STRATHFIELD GIRLS HIGH SCHOOL

2005

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Name: ______________________

Teacher: ______________________

Mathematics Extension 1

General Instructions

- All questions may be attempted.
- All questions are of equal value.
- Start each question on a new page.
- All necessary working should be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.

Total marks - 84

- Reading time - 5 minutes
- Working time - 2 hours

Exam Requirements

- 1 examination paper
- 1 answer sheet for Question 5c (detach from back of exam)
- 1 Standard Integrals sheet (detach from back of exam)
- 20 sheets of writing paper

Answer each question on a new page

Question 1 (12 marks) **Marks**

(a) Let A be the point (-4, -1) and B be the point (6, -6) **2**

Find the coordinates of the point P which divides the interval AB externally in the ratio 3:2

(b) Solve $\dfrac{3x-1}{x+1} \le 2$ **3**

(c) Use the table of standard integrals to find the exact value of **2**

$$\int_3^5 \frac{1}{\sqrt{x^2-4}}\,dx$$

(d) Evaluate $\lim\limits_{x \to 0} \dfrac{3x}{\sin 7x}$ **2**

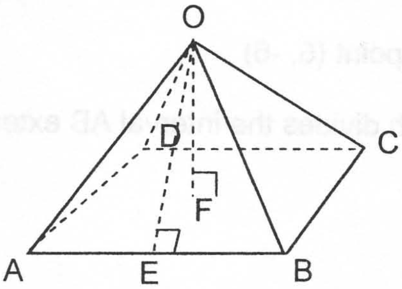
(e) Use the substitution $u=\sqrt{x}$ to find the exact value of **3**

$$\int_0^4 \frac{e^{\sqrt{x}}}{4\sqrt{x}}\,dx$$

End of Question 1

Question 2 (12 marks) START A NEW PAGE **Marks**

a) *ABCDO* is a right square pyramid.
The lengths of the sides of the square base are 6 cm. *F* is the foot of the altitude *OF*.



(i) If $< OAB = 60°$, show that the slant height *EO* is $3\sqrt{3}$ cm **1**

(ii) Hence, or otherwise, find in simplest form, the exact height, *OF*, of the pyramid **2**

b) Find $\int \sin^2 3x \, dx$ **2**

c) At SGHS there are 200 Year 12 students. 80 of these learn Chinese, 60 learn Italian and 100 learn neither language. **2**

If a student is chosen at random what is the probability that she learns Italian only?

d) Assume that the rate at which a body warms in the air is proportional to the difference between the temperature T and the constant temperature A of the surrounding air. This rate can be expressed by the differential equation

$$\frac{dT}{dt} = k(T - A)$$

Where *t* is the time in minutes and *k* is a constant.

(i) Show that $T = A + Ce^{kt}$ is a solution of the differential equation, where *C* is a constant. **1**

(ii) A cooled body warms from 5° Celsius to 10° Celsius in 20 minutes. The air temperature around the body is 25° C. Find the temperature of the body after a <u>further</u> 40 minutes have elapsed. Give your answer to the nearest degree. **3**

(iii) By referring to the equation for *T*, describe the behaviour of *T* as *t* becomes large. **1**

End of Question 2

Question 3 (12 marks) START A NEW PAGE **Marks**

(a) Let $P(x) = (x+5)(x-2) + a(x-2) + b$ where *Q(x)* is a polynomial and *a* and *b* are real numbers.

When *P(x)* is divided by $(x-2)$ the remainder is 3.
When *P(x)* is divided by $(x+5)$ the remainder is 52.

(i) What is the value of *b*? **1**

(ii) What is the remainder when *P(x)* is divided by $(x+5)(x-2)$? **2**

(b) The function $f(x) = e^{4x} - \sqrt{x+0.64}$ has a zero near $x = 0$. Use one application of Newton's method to find a second approximation to the zero. Write your answer correct to three significant figures. **3**

(c) Express $\dfrac{1-\cos 4\theta}{1+\cos 4\theta}$ in terms of $\tan 2\theta$ **3**

(d) Three dice are thrown. Find, as a fraction, the probability that: **3**

(i) there are three sixes

(ii) there is at least one six

(iii) there are exactly two sixes

End of Question 3

Question 4 (12 marks) START A NEW PAGE **Marks**

a) Use mathematical induction to prove that: **3**

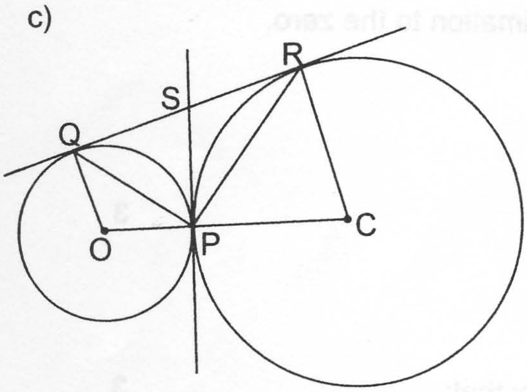
$$\sum_{r=1}^{n} \log\frac{r+1}{r} = \log(n+1)$$

for all positive integers n.

b) It is known that two of the roots of the equation $4x^3 + 8x^2 - 9x - k = 0$ are of the same size but have opposite signs. **2**

Find the value of k.

c) **7**



Two circles, centred O and C, touch externally in P. QR is a common tangent to the circles touching the circle centre O in Q and the circle centre C in R. The common tangent at P meets QR in S. Let $< PQO = \alpha$ and $< PRC = \beta$.

Copy or trace the diagram onto your answer sheet and include any additional information.

(i) Explain why SQOP is a cyclic quadrilateral.

(ii) Prove that SQOP is a kite.

(iii) Hence or otherwise prove that S is the midpoint of QR.

End of Question 4

Question 5 (12 marks) START A NEW PAGE **Marks**

(a) The two points $P(2p, p^2)$ and $Q(2q, q^2)$ are on the parabola $x^2 = 4y$.

(i) Prove that the equation of the tangent at P is $y = px - p^2$. **2**

(ii) The tangents at P and Q intersect at R. **2**
If $p^2 + q^2 = 1$, find, in Cartesian form, the locus of R.

b)

(i) Find $\frac{d}{dx}\sqrt{2x^3 - 1}$ **2**

(ii) Hence evaluate $\int_1^2 \frac{x^2}{\sqrt{2x^3 - 1}}\,dx$ **2**

(c)

(i) On the set of axes provided draw a neat sketch of $y = \sin 2x$ and $y = \frac{1}{2} + \sin x$ for $-\pi \le x \le \pi$. **3**
Ensure that your diagram has clearly labeled scales on the axes and show all x- and y-intercepts. Clearly label each function.

(ii) Hence or otherwise find an approximate solution to $\sin 2x = \frac{1}{2} + \sin x$ for $-\frac{\pi}{2} \le x \le 0$ **1**

End of Question 5

Question 6 (12 marks) START A NEW PAGE **Marks**

(a) A projectile is fired from ground level, and at any instant t seconds after firing, the position of the projectile is given by $x = 4000t$ metres and $y = 512t(10-t)$ metres.

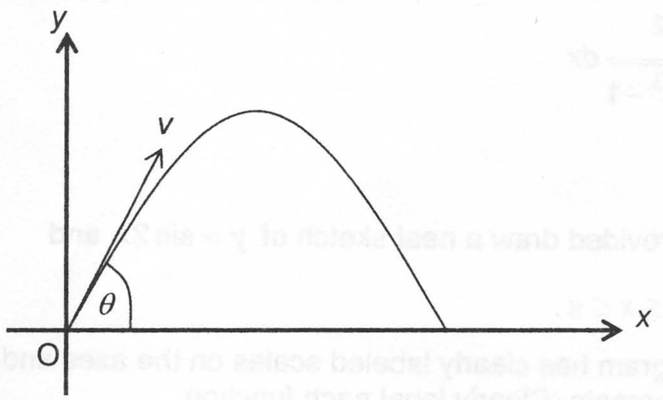
Find:

(i) the initial velocity and θ, the angle of projection. Give each answer to the nearest whole number. **3**

(ii) the time of flight **1**

(iii) the greatest height reached **2**

(iv) The height at which it would strike a plane 10000 metres away horizontally. **2**



(b)

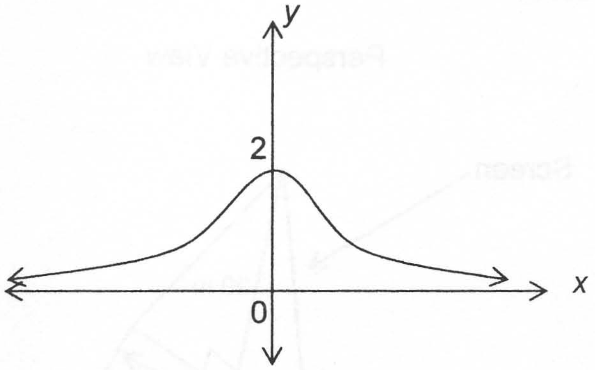
(i) Express $5\cos x + 12\sin x$ in the form $R\cos(x-\alpha)$ where α is in radians. Answer to 2 decimal places. **2**

(ii) Hence solve for $5\cos x + 12\sin x = 13$ or $0 \le x \le 2\pi$ **2**

End of Question 6

Question 7 (12 marks) START A NEW PAGE **Marks**

(a) The graph of $f(x)=\dfrac{2}{1+x^2}$ is shown in the diagram



(i) Explain why *f(x)* does not have an inverse function. **1**

(ii) Sketch the graph of the inverse function, $g^{-1}(x)$, of *g(x)* where **1**

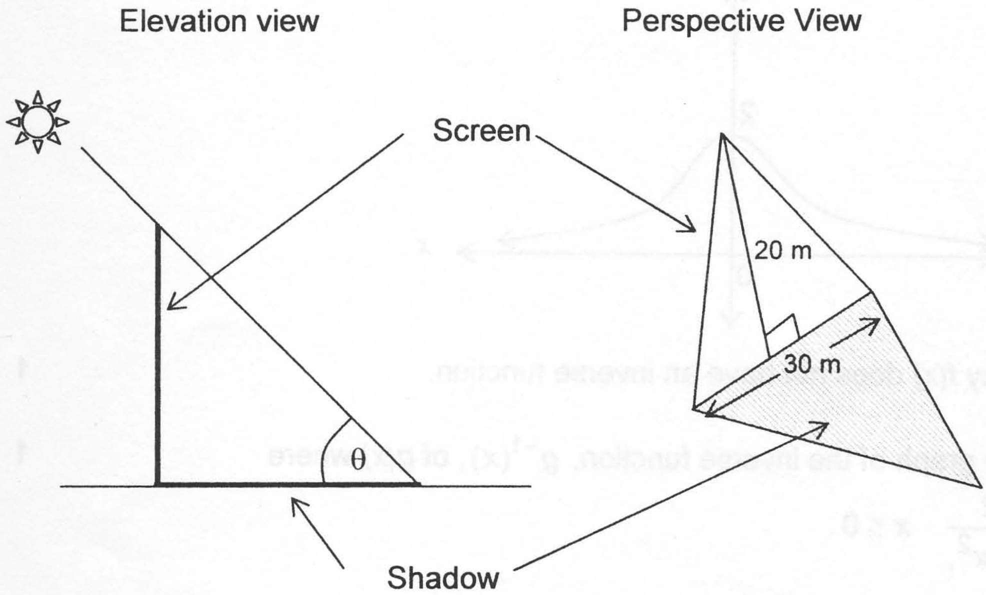
$g(x)=\dfrac{2}{1+x^2},\quad x\le 0$.

(iii) State the domain of $g^{-1}(x)$. **1**

(iv) Find an expression for $y=g^{-1}(x)$ in terms of *x*. **2**

Question 7 continues on page 8

(b) A thin, 20 metre high, triangular screen which has a 30 metre long base running north to south, stands on level ground. At noon it casts no shadow and, as the afternoon progresses, its shadow increases in length as the angle of elevation of the sun, θ, decreases.



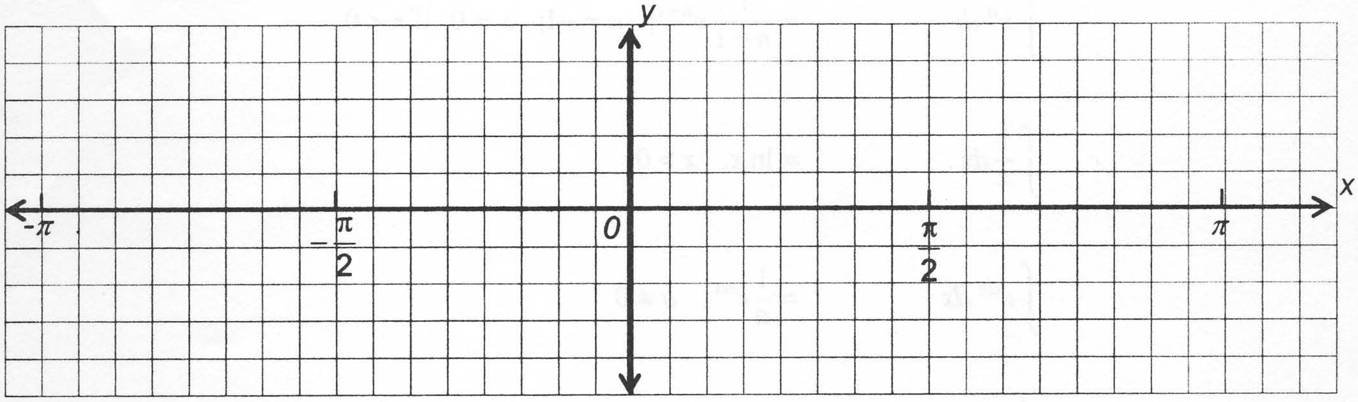
(i) Show that the area of the shadow is given by $a = \dfrac{300}{\tan\theta}$ **1**

(ii) Given that the angle of elevation of the sun decreases at a rate of $\dfrac{\pi^c}{12}$/hour (ie $\dfrac{d\theta}{dt} = -\dfrac{\pi}{12}$,) find the rate in m^2/hour at which the area of the shadow is increasing at 3 pm. **3**

(iii) Sketch $a = \dfrac{300}{\tan\theta}$ for $0 \le \theta \le \dfrac{\pi}{2}$. Use this graph to help describe what happens to the area of the shadow as the afternoon progresses and also what happens to the rate of change in the area. In particular, describe what happens just after midday and as the time approaches 6 pm. **3**

End of paper

Question 5c)



STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$