STRATHFIELD GIRLS HIGH SCHOOL

Name: ______________________

Teacher: ______________________

STRATHFIELD GIRLS HIGH SCHOOL

2006

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- All questions may be attempted.
- All questions are of equal value.
- Start each question on a new page.
- All necessary working should be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Approved calculators may be used.

Total marks - 84

- Reading time - 5 minutes
- Working time - 2 hours

Exam Requirements

- 1 examination paper
- 1 Standard Integrals sheet (detach from back of exam)
- 20 sheets of writing paper

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This is a trial examination paper only and does not necessarily reflect the format of the HSC paper

Question 1 (12 marks)

Marks

(a) (i) Sketch the graph of $y = x^3 - 4x$ **1**

(ii) Hence or otherwise, solve $x^3 - 4x > 0$ **2**

(b) Divide the interval AB externally in the ratio $4:3$, where A is the point $(2,-1)$ and B is $(1,-3)$. **3**

(c) Find the exact value of $\cos 15^0$ **2**

(d) (i) Sketch the graph of $y = \tan x$ for $0 \leq x \leq \pi$ **1**

(ii) Hence or otherwise, find values of x in the domain $0 \leq x \leq \pi$ such that $1 + \sqrt{3}\tan x + 3\tan^2 x + 3\sqrt{3}\tan^3 x + ...$ has a limiting sum **3**

Question 2 (12 marks) **(Start a new page)**

(a) For the parabola defined by the parametric equations $x = 4t \quad , \quad y = 2t^2$

(i) Show, by differentiation, that the gradient of the tangent at the point, P, where $t = 3$, is 3. **1**

(ii) Find the gradient of the focal chord through P. **1**

(iii) Calculate the acute angle between the tangent at P and the focal chord through P. **2**

Question 2 continued

Marks

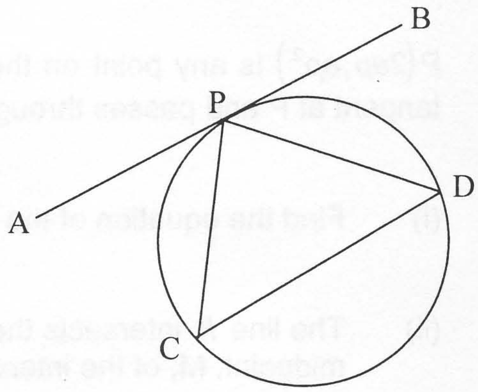
(b) Use one iteration of Newton's method to find an approximation of the root of the equation $x\log_e x - 2x = 0$ near $x = 7$. Give your answer to one decimal place. **3**

(c) Calculate $\lim_{x\to 0}\frac{2\sin 2x}{x}$ **2**

(d) A dice is biased so that to throw a 2 is twice as likely as any other number. Find the probability of throwing a 2. **1**

(e) **2**

PC and PD are equal chords of a circle. A tangent, AB, is drawn at P. Prove that AB is parallel to CD.



Question 3 (12 marks) **(Start a new page)**

(a) Differentiate $\cos^{-1}(\sin x)$ **3**

(b) The diameter of a circle meets the points $A(x_1, y_1)$ and $B(x_2, y_2)$ on the circumference. Show that the equation of the circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$. **2**

(c) Use the method of mathematical induction to prove that $2^{3n} - 3^n$ is divisible by 5 for all positive integers n. **4**

(d) The arc, between $x = 0$ and $x = \frac{\pi}{6}$, of the curve $y = \cos 2x$, is rotated through 360^0 about the x-axis. Find the exact volume of the solid formed. **3**

Question 4 (12 marks) **(Start a new page)**

Marks

(a) The polynomial $P(x) = 2x^3 + ax^2 + x + 2$ has a factor $(2x+1)$. Find the value of a. **2**

(b) Solve the inequality $\dfrac{1}{x-2} \geq 2$ **2**

(c) Using the substitution $u = x + 2$, find $\int \frac{x}{3}\sqrt{x+2}\, dx$. **3**

(d) $P(2ap, ap^2)$ is any point on the parabola $x^2 = 4ay$. The line k is parallel to the tangent at P and passes through the focus, S, of the parabola.

(i) Find the equation of the line k. **2**

(ii) The line k intersects the x-axis at the point Q. Find the coordinates of the midpoint, M, of the interval QS. **2**

(iii) What is the equation of the locus of M? **1**

Question 5 (12 marks) **(Start a new page)**

Marks

(a) The function $f(x) = \sec x$ for $0 \le x < \frac{\pi}{2}$, and is not defined for other values of x.

(i) State the domain of the inverse function $f^{-1}(x)$. **1**

(ii) Show that $f^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$. **2**

(iii) Hence find $\frac{d}{dx}f^{-1}(x)$. **1**

(b) An amount $\$A$ is borrowed at $r\%$ per annum reducible interest, calculated monthly. The loan is to be repaid in equal monthly repayments of $\$M$.

Let $R = \left(1 + \frac{r}{1200}\right)$ and let $\$B_n$ be the amount owing after n monthly repayments have been made.

(i) Show that $B_n = AR^n - M\left(\frac{R^n - 1}{R - 1}\right)$. **2**

Pat borrows \$30 000 at 9% per annum reducible interest, calculated monthly. The loan is to be repaid in 60 equal monthly instalments.

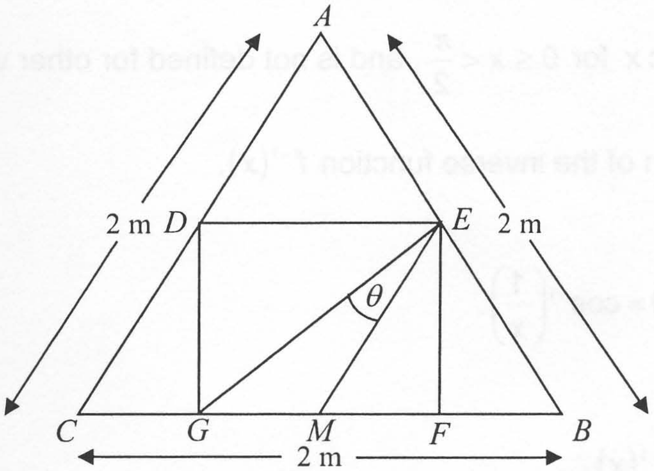
(ii) Show that the monthly repayments should be \$622.75 **2**

(iii) With the twelfth repayment, Pat pays an additional \$5000, so this payment is \$5622.75. After this, repayments continue at \$622.75 per month. How many more repayments will be needed? **4**

Question 6 (12 marks) **(Start a new page)**

Marks

(a)



The equilateral triangle ABC has side lengths of 2 metres. The points D and E are the mid-points of AC and AB respectively and GF lies on CB. Also, $DEFG$ is a rectangle. Point M is the midpoint of GF. $\angle GEM = \theta$.

(i) Show that $GE = \dfrac{\sqrt{7}}{2}$ metres. **3**

(ii) Show that $\cos\theta = \dfrac{5\sqrt{7}}{14}$. **2**

(iii) Hence show that the area of ΔGEM is $\dfrac{\sqrt{3}}{8}$ square metres. **1**

(b) Find $\displaystyle\int \frac{\cos x \sin x}{\sqrt{2-\sin^2 x}}\,dx$ using the substitution $u = \sin x$. **3**

(c) (i) Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$ where $0 < \alpha < \dfrac{\pi}{2}$ and $R > 0$. **2**

(ii) Hence, solve $\sqrt{3}\cos x - \sin x = 1$ for $0 \le x \le \dfrac{\pi}{2}$. **1**

Question 7 (12 marks) **(Start a new page)**

Marks

(a) A melting snowball which is always spherical in shape is decreasing in volume at a constant rate of 8 cm^3 / min. Find the rate at which the radius is changing when the radius is 4 cm. **2**

(b) (i) If $\frac{dy}{dt} = ky$, show that $y = Ae^{kt}$ where A and k are positive constants. **2**

(ii) Given that $\frac{dM}{dt} = -kM$ where k is a positive constant and when $t = 0,\quad M = M_0$ find an expression for M in terms of t. **1**

(c) (i) If $x = a + b$ and $y = a - b$, show that $a = \frac{x+y}{2}$ and $b = \frac{x-y}{2}$. **1**

(ii) Show that $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ **2**

(iii) Hence find all solutions to $\cos 4\theta + \cos 3\theta + \cos 2\theta + \cos\theta = 0$ for $0 \le \theta \le 2\pi$ **4**

END OF EXAM