Name: ____________________________

Teacher: __________________________

STRATHFIELD GIRLS HIGH SCHOOL

2006

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks - 120

Attempt questions 1 - 10

All questions are of equal value

Exam Requirements

1 exam paper

20 sheets writing paper

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the HSC examination

Question 1 **Marks**

(a) Evaluate $\sqrt{\dfrac{4.3^2-1}{1.7^3}}$ correct to 3 significant figures **2**

(b) Solve the equation $\dfrac{2x-1}{2}-\dfrac{x+1}{3}=\dfrac{1}{2}$ **2**

(c) Express $4\sqrt{27}-2\sqrt{12}$ in simplest surd form **2**

(d) Find the primitive of $5x+\dfrac{5}{x}$ **2**

(e) The angle θ is subtended at the centre of a circle, by an arc of length 10cm. The centre of a circle has a radius of 15cm. Find the value of θ to the nearest minute. **2**

(f) Factorise fully $40-5y^3$ **2**

Question 2 **– (Start a new page)**

(a) Differentiate, with respect to x

(i) $x^2\cos x$ **2**

(ii) $\left(e^{2x}-4\right)^4$ **2**

(b) Evaluate $\int_0^{\frac{\pi}{3}}\cos 3x\,dx$ **2**

(c) Find $\int\dfrac{x^2}{x^3-5}\,dx$ **2**

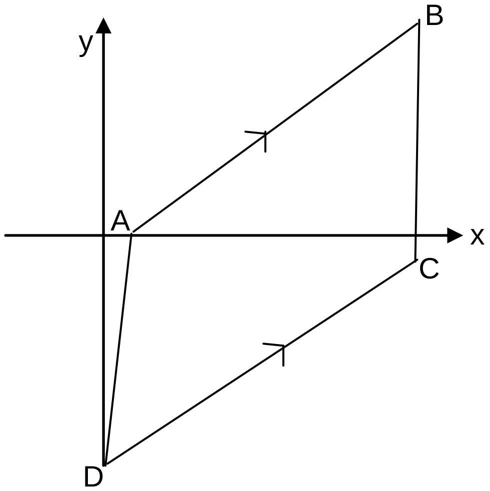
(d) Find p, if the roots of $9x^2-3x+p=0$ are real **2**

(e) Find the equation of the parabola which has its vertex at (0, 2) and its directrix is given by y=5 **2**

Marks

<u>Question 3</u> – (Start a new page)

(a) The coordinates of the points A, B and C on the number plane below are (2, 0) (6, 2) and (6, -1) respectively. AB is parallel to CD.



(Not to scale)

Copy the diagram onto your answer paper

(i) Find the exact length of AB **2**

(ii) Find the gradient of AB **1**

(iii) Show that the equation of the line through C, parallel to AB is $x - 2y - 8 = 0$ **1**

(iv) Find the coordinates of point D (where DC cuts the y-axis) **1**

(v) Find the perpendicular distance from A to the line DC **2**

(vi) Find the length of DC **1**

(vii) Hence find the exact area of the quadrilateral ABCD **2**

(b) Solve the equation $2 \sin x = \sqrt{3}$ for $0 \le x \le 2\pi$ **2**

<u>Question 4</u> – (Start a new page)

(a) For the function $f(x) = 6x^2 - x^3$

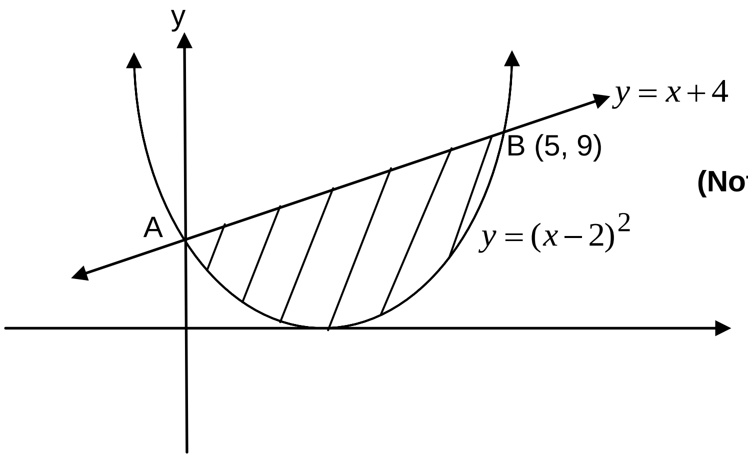
(i) Find the coordinates of the stationary point(s) and their nature **4**

(ii) Find any points of inflexion **2**

Question 4 continues next page

Marks

(b)



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The graphs of $y = (x-2)^2$ and $y = x + 4$ intersect at the point A and the point B (5, 9)

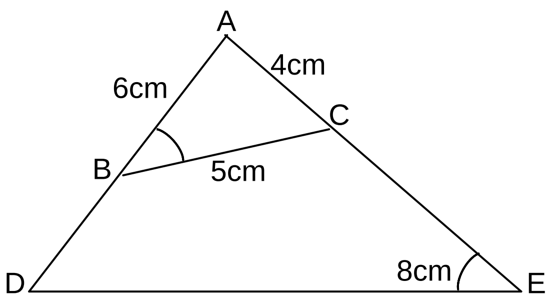
(i) Show that the point A lies on the y-axis **2**

(ii) Find the area of the shaded region bounded by the graphs of $y = (x-2)^2$ and $y = x + 4$ **4**

<u>Question 5</u> – (Start a new page)

(a) For what value of x is the tangent to the curve $y = e^{4x}$ parallel to the line $8x - y = 0$ **3**

(b)



(Not to scale)

In the above diagram $\angle ABC = \angle AED$, AB=6cm, AC=4cm, CE=8cm and BC=5cm

(i) Copy the diagram onto your answer page

(ii) Prove Δ ABC is similar to Δ AED **2**

(iii) Find the length of BD **2**

Question 5 continues next page

Marks

(c) Organisers of a music festival issued 750 tickets in the first year of the festival.

The number of tickets issued increased by 150 each year after that.

(i) What was the total number of tickets issued in the first fifteen years? **2**

(ii) In which year of the festival did the number of tickets issued first exceed 5000? **3**

Question 6 – (Start a new page)

(a) Consider the function $f(x)=\frac{1}{x}+\ln x \quad (x>0)$

(i) Find the first derivative **1**

(ii) Find the stationary point and its nature **4**

(b) The curve y = sec x for $0 \le x \le \frac{\pi}{3}$ is rotated around the x-axis **3**

Find the exact volume of the solid generated.

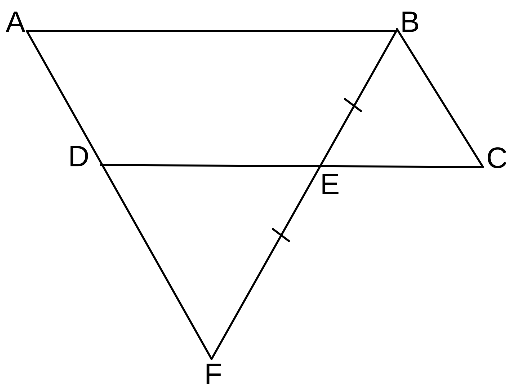
(c) (i) State the period of $y=3\sin 2x$ **1**

(ii) Draw the graph, <u>neatly</u>, of $y=3\sin 2x$ in the domain $0 \le x \le 2\pi$ **2**

(iii) How many solutions are there if $3\sin 2x = -1$ **1**

Question 7 – (Start a new page)

(a) In the diagram below ABCD is a parallelogram and BE=EF. AD is produced to F.



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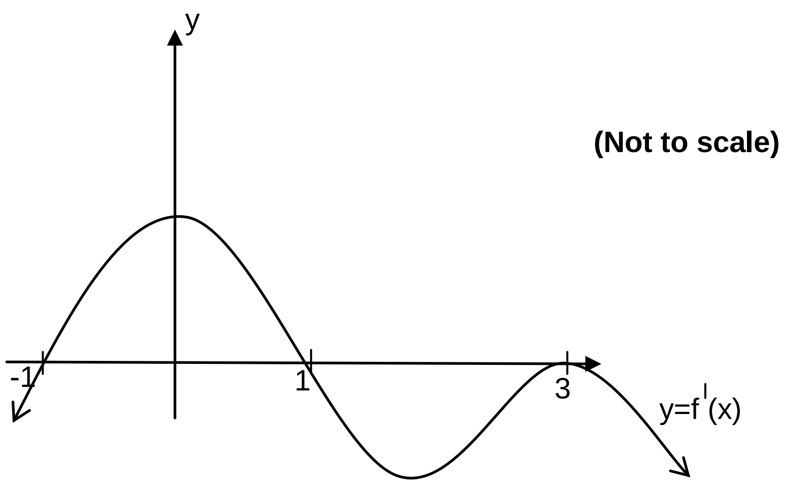
(i) Prove that $\triangle$DEF is congruent to $\triangle$CEB **3**

(ii) Hence prove that DE = $\frac{1}{2}$DC **2**

Question 7 continues next page

Marks

(b) Copy the diagram of $y = f^{I}(x)$ below neatly onto your answer paper. Then sketch $y = f(x)$ below it, given that it passes through (0, 0). Ensure that both graphs are clear and neat. **4**



(c) The following table lists three values of a function.

x	2.0	3.0	4.0
f(x)	1.7	9.0	4.3

Use these three function values to estimate $\int_2^4 f(x)\,dx$ by: **3**

(i) Simpson's Rule

(ii) The Trapezoidal Rule

<u>Question 8</u> – (Start a new page)

(a) Consider the series, $\text{Sin}^2 x + \text{Sin}^4 x + \text{Sin}^6 x$ …… where $0 < x < \frac{\pi}{2}$

(i) Show that a limiting sum exists for this series **2**

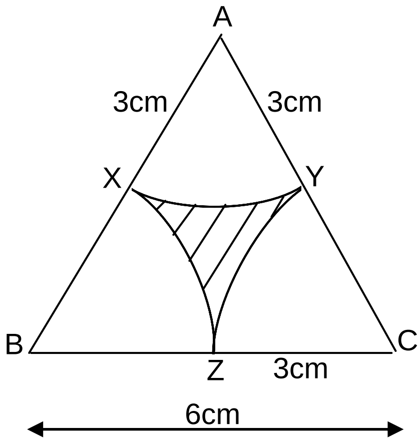
(ii) Find the limiting sum, expressing it in simplest form **2**

(b) Express $x^2 - 3$ in the form $a(x-2)^2 + b(x-2) + c$ **3**

Question 8 continues next page

Marks

(c) ABC is an equilateral triangle with sides of length 6cm. An arc, centre A, and radius 3cm, cuts AB and AC at X and Y respectively. This is repeated at B and C, as shown in the diagram below.



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(i) Explain why $\angle ABC = \frac{\pi}{3}$ **1**

(ii) Find the exact area of sector AXY **1**

(iii) Find the shaded area enclosed by the arcs XY, YZ and ZX (as shown in the diagram) **3**

<u>Question 9</u> – (Start a new page)

(a) A closed cylindrical aluminium can is to have a volume $432\pi \text{cm}^3$

(i) If r is the radius of the can, show that the surface area, A, of the can is given by $A = 2\pi r^2 + \frac{864\pi}{4}$ **1**

(ii) If this can is to be manufactured at minimum cost, find the least amount of aluminium required. Answer in terms of π. **5**

Show that this minimum surface area occurs when the height and diameter of the can are equal.

Question 9 continues next page

(b) Julian borrowed $20000 from a finance company to buy a new car. Interest is charged on the loan and is calculated at the rate of 2.5% per quarter and is charged immediately prior to Julian making his quarterly payment of $M.

Let An be the amount of dollars owing on the loan after the n[th] repayment has been made.

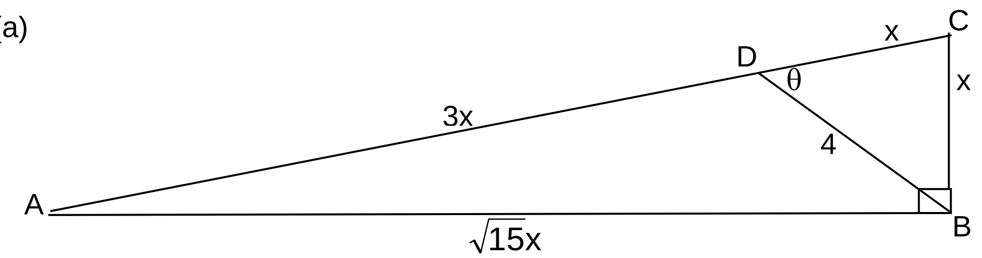
(i) Show that $A_2 = 20000x1.025^2 - M(1+1.025)$ **2**

(ii) Show that $An = 20000x1.025^n - 40M(1.025^n - 1)$ **2**

(iii) If the loan were to be paid out after 7 years, what would the value of M be? **2**

<u>Question 10</u> – (Start a new page)

(a)



(Not to scale)

In the diagram, ABC is a right angled triangle where AB = $\sqrt{15}x$ cm

The point D lies on AC and CD = BC =xcm, AD = 3xcm and BD = 4cm.

Let $\angle BDC = \theta$

(i) Use the cosine rule to show the $\cos\theta = \dfrac{2}{x}$ **2**

(ii) Use the sine rule in triangle BCD to show that $\sin\theta = \dfrac{\sqrt{15}x}{16}$ **2**

(iii) By using the formula the formula $\sin^2\theta + \cos^2 = 1$ or otherwise, show that $5x^4 - 256x^2 + 1024 = 0$ **1**

Question 10 continues next page

Marks

(b) David has a superannuation fund, which pays 5% per annum interest compounding annually. David pays $12000 into the fund on 1 July each year.

(i) What is the value of David's superannuation fund on 30 June one year after he makes his first payment. **1**

(ii) What is the value of David's superannuation fund on 30 June ten years after he makes his first payment? **3**

(iii) After making his tenth payment, David considers increasing his payment to M dollars per year. **3**

Show that if David does this, then the value of his superannuation fund twenty years after his first payment of $12000 was made, would be approximately given by $13\cdot 2068\left(12000\text{x}1.05^{10} + \text{M})\right)$

- END OF EXAM -