C) $y = \ln(x + 1)$

(D) $y = \dfrac{1}{e^x}$

6.



In the diagram above, which of the values is closest to the length of the side *BC*?

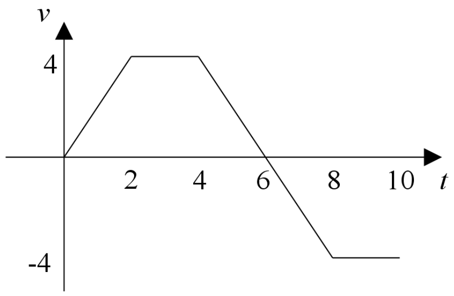
(A) 16 (B) 18 (C) 24 (D) 322

---

7. What is the value of $\displaystyle\int_{-2}^{2} \sqrt{4 - x^2}\, dx$?

(A) $\dfrac{3\pi}{2}$ (B) 2π (C) 3π (D) 4π

8.



The graph above shows the velocity of a particle for the first 10 seconds of its movement. If the particle starts at 2 *m* to the left of the origin, where is the particle after 10 seconds?

(A) At the origin

(B) 4 metres to the left of the origin

(C) 4 metres to the right of the origin

(D) 2 metres to the right of the origin

---

9. What is the approximate value of $\log_5 37$?

(A) 1.26 (B) 2.24 (C) 2.99 (D) 3.48

---

10. Which of the following functions describe a curve with amplitude of 2 and a period of 4π ?

(A) $y = 1 + 2\cos\frac{1}{2}x$

(B) $y = 2 - \sin\frac{1}{2}x$

(C) $y = 2\cos 4x$

(D) $y = 2 + 2\cos 2x$

Section 2

Question 11 (Start a new Booklet) Marks

(a) Calculate the value of $\dfrac{3.7 + 2.11}{1.45 \times 2.22}$ correct to 4 significant figures. 2

(b) Solve $|4x - 2| = 14$. 2

(c) Write the fraction $\dfrac{2}{3 + \sqrt{5}}$ with a rational denominator. 2

(d) Write down the domain and range of the function $y = \dfrac{3}{x + 1}$ 2

(e) $ABCD$ is a parallelogram. The coordinates of A, B and D respectively are $(1,4)$, $(5,7)$ and $(-2,-3)$.

(i) Show that the equation of the line AB is $3x - 4y + 13 = 0$. 2

(ii) Calculate the distance of the interval AB. 1

(iii) What are the coordinates of the point C. 1

(iv) Calculate the distance from D to the line AB. 2

(v) Hence find the area of the parallelogram $ABCD$. 1

Question 12 (Start a new Booklet) Marks

(a) Differentiate with respect to x.

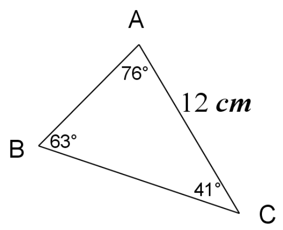
(i) $\left(6e^{2x}+2\right)^5$ 2

(ii) $3x^2 \cos 2x$ 2

(b) (i) Find $\int 3\sec^2 4x \, dx$ 1

(ii) Calculate $\int_1^3 \frac{x}{2x^2+5}\, dx$, leaving your answer correct to 2 decimal places. 2

(c) Consider the triangle below.



(i) Calculate the length of the smallest side (write your answer correct to 3 significant figures). 2

(ii) Calculate the area of ΔABC (write your answer correct to 3 significant figures). 2

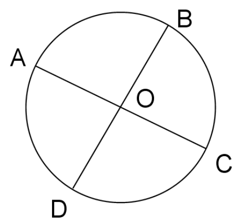
(d) Given the function $y = 27 - x^3$. Find the equation of the tangent at the point where the curve cuts the x-axis. 4

Question 13 (Start a new Booklet) Marks

(a) (i) Show that the coordinates of the vertex of the parabola $y = 2x^2 + 8x + 16$ are $(-2,8)$. 1

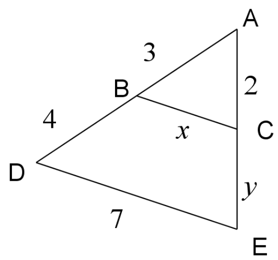
(ii) Find the focus of the parabola. 2

(b) 3



Given that AC and BD are diameters of the circle. Prove that $AB = CD$.

(c) 2



If $\Delta ACB \ ||| \ \Delta AED$, find the values of x and y.

(d) A bowl is formed by rotating the curve $y = \dfrac{x^2}{3}$ between $x = 0$ and $x = 2$ about the y-axis. Find the volume of the solid formed. 3

(e) (i) Copy and complete the table below for the function $y = \log_e x$. Write your answers correct to 2 decimal places. 1

x	1	2	3	4	5
y					

(ii) Using the Simpson's Rule find an approximation for 3

$$\int_1^5 \log_e x \, dx \ .$$

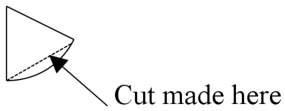
Leave your answer correct to 2 decimal places.

Question 14 (Start a new Booklet) Marks

(a) Solve the equation $2\sin x + 1 = 0$ for $0 \le x \le 2\pi$ 2

(b) Ricardo's Pizzeria makes pizzas that have an area of $36\pi\ cm^2$. They slice their pizzas into 8 equal sectors. 3

Ben does not like the crust of his pizza. His mother cuts the end off each slice of the pizza as shown in the diagram.



How much pizza does Ben's mother cut off his pizza?

(c) Solve the equation $x - xe^{5x+1} = 0$ for x. 2

(d) Calculate the area between the curve $y = \ln(x - 1)$, the line $x = 4$ and the x-axis. 3

(e) (i) Show that $x = \frac{\pi}{3}, \frac{2\pi}{3}$ are the solutions of the equation $1 + 2\cos 2x = 0$ for $0 \le x \le \pi$. 1

(ii) Draw a graph of $y = 1 + 2\cos 2x$ for $0 \le x \le \pi$ 2

(iii) Find the area between the curve $y = 1 + 2\cos 2x$ and the x-axis for $\frac{\pi}{3} \le x \le \frac{2\pi}{3}$. 2

Question 15 (Start a new Booklet) Marks

(a) Simon collects Olympic pins at the rate given by the formula 3

$R = 3 + \dfrac{4}{t+1}$, where R is the number of Olympic pins collected per day.

If Simon has 4 pins to start with, how many pins does he have after 16 days?

(b) In her training for the Olympics, Susie swims 800 m on the first day of training. She increases her distance swum by 20 m each day. She continues her training for 200 days in total.

(i) How far does Susie swim on the 200th day of training? 1

(ii) What is the total distance Susie swims in her 200 days of training? 2

(c) The formula for the sum of a series is given as $S_n = 3n + n^2$. Calculate the 15th term of the series. 2

(d) Karen borrows \$450 000 to buy a house. The loan is charged 9% p.a. interest, compounded monthly over 25 years. Karen makes monthly repayments of \$$M$.

(i) Show that the amount owing after 2 months (A_2) is 1

$$A_2 = 450\,000\,(1.0075)^2 - M(1.0075) - M$$

(ii) Show that the amount of each repayment is \$3 776.38 . 3

After 10 years (i.e. 120 repayments) the interest rate is lowered to 6% p.a.

(iii) Calculate the amount that Karen still owes after 10 years. 1

(iv) Calculate the new repayment amount if the loan will still be paid in the 25 year period. 2

Question 16 (Start a new Booklet) Marks

(a) Consider the curve $y = x^3 - 12x + 4$.

(i) Find the coordinates of any stationary points and determine their nature. 3

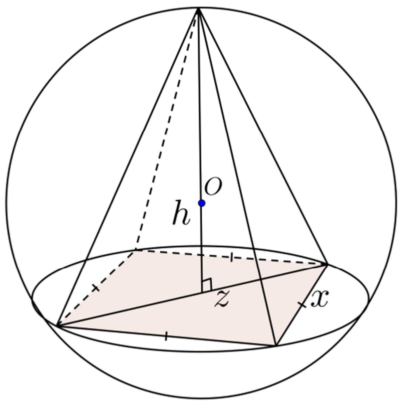
(ii) Hence sketch the graph of the curve showing the stationary points and the y-intercept. 2

(b) A radioactive substance decays according to the formula $Q = Q_0 e^{-kt}$. Initially there is 250 kg of the radioactive substance and it has a half-life of 150 years.

(i) Calculate the exact values of Q_0 and k . 2

(ii) Find the amount of time to pass before there is only 50 kg remaining of the substance (leave your answer rounded to the nearest year). 2

(c) A pyramid with a square base is inscribed in a sphere of radius $4\ cm$. Let the base length of the pyramid be x and its height be h.



(i) If the diagonal of the base of the pyramid is $z\ cm$, show that 1

$$z^2 = 2x^2 .$$

(ii) Hence show that $x^2 = 16h - 2h^2$ and that the volume of the pyramid is 3

$$V = \frac{1}{3}\left(16h^2 - 2h^3\right) .$$

(iii) Show that the pyramid with largest volume that can be inscribed in this sphere has the height $h = \frac{16}{3}\ cm$. 2

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

SUGGESTED SOLUTIONS

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Working for Multiple Choice Answers

1. $\operatorname{cosec} \frac{4\pi}{3} = \frac{1}{\sin \frac{4\pi}{3}}$

$= \frac{1}{-\frac{\sqrt{3}}{2}}$

$= -\frac{2}{\sqrt{3}}$ (D)

2. (C) $y = x^2 - 4x - 4$

$\Delta = (-4)^2 - 4(1)(-4)$

$= 16 + 16$

$= 32 > 0$

$\therefore$ two real distinct roots.

3. $\sum_{r=1}^{3} r\, 2^r = 1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3$

$= 34$ (B)

4. $170 + 2 \times \frac{113\frac{1}{3}}{1 - \frac{2}{3}} = 850\text{m}$ (D)

5. C $y = 1 + \ln x$. Asymptote on y-axis, passing through (1,1)

6. $BC^2 = 24^2 + 18^2 - 2(24)(18)\cos 48^\circ$

$BC^2 = 321.8711\ldots$

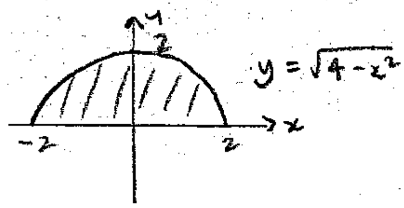
$BC = 17.94\ldots$

$\therefore BC = 18$ (B)

7. $A = \frac{1}{2} \times \pi \times (2)^2$

$= 2\pi$

(B)



8. (D) Distance travelled is equivalent to area under curve.

From 0 to 6, $A = \frac{1}{2} \times 2\times4 + 2\times4 + \frac{1}{2} \times 2\times4 = 16m \rightarrow$,

From 6 to 10, $A = \frac{1}{2} \times 2\times4 + 2\times4 = 12\,m \leftarrow$

$\therefore$ displ. $= -2+16-12$
$= 2m \rightarrow$.

9. $\frac{\log 37}{\log 5} = 2.24$ (B)

10. period $= 4\pi = \frac{2\pi}{n}$

$\therefore n = \frac{2\pi}{4\pi}$

$= \frac{1}{2}$

$\therefore$ (A) $y = 1 + 2\cos\frac{1}{2}x$

amplitude

SIC Mathematic Trial Examination 2012

1. D.

2. C

3. B.

4. D.

5. B.

6. B.

7. B.

8. D.

9. B.

10. A

Marker : MXF

Q 11.

(a) $1.80490\ldots = 1.805$ ✓✓

(b) $4x - 2 = 14$ $\quad$ $4x - 2 = -14$

$4x = 16$ $\quad$ $4x = -12$

$x = 4$ $\quad$ $x = -3$ ✓✓

(c) $$\frac{2}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{6-2\sqrt{5}}{9-5}$$

$$= \frac{6-2\sqrt{5}}{4}$$

$$= \frac{3-\sqrt{5}}{2}$$ ✓✓

(Had to be simplified)

(d) D: $x \in R$, $x \neq -1$

R: $y \in R$, $y \neq 0$

Both conditions had to be right for full marks.

(e) (i) $m(AB) = \frac{7-4}{5-1}$

$= \frac{3}{4}$ ✓

$\therefore$ equation

$y - 4 = \frac{3}{4}(x - 1)$

$4y - 16 = 3x - 3$

$\therefore 3x - 4y + 13 = 0$ ✓

(ii) $d(AB) = \sqrt{(1-5)^2 + (4-7)^2}$

$= \sqrt{25}$

$= 5u.$ ✓

(iii) $C(2, 0)$ ✓

(iv) $d = \left| \frac{3(-2) - 4(-3) + 13}{\sqrt{(3)^2 + (-4)^2}} \right|$

$= \left| \frac{19}{5} \right|$ ✓✓

$= \frac{19}{5} u.$

(v) $A = 5 \times \frac{19}{5}$

$= 19 u^2$ ✓

Marker: CJF.

Q12.

(a) (i) $5\left(6e^{2x}+2\right)^4 \times 12e^{2x}$

$= 60e^{2x}\left(6e^{2x}+2\right)^4$

(ii) $6x\cos 2x + 3x^2 \times -2\sin 2x$

$= 6x\cos 2x - 6x^2\sin 2x$

Q12 a) i) $12e^{2x}$ 1 mark

OR $5(6e^{2x}+2)^4$ 1 mark

$60e^{2x}(6e^{2x}+2)^4$ 2 m

ii) $6x\cos 2x$

OR $-6x^2\sin 2x$ 1 m

b) ii) $\frac{1}{4}(\ln 23 - \ln 7)$ 2 m

c) i) shortest side opposite smallest angle. Waste time finding BC

(b) (i) $\frac{3}{4}\tan 4x + C$

(ii) $\frac{1}{4}\int_1^3 \frac{4x}{2x^2+5}\,dx = \frac{1}{4}\left[\ln(2x^2+5)\right]_1^3$

$= \frac{1}{4}\left\{\left[\ln(2(3)^2+5)\right] - \left[\ln(2(1)^2+5)\right]\right\}$

$= \frac{1}{4}\left[\ln 23 - \ln 7\right]$

$= 0.30$

(c) (i) $\frac{AB}{\sin 41^\circ} = \frac{12}{\sin 63^\circ}$

$AB = \frac{12\sin 41^\circ}{\sin 63^\circ}$

$= 8.84$ cm.

(ii) $A = \frac{1}{2}(8.84)(12)\sin 76^\circ$

$= 51.5\ \text{cm}^2$

Q12 c) ii) Cannot use $\frac{1}{2}$ base. height because not a right angled Δ

d) $x = 3$ (no working from $x^3 = 27$ needed) Any working is waste of time

(d) at x-axis, $y = 0$

$0 = 27 - x^3$

$x^3 = 27$

$x = 3$

$\therefore y' = -3x^2$

at $x = 3$

$M = -3(3)^2$

$= -27$

$\therefore$ equation.

$y - 0 = -27(x-3)$

$y = -27x + 81$

$27x + y - 81 = 0$

Name: Q13 Teacher: GJA

Suggested Solutions | Marks | Marker's Comments

a i) $x = -b/2a = \frac{-8}{2(2)} = \frac{-8}{4} = -2$
$y = 2(-2)^2 + 8(-2) + 16 = 8$ ✓

Note: you can't just show that (-2,8) lies on the parabola you must show that the co-ordinates are the Vertex.

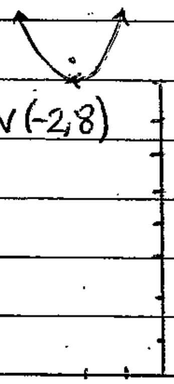
OR | OR

$\frac{1}{2}y - 8 = x^2 + 4x$
$\frac{1}{2}y - 4 = x^2 + 4x + 4$
$\frac{1}{2}(y-8) = (x+2)^2$ ✓
Vertex $= (-2, 8)$

OR | OR

$\frac{dy}{dx} = 4x + 8 = 0 \quad x = -2$
$y = 2(-2)^2 + 8(-2) + 16 = 8$ ✓

ii) Using the "completing the square" technique



$4a = \frac{1}{2}$
$a = \frac{1}{8}$ ✓
focus $(-2, 8\frac{1}{8})$ ✓

b) 1. $\angle AOB = \angle COD = \theta$ (vertically opposite angles are equal)
2. $AO = BO = DO = CO$ (radii) $= r$ ✓

3. $\therefore \triangle AOB \equiv \triangle DOC$ (SAS) ✓
4. $\therefore AB = CD$ (corresponding sides of congruent triangles) ✓

OR

use steps ① and ②
$l = r\theta$
arc AB = arc CD
($\therefore$ $AB = CD$) (equal arcs cut off equal chords)

Name:.. Teacher:..

	Marks	Marker's Comments

c) $\triangle ACB \parallel\!| \triangle AED$

$\frac{x}{7} = \frac{3}{7}$ (ratio of sides in similar triangles)

$x = 3$ ✓

$\frac{2+y}{2} = \frac{7}{3}$ (ratio of sides in similar triangles).

$6 + 3y = 14$

$3y = 8$

$y = 8/3$ ✓

d) Volume around the "y-axis"

$V = \pi \int_a^b x^2 \, dy$

Note when $x=0$ $y=0$
when $x=2$ $y=4/3$ ✓

$y = \frac{x^2}{3}$

$x^2 = 3y$ ✓

Note: Many forgot to change the ordinates

$V = \pi \int_0^{4/3} 3y \, dy$

$= \pi \left[\frac{3y^2}{2} \right]_0^{4/3}$

$= \frac{3\pi}{2} \left[y^2 \right]_0^{4/3}$

$= \frac{3\pi}{2} \left[16/9 - 0 \right] = \frac{8\pi}{3}$ units^3. ✓

e

x	1	2	3	4	5
y	0	0.69	1.10	1.39	1.61

✓

Simpson's Rule.

✓ ✓

$\int_1^5 \log_e x \, dx \doteq \frac{1}{3} \left[0 + 1.61 + 4(0.69 + 1.39) + 2(1.10) \right]$

$\doteq 4.04$ ✓

Marker : MXF

Q14.

(a) $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ ✓✓

(b) $\theta = \frac{2\pi}{8} = \frac{\pi}{4}$ $\pi r^2 = 36\pi$ ✓

$\therefore r = 6$

A (segment) $= \frac{1}{2}(6)^2\left[\frac{\pi}{4} - \sin\frac{\pi}{4}\right]$

$= \left(\frac{9\pi}{2} - \frac{18}{\sqrt{2}}\right)$ cm^2 ✓

$\therefore$ total pizza cut off

$= 8\left(\frac{9\pi}{2} - \frac{18}{\sqrt{2}}\right)$

$= 36\pi - \frac{8 \times 18 \times \sqrt{2}}{2}$

$= (36\pi - 72\sqrt{2})$ cm^2 ✓

(Some missed out on the 8 slices)

(c) $x - xe^{5x+1} = 0$

$x(1 - e^{5x+1}) = 0$

$x = 0$ ✓ $e^{5x+1} = 1$ → common error, forgot about the $x = 0$

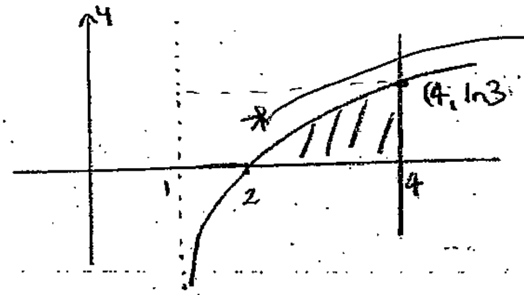
$\therefore 5x+1 = \ln 1$

$5x + 1 = 0$

$5x = -1$

$x = -\frac{1}{5}$ ✓

(d)



Many did not get this right → Worked out this area. Some used Simpson's rule, accepted if answer was right.

$y = \ln(x-1)$

$e^y = x - 1$

$x = e^y + 1$ ✓

$\therefore A = 4 \times \ln 3 - \int_0^{\ln 3} e^y + 1 \, dy$ ✓

$= 4\ln 3 - \left[e^y + y\right]_0^{\ln 3}$

$= 4\ln 3 - \left\{\left[e^{\ln 3} + \ln 3\right] - \left[e^0 + 0\right]\right\}$

$= 4\ln 3 - (3 + \ln 3 - 1)$

$= (3\ln 3 - 2)\ u^2$ ✓

5/

(e) (i) $2\cos 2x = -1$

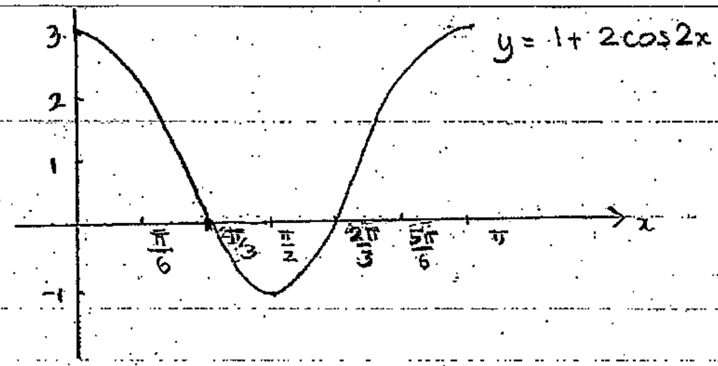
$\cos 2x = -\frac{1}{2}$

$2x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$ ✓

Both values of x had to be given

(ii)



✓✓

All points of intersection had to be shown.

(iii) $\left| \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} 1 + 2\cos 2x \ dx \right|$

$= \left| \left[x + \sin 2x \right]_{\pi/3}^{2\pi/3} \right|$

$= \left| \left[\frac{2\pi}{3} + \sin 2\left(\frac{2\pi}{3}\right) \right] - \left[\pi/3 + \sin\left(\frac{2\pi}{3}\right) \right] \right|$

$= \left| \frac{2\pi}{3} - \frac{\sqrt{3}}{2} - \pi/3 - \frac{\sqrt{3}}{2} \right|$

$= \left| \frac{\pi}{3} - \sqrt{3} \right|$

$= \left(\sqrt{3} - \frac{\pi}{3}\right) \ u^2$

$\doteqdot 0.68 \ u^2$ ✓✓

6

Marker: BDD.

Q15.

(a) $R = 3 + \frac{4}{t+1}$

$\int R\,dt = \int 3 + \frac{4}{t+1}\,dt$

$= 3t + 4\ln(t+1) + C$ ✓

① A number did not realise this was a rate and didn't integrate the function.

② Some tried to generate a series by substituting $t=0,1,2,$ etc.

③ Some integrated but didn't identify the log function.

when $t=0$ he has 4 pins

$4 = 3(0) + 4\ln(0+1) + C$

$\therefore C = 4$ ✓

$\therefore$ After 16 days

number of pins $= 3(16) + 4\ln(16+1) + 4$

$\doteqdot 63.3\ldots$

$\doteqdot 63$ ✓ needed to round off to nearest whole no.

(b) (i) $T_{200} = 800 + (200-1)(20)$

$= 4780$ ✓

(ii) $S_{200} = \frac{200}{2}\left[2(800) + (200-1)(20)\right]$ ✓

$= 558\,000$ m. ✓

well done

(c) $S_{15} - S_{14} = (3(15) + 15^2) - (3(14) + 14^2)$ ✓

$= 32.$ ✓

Well done

(d) (i) $A_1 = 450\,000(1.0075) - M$

$A_2 = [450\,000(1.0075) - M](1.0075) - M$ ← must show this line.

$= 450\,000(1.0075)^2 - M(1.0075) - M.$

✓

(ii) $A_{300} = 450\,000(1.0075)^{300} - M(1.0075)^{299} - M(1.0075)^{298} - \dots - M$

$\therefore 0 = 450\,000(1.0075)^{300} - M[1 + 1.0075 + 1.0075^2 + \dots + 1.0075^{299}]$ ✓

$M\left[\dfrac{1(1.0075^{300} - 1)}{1.0075 - 1}\right] = 450\,000(1.0075)^{300}$

Well done

$M = \dfrac{450\,000(1.0075)^{300}(0.0075)}{1.0075^{300} - 1}$ ✓

$= \$3\,776.38$ ✓

(iii) $A_{120} = 450\,000(1.0075)^{120} - 3776.38\left[\dfrac{1(1.0075^{120} - 1)}{1.0075 - 1}\right]$

$= \$372\,327.24$ ✓ Well done

(iv) $M = \dfrac{372\,327.24(1.005)^{180}(0.005)}{1.005^{180} - 1}$ ✓

$= \$3141.91$ ✓

Some common mistakes were:

1. incorrect time. Using $n=300$ instead of 180
2. incorrect interest rate. using 7.5% not 5%

Question 16 Marker: NHM.

a) $y = x^3 - 12x + 4$ $y' = 3x^2 - 12$ $y'' = 6x$

i) Let $y' = 0$ for stat pts

$3x^2 - 12 = 0$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

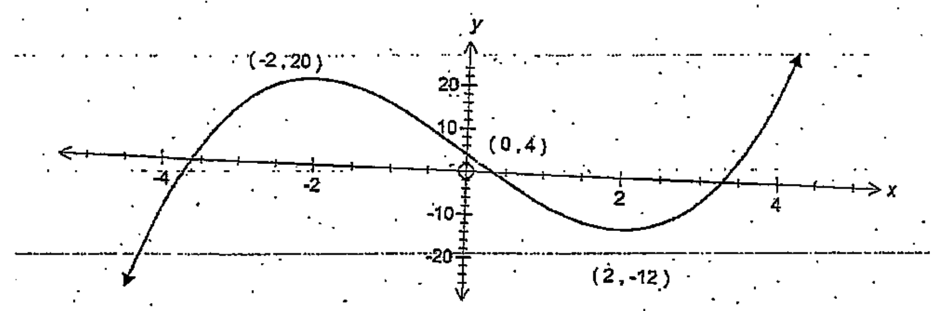
$x = 2$ $y = -12$ ← 1mk

$x = -2$ $y = 20$ ← 1mk

at $x = -2$ $y'' < 0$ $\therefore$ Maximum ← 1mk

$x = 2$ $y'' > 0$ $\therefore$ Minimum

Generally well done. Some careless factoring.

ii) 

← 1mk correct intercepts

← 1mk Correct shape.

Some poorly drawn/lazy graphs.

b) i) $Q = Q_0 e^{-kt}$

$t = 0$ $250 = Q_0 e^0$

$250 = Q_0$ ← 1mk

$t = 150$ $Q = 125$

$125 = 250 e^{-k.150}$

$\ln \frac{1}{2} = \ln e^{-k.150}$

$-k.150 = \ln \frac{1}{2}$

$k = \dfrac{\ln \frac{1}{2}}{-150}$ OR $\dfrac{\ln 2}{150}$ ← 1mk

Well done.

ii) $50 = 250 e^{k.t}$

$\frac{1}{5} = e^{-k.t}$ ← 1mk

$-kt = \ln(\frac{1}{5})$

$t = \dfrac{\ln(\frac{1}{5})}{-k}$

$= 348.28$ $\therefore$ 348yrs ← 1mk

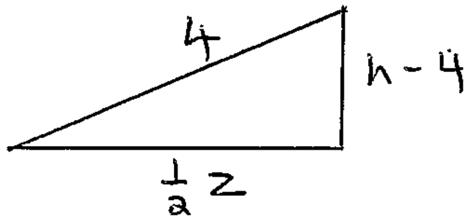
Question 16

c) i) $Z^2 = x^2 + x^2$ (Pythagorus)

$= 2x^2$

← 1mk
Well done

ii)



← 1mk
Pretty poorly done.
Quite difficult 2 unit question

$4^2 = (h-4)^2 + (\frac{1}{2}z)^2$

$16 = h^2 - 8h + 16 + \frac{z^2}{4}$

$0 = h^2 - 8h + \frac{x^2}{2}$ ← 1mk

$x^2 = 16h - 2h^2$

$V = \frac{1}{3}x^2 h$

$= \frac{1}{3}(16h - 2h^2)h$

$= \frac{1}{3}(16h^2 - 2h^3)$

← 1mk

iii) $V = \frac{1}{3}(16h^2 - 2h^3)$

$\frac{dV}{dh} = \frac{1}{3}(32h - 6h^2)$ let $\frac{dV}{dh} = 0$

$\frac{2h}{3}(16 - 3h) = 0$ ← 1mk

$3h = 16$

$h = \frac{16}{3}$

Pretty well done by most.
Some careless errors.

$\frac{d^2V}{dh^2} = \frac{1}{3}(32 - 12h)$

at $h = \frac{16}{3}$ $\frac{d^2V}{dh^2} < 0$ ← 1mk

$\therefore$ V is a maximum

SM-Marks Scatter Chart
12MTH12
Total Task 4