

## Question 1

Marks

(a) Evaluate  $\lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{h}$  1

(b) P divides the interval from (-4,2) to (2,-1) externally in the ratio 5:2. 3

Find the coordinates of P.

(c) Use the substitution  $u = x^2 + 1$  to evaluate

$$\int_1^7 \frac{x}{\sqrt{x^2 + 1}} dx$$
 3

Give your answer in simplest surd form.

(d) The angle between the lines  $y = mx + 2$  and  $x = 2y$  is  $\frac{\pi}{4}$ . Find the exact value(s) of  $m$ . 3

(e) Evaluate  $\sin\left(2 \tan^{-1} \frac{1}{2}\right)$  2

Question 2 (Start a new page)

Marks

(a) Solve  $\frac{6}{x-2} \leq 3$  3

(b) Find the exact value of  $\int_0^3 \frac{1}{\sqrt{12-x^2}} dx$  3

(c) Evaluate  $\int_0^{\frac{\pi}{6}} \sin^2 3x dx$  3

(d) The equation  $f(x) = \sin x + \frac{x}{2} - 1$  has a root near  $x = 0.6$ . 3  
Find a better approximation to the root, by using Newton's method once, giving your answer to 3 decimal places.

Question 3      (Start a new page)      Marks

(a) Sketch the graph of  $y = 2\cos^{-1} 3x$ .      3  
What is the domain and range of the graph?

(b) A spherical balloon is inflated with gas at the rate of  $10\text{m}^3/\text{min}$ . How fast is the radius of the balloon increasing at the instant when the radius is 2 metres?      3

(c) A particle is projected in a straight line with velocity  $2\text{ m/s}$ . When  $x$  metres from the origin, its acceleration is  $\left(2 - e^{-\frac{x}{2}}\right) \text{ m/s}^2$       6

(i) Show that, when  $x\text{m}$  from the origin, its velocity,  $v\text{ m/s}$ , is given by

$$v^2 = 4x + 4e^{-\frac{x}{2}}$$

(ii) Explain why, for large positive values of  $x$

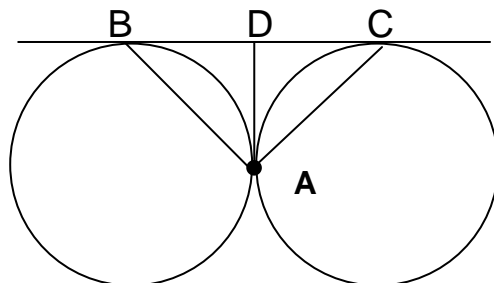
$$v \approx 2\sqrt{x}$$

(iii) Prove that the particle will move from  $x=100$  to  $x=121$  in one second.

Question 4 (Start a new page)

Marks

(a)



4

BC and DA are common tangents drawn to both circles.  
Prove that

(i)  $BD = DC$

(ii)  $\angle BAC = 90^\circ$

(b) (i) Sketch a graph of  $y = \frac{1}{x^2 + 1}$ , showing its stationary points, and state its range. 8

(ii) A function  $f(x)$  is defined by  $f(x) = \frac{1}{x^2 + 1}, x \geq 0$ .  
Explain why  $f(x)$  has an inverse function and state its domain.

(iii) On your graph of (i) sketch  $y = f^{-1}(x)$

(iv) Find  $f^{-1}(x)$  with  $y$  as the subject.

- (a) Use the method of Mathematical Induction to prove that  $9^{n+2} - 4^n$  is divisible by 5, for all positive integers,  $n$ . 4
- (b) A particle moves in such a way that its displacement  $x$  metres from the origin is given by  $x = \sqrt{3} \cos 3t - \sin 3t$ . 8
- (i) Show that the equation can be expressed in the form  $x = 2 \cos\left(3t + \frac{\pi}{6}\right)$
- (ii) Show that the motion of the particle is simple harmonic.
- (iii) Find the period and amplitude of the motion.
- (iv) Find the time at which the particle first passes through the origin.
- (v) Find the velocity of the particle when it is 1 metre from the origin.

(a) Write down the general solution of  $\cos \frac{x}{2} = \frac{1}{2}$  1

(b) Solve for  $\theta$ , given that  $0 \leq \theta \leq 2\pi$ ,  $\sin 2\theta = \cos^2 \theta$  3

(c) The rate of cooling of a hot body is proportional to the difference between the temperature of the body and room temperature. 4

Hot molten gold is poured into a mould to form ingots. The molten gold is at a temperature of  $3000^\circ\text{C}$  and when poured into the moulds it cools to  $2000^\circ\text{C}$  in 20 seconds.

If the temperature after  $t$  seconds is  $T^\circ\text{C}$ , then

$\frac{dT}{dt} = -k(T - R)$  where  $k$  is a constant and  $R$  is the room temperature.

(i) Show that  $T - R = (3000 - R)e^{-kt}$  is a possible solution of  $\frac{dT}{dt} = -k(T - R)$

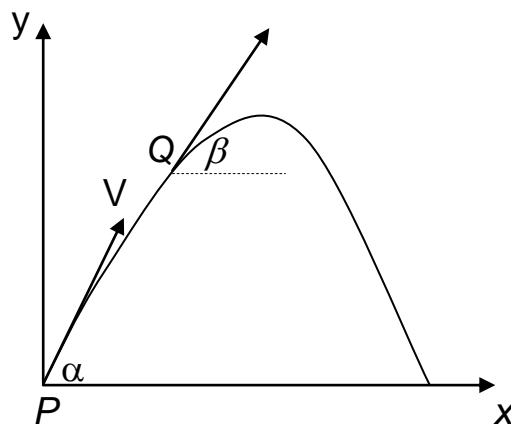
(ii) Find the value of  $k$  if the room temperature is  $20^\circ\text{C}$  (to 4 significant figures)

(iii) Find the temperature of the ingot after 1 minute.

(d) Two of the three roots of the cubic equation  $x^3 - 15x + 4 = 0$  are reciprocals. 4

Find all three roots and verify that two of them are reciprocals.

- (a) In how many ways can 4 men and 2 women sit in a row if the 3
- (i) 2 women are to sit together
- (ii) 2 women are not to sit together
- (iii) 2 women are to be separated by 2 men?
- (b) Obtain the term independent of  $x$  in  $\left(3x^2 - \frac{1}{2x}\right)^9$  2
- (c) A particle is projected from a point  $P$  on horizontal ground, 7  
with speed  $V$  m/s at an angle of elevation to the horizontal of  $\alpha$ .



Its equations of motion are  $\ddot{x} = 0$ ,  $\ddot{y} = -g$ .

- (i) Write down expressions for its horizontal ( $x$ ) and vertical ( $y$ ) displacements from  $P$  after  $t$  seconds.
- (ii) Write down the time of flight of the particle.

Question 7 continues over the page

Question 7 continued

- (iii) The particle reaches a point  $Q$ , as shown, where the direction of the flight makes an angle  $\beta$  with the horizontal.

Show that the time taken to travel from  $P$  to  $Q$  is

$$\frac{V \sin(\alpha - \beta)}{g \cos \beta} \text{ seconds}$$

- (iv) Consider the case where  $\beta = \frac{\alpha}{2}$ . If the time taken to travel from  $P$  to  $Q$  is then one-third of the total time of flight, find the value of  $\alpha$ .

END OF EXAMINATION



