NAME..

TEACHER...................................

STRATHFIELD GIRLS HIGH SCHOOL

2002

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Calculators may be used
- Write your name and teacher's name at the top of each page
- A table of standard integral is provided at the back of this paper
- All necessary work should be shown in every question

Total marks - 84

- **Attempt questions 1 to 7**
- **All questions are of equal value**

Exam Requirements

- 1 exam paper
- 20 sheets of writing paper

Disclaimer

This is a Trial Examination only and does not necessarily reflect the form of this paper in the Higher School Certificate

Question 1 **(Start a new page)** **Marks**

(a) Differentiate $x^2 \cos^{-1} x$ 2

(b) $x-3$ divides $x^3+3x^2+px-14$ with a remainder of 1. 2
Find the value of p

(c) Solve the simultaneous equations 3
$|x-3|<4$ and $|x-1|>1$

(d) The point P (5,7) divides the interval joining the points A(-1,1) and B(3,5) externally in the ratio k:1 2

Find the value of k.

(e) Write $x^2+6x+13$ in the form $(x+b)^2+c$ 3
And hence find $\int \frac{dx}{x^2+6x+13}$

Question 2 (Start a new page) **Marks**

(a) Find the exact value of $\cos\left[2\tan^{-1}\frac{5}{12}\right]$ 3

(b) (i) Show that the equation $e^x = x + 2$ has a solution in the interval $1 < x < 2$ 4

(ii) Letting $x_1 = 1.5$, use one application of Newton's Method to approximate that solution, correct to 3 decimal places

(c) Show that $\sin^{-1} x$ is an odd function 2

(d) Use the substitution $x = \ln u$ to find $\int \frac{e^x}{\sqrt{1 - e^{2x}}}\,dx$ 3

Question 3 (Start a new page) **Marks**

(a) Use the method of Mathematical Induction to prove that 3

$$9^{n+2} - 4^n$$

is divisible by 5, for all positive integers, n

(b) (i) Using $t = \tan\frac{x}{2}$ 5

write expressions for $\sin x$ and $\cos x$ in terms of t

(ii) Hence , or otherwise, solve

$$3\cos x + 5\sin x = 5, \quad 0^{0} \le x \le 360^{0}$$

to the nearest degree

(c) Find the general solution of $\cos\frac{x}{2} = \frac{1}{2}$ 2

(d) Solve for θ, $\sin 2\theta = \cos^2\theta, \quad 0 \le \theta \le 2\pi$ 2

Question 4 **(Start a new page)** **Marks**

(a) Find the primitive of $\cos^2\theta$ 2

(b) Solve the following inequality 2

$$\frac{4}{x-1} > 1$$

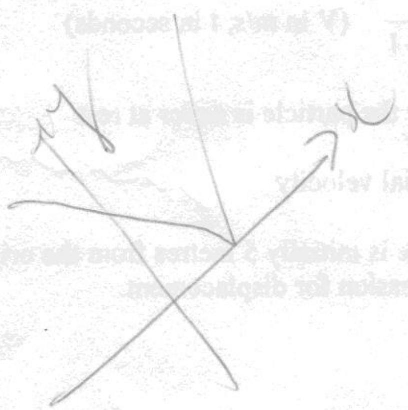
(c) (i) Sketch the function $f(x) = |x-1|$ over its natural domain 4

(ii) Explain why $f(x)$ does not have an inverse over this domain

(iii) If $f_1(x)$ is the restriction of $f(x)$ to the domain $x \geq 1$ find $f_1^{-1}(x)$, stating its domain and range

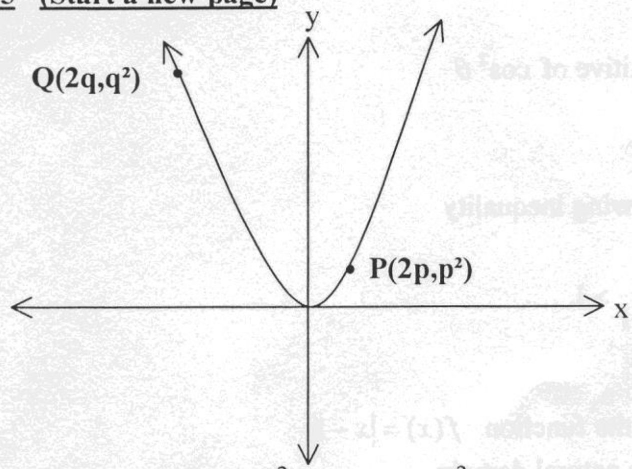
(d) The line L_1 has equation $2x - y + 5 = 0$ and P is a point with coordinates (-1,2). L_2 goes through P and makes an angle, θ, with L_1 such that $\tan\theta = \frac{1}{3}$ 4

Find the equation(s) of L_2



Question 5 **(Start a new page)** **Marks**

(a) 5



The points $P(2p, p^2)$ and $Q(2q, q^2)$ lie on the parabola $x^2 = 4y$.

The chord PQ subtends a right angle at the origin

i) Show that $pq = -4$

ii) If M is the midpoint of PQ, find the locus of M as P and Q move on the parabola

(b) If α, β and γ are the roots of the polynomial $x^3 + 4x - 9 = 0$, find the value of $\alpha(\beta + 1) + \beta(\gamma + 1) + \gamma(\alpha + 1)$ 3

(c) A particle moves in a straight line with velocity given by 4

$$V = \frac{1}{2t + 1} \quad \text{(V in m/s, t in seconds)}$$

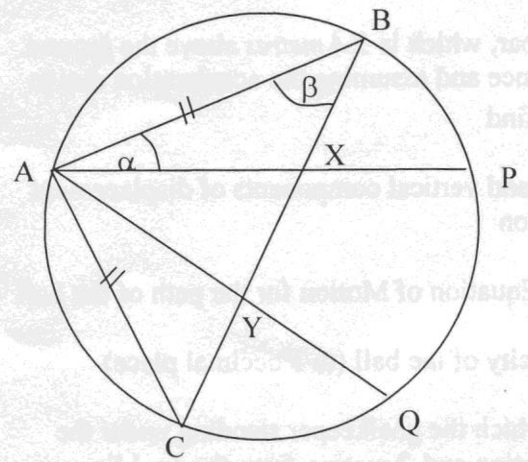
(i) Explain why the particle is never at rest

(ii) Find the initial velocity

(iii) If the particle is initially 5 metres from the origin, find an expression for displacement.

Question 6 **(Start a new page)** **Marks**

(a) 6



Let ABPQC be a circle such that AB=AC, AP meets BC at X, and AQ meets BC at Y, as in the diagram
Let $\angle BAP = \alpha$ and $\angle ABC = \beta$

i) Copy the diagram onto your answer page and state why $\angle AXC = \alpha + \beta$

ii) Prove that $\angle BQP = \alpha$

iii) Prove that PQYX is a cyclic quadrilateral

(b) An object hanging from the end of a light spring is undergoing Simple Harmonic Motion between points P and Q, which are 6 *cm* apart. Initially, the object is at rest at P. After $\frac{\pi}{2}$ seconds, the particle is first at Q. 6

(i) Write down an expression for the velocity of the object as a function of displacement from the centre of the motion.

(ii) Find the exact value of the speed of the object when its displacement from the centre of the motion is 1.5 *cm*.

(iii) What is the magnitude of the object's maximum acceleration and at what point does this occur?

Question 7 (Start a new page) **Marks**

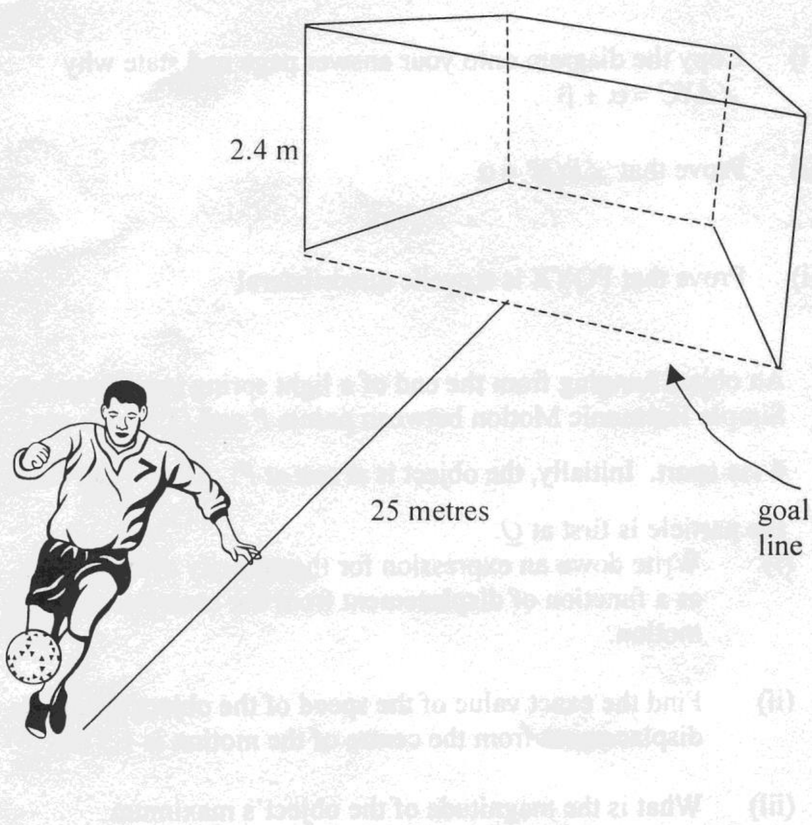
(a) Totti kicks a soccer ball off the ground 25 *metres* out from the goal at an angle of 30^0 to the horizontal. V m/s is the initial velocity.
The ball hits the top bar, which is 2.4 *metres* above the ground. Neglecting air resistance and assuming the acceleration due to gravity is $10\, m/s^2$, find

(i) the horizontal and vertical components of displacement using integration

(ii) the Cartesian Equation of Motion for the path of the ball

(iii) the initial velocity of the ball (to 1 decimal place)

(iv) the height to which the goalkeeper standing under the path of the motion and 2 *metres* from the goal line would have to jump to touch the ball.
(to the nearest metre)

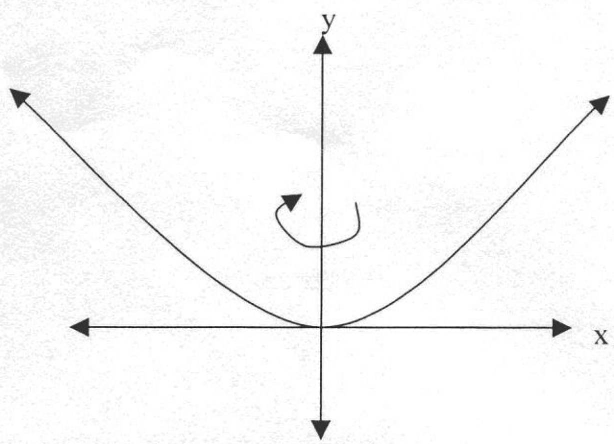


Question 7 continued **Marks**

(b) A water tank is generated by rotating the curve

$$y = \frac{x^4}{16} \qquad \text{around the y-axis}$$

6 5



i) Show that the volume of water, V as a function of its depth h, is given by

$$V = \frac{8}{3}\pi h^{\frac{3}{2}}$$

ii) Water drains from the tank through a small hole at the bottom.
The rate of change of the volume of water in the tank is proportional to the square root of the water's depth.
Use this fact to show that the height of the water in the tank falls at a **constant rate**.

END OF EXAMINATION