Sydney Girls High School
2013

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Extension 1 Mathematics

General Instructions

- Reading Time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I Pages 3 – 6

10 Marks

- Attempt Questions 1 – 10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II Pages 7 – 13

60 Marks

- Attempt Questions 11 – 14
- Answer on the blank paper provided
- Begin a new page for each question
- Allow about 1 hours and 45 minutes for this section

Name:	**THIS IS A TRIAL PAPER ONLY**
Teacher:	It does not necessarily reflect the format or the content of the 2013 HSC Examination Paper in this subject.

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

(1) The acute angle between the straight lines $y = \sqrt{3}x + 2$ and $y = 2$ is :

(A) 30°

(B) 60°

(C) 47°

(D) 68°

(2) The value of $\lim_{n \to \infty} \frac{5(10^n) + 3}{2(10^n) + 5}$ is:

(A) $\frac{3}{5}$

(B) 0

(C) 1

(D) $\frac{5}{2}$

(3) The exact value of k given $\int_0^1 \frac{dx}{x^2 + 3} = k\pi$ is:

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}}{9}$

(C) $\frac{\sqrt{3}}{18}$

(D) $6\sqrt{3}$

(4) Which of the following is the derivative of $x^2 \cos^{-1} 3x$?

(A) $2x\sin^{-1} 3x$

(B) $2x\cos^{-1} 3x + x^2 \sin^{-1} 3x$

(C) $2x\cos^{-1} 3x - \dfrac{x^2}{\sqrt{1-9x^2}}$

(D) $2x\cos^{-1} 3x - \dfrac{3x^2}{\sqrt{1-9x^2}}$

(5) The solution to $\ln(x^3 + 19) = 3\ln(x+1)$ is:

(A) $x = -3$ or $x = 2$

(B) $x = 3$

(C) $x = -2$

(D) $x = 2$

(6) The exact value of $\int_0^{\frac{\pi}{4}} \cos^2 \frac{1}{2}x \, dx$ is :

(A) $\frac{1+\pi}{\sqrt{2}}$

(B) $\frac{2\sqrt{2}+\pi}{8}$

(C) $\frac{2\sqrt{2}+\pi}{4}$

(D) $\frac{\sqrt{2}+\pi}{8}$

(7) The domain of $y=\cos^{-1}\sqrt{\frac{1}{4}-x^2}$ is :

(A) $0 \le x \le \frac{1}{2}$

(B) $\frac{-1}{4} \le x \le \frac{1}{2}$

(C) $\frac{-1}{2} \le x \le \frac{1}{2}$

(D) $\frac{1}{4} \le x \le \frac{1}{2}$

(8) A metal disc , 5 cm radius , expands when heated. If the radius is increasing at the rate of $0.01\ cm/\sec$, the rate at which the area of one of the faces is increasing is given by:

(A) $\frac{\pi}{10}\ cm^2/\sec$

(B) $\frac{\pi}{5}\ \mathrm{cm}^2/\sec$

(C) $\frac{2\pi}{5}\ \mathrm{cm}^2/\sec$

(D) $\frac{5\pi}{2}\ \mathrm{cm}^2/\sec$

(9) Two roots of the equation $x^3 - 2x^2 + kx + 18 = 0$ are opposites. The value of k is :

(A) -9

(B) 9

(C) -6

(D) 6

(10) A point moving with simple harmonic motion starts from a point $5cm$ from the centre of the motion with a speed of $1cm/s$. The period is 8 seconds. The maximum acceleration is:

(A) $4.9ms^{-2}$

(B) $5.2ms^{-2}$

(C) $24.4ms^{-2}$

(D) $25.6ms^{-2}$

Section II

60 marks

Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer on the blank paper provided. Begin a new page for each question
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (Begin a New Page) (15 Marks)

(a) By making the substitution $u^2 = x+1$, find $\int \frac{x+2}{\sqrt{x+1}}\,dx$ [2]

(b) Solve : $x+2 < \frac{4}{x-1} \quad (x \neq 1)$ [3]

(c) Find the general solution (in radian form) of the equation $\cos 2x = \cos x$ [3]

(d)

i) Sketch the graph of the curve $y = 3\sin^{-1}(x/2)$, clearly indicating the domain and range. [1]

ii) Find the area enclosed between the curve $y = 3\sin^{-1}(x/2)$, the line $x = 1$ and the positive x axis. [3]

(e) Consider the series $\tan x + \tan^3 x + \tan^5 x + \ldots\ldots\ldots$, where $0 \le x \le \frac{\pi}{4}$

i) Explain why this series has a limiting sum [1]

ii) Show that $S_\infty = \frac{1}{2}\tan 2x$ [2]

End of Question 11

Question 12 (Begin a New Page) (15 Marks)

(a) Use mathematical induction to show that $5^n + 2(11^n)$ is a multiple of 3 for all positive integers n. [3]

(b) At time t minutes the number of individuals in each of population

A and B is given by $N_A = 15 + 20e^{-0.5t}$ and $N_B = 5 + 40e^{-0.5t}$ respectively.

i) Find the initial size of population A [1]

ii) Find the initial rate of change of population B [1]

iii) Find the time at which the two population sizes are equal. [2]

(c) A particle moves along the x axis according to the equation $x = 6\sin 2t - 2\sqrt{3}\cos 2t$.

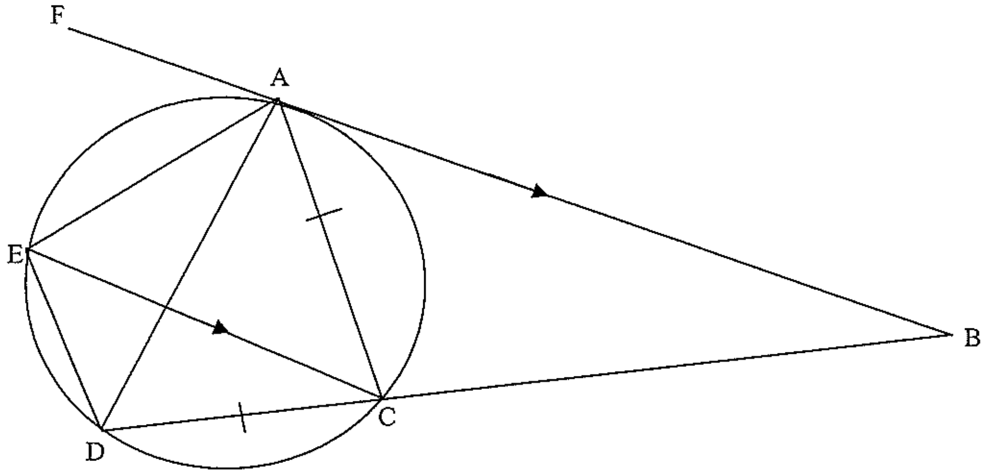
i) Express x in the form $R\sin(2t - \alpha)$ where $R > 0$ and $0 \le \alpha \le \pi/2$. [2]

ii) Prove that the particle moves in simple harmonic motion. [2]

iii) Find when the particle is 2m to the right of the origin. (correct to 2 decimal places) [2]

Question 12 continues on the next page

(d) AB is a tangent to the circle. $AB \parallel EC$ and $CD = AC$.



i) Copy the diagram on your answer sheet

ii) Prove that $AC \parallel ED$ [2]

End of Question 12

Question 13 (Begin a New Page) (15 Marks)

(a) The function $f(x)$ is given by $f(x)=\sqrt{x+6}$ for $x \geq -6$

i) Find the inverse function $f^{-1}(x)$ and find its domain. [2]

ii) On the same diagram , sketch the graphs of $y=f(x)$ and $y=f^{-1}(x)$. Showing Clearly all the intercepts on the coordinates axes. [2]

iii) Show that the x coordinates of any points of intersection of the graphs of $y=f(x)$ and $y=f^{-1}(x)$ satisfy the equation $x^2-x-6=0$. [1]

iv) Hence find the point of the intersection of the two graphs. [1]

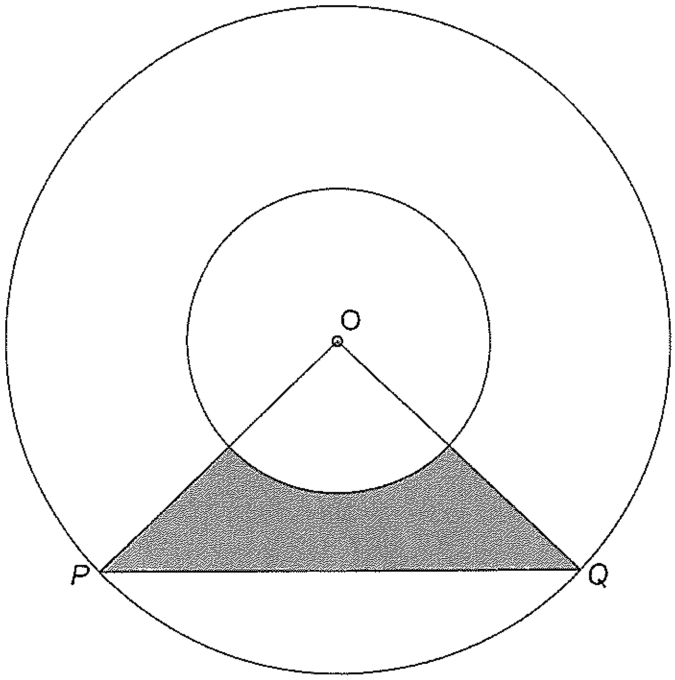
(b) A vertical flagpole CD of height h metres stands with its base C on horizontal ground. A is a point on the ground due west of C and B is a point on the ground 40 metres due south of A . From A and B the angles of elevation of the top D of the flagpole are 20° and 10° respectively.

i) Draw a diagram for the information given [1]

ii) Find the height of the flagpole to the nearest metre. [3]

Question 13 continues on the next page

(c) Two concentric circles with centre O have radii $2\ cm$ and $4\ cm$. The points P and Q lie on the larger Circle and $\angle POQ = x$, where $0 \le x \le \frac{\pi}{2}$



i) If the area $A\ cm^2$ of the shaded region is $\frac{1}{16}$ the area of the larger circle, show that x satisfies the equation $8\sin x - 2x - \pi = 0$. [1]

ii) Show that this equation has a solution $x = \alpha$, where $0.5 \le \alpha \le 0.6$ [2]

iii) Taking 0.6 as a first approximation for α, use one application of Newton's method to find a second approximation, giving the answer correct to 2 decimal places. [2]

End of Question 13

(15 Marks)

line. At time t seconds its displacement is x

n the line, its acceleration is a ms^{-2}, and its

given by $v=\frac{32}{x}-\frac{x}{2}$. Initially the particle is at

xpression for a in terms x. [2]

at $t=\int \frac{2x}{64-x^2}\,dx$, and hence show that

$-60e^{-t}$. [3]

e graph of x^2 against t and describe the limiting

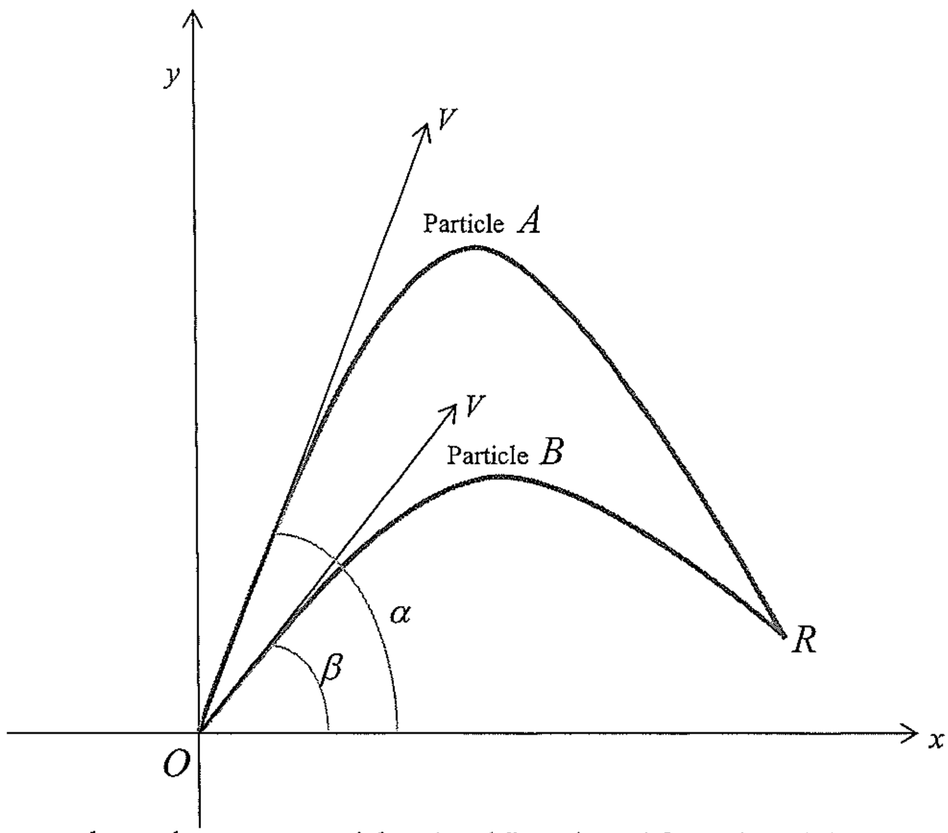
r of the particle. [1]

arabola $x^2=4y$ with focus F. The point M

rnally in the ratio $3:1$. Show that as P moves

n the locus of M is given by $x^2=6y+3$. [3]

Question 14 continues on the next page



(c) The diagram above shows two particles A and B projected from the origin. Particle A is projected with initial velocity V m/s at an angle α and Particle B is projected T seconds later with the same initial velocity V m/s but an angle of β. The particles collide at the point R.

i) You may assume that the equation of the path of A is given by

$$y = -\frac{gx^2}{2V^2}\sec^2\alpha + x\tan\alpha$$

Write down the equation of the path of B. [1]

ii) Show that the x-coordinate of the collision point R is given by

$$x = \frac{2V^2\cos\alpha\cos\beta}{g\sin(\alpha+\beta)}$$ [2]

iii) You may assume that the horizontal displacement of A after t seconds is given by

$$x = Vt\cos\alpha$$

Write down the equation for the horizontal displacement of B. [1]

iv) Show that, for the collision to take place, the value of T is given by

$$T = \frac{2V(\cos\beta - \cos\alpha)}{g\sin(\alpha+\beta)}$$ [2]

End of paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Sydney Girls High School

Mathematics Faculty

Years 12 HSC Mathematics Extension 1 2013 Trial

Question	Marker's Comment
11	This question overall was completed very average. Some students answered it very well and others very poorly, there was a large range. a) Substitution and integration was done well but substituting back in terms of x was very poor. b) Most students realised to multiply both sides by $(x-1)^2$ but then most couldn't complete and solve. c) Most students didn't know correct general solution and some even failed to recognise to substitute $\cos 2x = 2\cos^2 x - 1$ to be able to factorise and solve. d) Overall was done well, most knew domain and range. Shaded area was also done well. e) i) To obtain the mark both : $r = \tan^2 x$ and $\text{if } 0 < x < \frac{\pi}{4} \text{ then } 0 < \tan^2 x < 1$ had to be stated. ii) Was done very well.
12	b) iii) Many students successfully found the derivative but forgot to substitute for *t* and thus did not answer the question. c) ii) All calculus answers involving trigonometric functions must be in terms of radians. Radians are a linear measure, degrees are a measure of turning. d) There were many solutions to this question because of the number of equal angles in the diagram. Deductive solutions should be a logical progression of statements to a conclusion. Students who simply wrote down everything they could see from the diagram without a logical progression failed to gain full marks.
13	This question was done well. Only some students had difficulties with inverse functions and sketching these functions. A number of students couldn't do the three dimensional trigonometry.
14	This question was done very poorly. Many students didn't know the formula $a = \frac{d}{dx}\frac{1}{2}v^2$ or didn't know how to use it properly. Many students have no idea how to show a given formula in all parts of this question.

Sydney Girls High School Mathematics Faculty

Multiple Choice Answer Sheet –Trial HSC 2013
Extension 1

Student Number: Answers

Completely fill the response oval representing the most correct answer.

1.	A ○	B ●	C ○	D ○
2.	A ○	B ○	C ○	D ●
3.	A ○	B ○	C ●	D ○
4.	A ○	B ○	C ○	D ●
5.	A ○	B ○	C ○	D ●
6.	A ○	B ●	C ○	D ○
7.	A ○	B ○	C ●	D ○
8.	A ●	B ○	C ○	D ○
9.	A ●	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○ No answer

Section II

Question 11.

a) $u^2 = x+1$

$u = (x+1)^{1/2}$

$\frac{du}{dx} = \frac{1}{2}(x+1)^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x+1}}$

$2du = \frac{dx}{\sqrt{x+1}}$

$u^2 = x+1$

$u^2 - 1 = x$

$x+2 = u^2 - 1 + 2$

$= u^2 + 1$

$\therefore \int \frac{x+2}{\sqrt{x+1}}\,dx = 2\int (u^2+1)\,du$

$= 2\left[\frac{u^3}{3} + u\right] + C$

$= 2\left[\frac{(x+1)^{3/2}}{3} + (x+1)^{1/2}\right] + C$

$= 2\sqrt{x+1}\left[\frac{(x+1)^3}{3} + 1\right] + C$

b) $(x-1)^2\, x+2 < \frac{4}{(x-1)}(x-1)^2$

$(x-1)^2(x+2) < 4(x-1)$

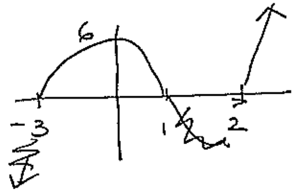
$(x-1)^2(x+2) - 4(x-1) < 0$

$(x-1)\left[(x-1)(x+2) - 4\right] < 0$

$(x-1)\left[x^2 - x + 2x - 2 - 4\right] < 0$

$(x-1)\left[x^2 + x - 6\right] < 0$

$(x-1)(x+3)(x-2) < 0$



$\therefore x < -3$

$1 < x < 2$

c) General solution

$\cos 2x = \cos x$

$\cos 2x - \cos x = 0$

$2\cos^2 x - 1 - \cos x = 0$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x = -1$

$\cos x = -\frac{1}{2}$

$x = \cos^{-1}(-\frac{1}{2})$

$x = \frac{2\pi}{3}$ (120°)

$\therefore x = 2n\pi \pm \frac{2\pi}{3}$

$\cos x = 1$

$x = \cos^{-1}(1)$

$x = 0$

$x = 2n\pi$

d) i) $y = 3\sin^{-1}\left(\frac{x}{2}\right)$

Domain: $y = \sin^{-1} x$ $\quad -1 \le x \le 1$

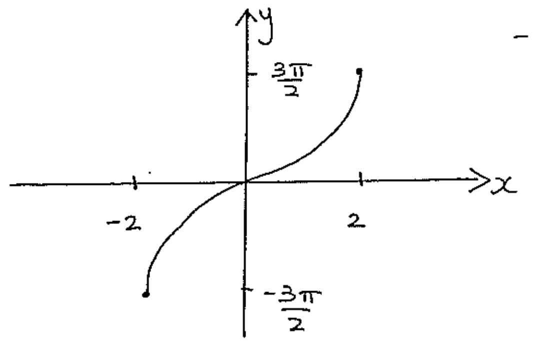
Domain: $y = 3\sin^{-1}\frac{x}{2}$ $\quad -1 \le \frac{x}{2} \le 1$

$-2 \le x \le 2$.

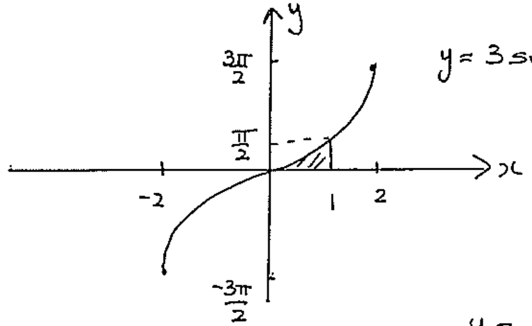
Range: $y = \sin^{-1} x$ $\quad -\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$

Range: $y = 3\sin^{-1}\frac{x}{2}$ $\quad -\frac{3\pi}{2} \le 3\sin^{-1}\frac{x}{2} \le \frac{3\pi}{2}$

$-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$



d) ii)



$y = 3\sin^{-1}\frac{x}{2}$

at $x = 1$, $y = 3\sin^{-1}\frac{x}{2}$

$y = 3\sin^{-1}\frac{1}{2}$

$y = \frac{\pi}{2}$

$y = 3\sin^{-1}\frac{x}{2}$

$\frac{y}{3} = \sin^{-1}\frac{x}{2}$

$\sin\left(\frac{y}{3}\right) = \frac{x}{2}$

$2\sin\left(\frac{y}{3}\right) = x$

Shaded Area = Area of rectangle $- \int_0^{\frac{\pi}{2}} 2\sin\frac{y}{3}\,dy$

$= \left(1 \times \frac{\pi}{2}\right) - 2 \times 3\left[-\cos\frac{y}{3}\right]_0^{\pi/2}$

$= \frac{\pi}{2} - 6\left[-\cos\frac{\pi}{6} - -\cos 0\right]$

$= \frac{\pi}{2} - 6\left[-\frac{\sqrt{3}}{2} + 1\right]$

$= \frac{\pi}{2} + \frac{6\sqrt{3}}{2} - 6$

$= \frac{\pi}{2} + 3\sqrt{3} - 6$ units^2

$\doteqdot 0.77$ units^2

e) i) $\tan x + \tan^3 x + \tan^5 x + \ldots \quad 0 \le x \le \frac{\pi}{4}$

Common ratio $\frac{\tan^3 x}{\tan x} = \frac{\tan^5 x}{\tan^3 x}$

$\therefore r = \tan^2 x$

limiting sum exists $-1 < |r| < 1$

if $0 < x < \frac{\pi}{4}$ then

$0 < \tan^2 x < 1 \quad \therefore$ limiting sum exists

ii) $S_\infty = \frac{a}{1-r}$

$= \frac{\tan x}{1-\tan^2 x}$

$= \frac{1}{2}\left(\frac{2\tan x}{1-\tan^2 x}\right)$

$= \frac{1}{2}\tan 2x$

Question 12

a) When $n=1$

$\therefore 5^1 + 2(11^1) = 27$ which is a multiple of 3 ✓

Assume true for $n=k$

$\frac{5^k + 2(11^k)}{3} = m$ (an integer)

$5^k + 2(11^k) = 3m \Rightarrow 3m - 2(11^k) = 5^k$

Prove true for $n=k+1$ ie

$5^{k+1} + 2(11^{k+1})$ ✓

$= 5 \times 5^k + 22 \times 11^k$

$= 5(3m - 2(11^k)) + 22 \times 11^k$

$= 15m - 10(11^k) + 22 \times 11^k$

$= 15m + 12(11^k)$

$= 3(5m + 4(11^k))$ ✓

which is a multiple of 3

Hence if true for $n=k$, true for $n=k+1$

True for $n=1$, hence true for $n \geq 1$

b) i) initially $t=0$

then $N_A = 15 + 20e^0$

$= 35$ ✓

ii) $\frac{dN_B}{dt} = -20e^{-0.5t}$

when $t=0$

$\frac{dN_B}{dt} = -20$ ✓

iii) $N_A = N_B$ ie $15 + 20e^{-0.5t} = 5 + 40e^{-0.5t}$ ✓

$10 = 20e^{-0.5t}$

$-0.5t = \log_e(\frac{1}{2})$ (2)

$t = 2\log_e(2)$ ✓

$= 1.39$ min

c) i) $R = \sqrt{6^2 + (2\sqrt{3})^2}$

$= \sqrt{48}$ ✓

$= 4\sqrt{3}$

$R\sin(2t - \alpha) = 4\sqrt{3}\sin 2t\cos\alpha - 4\sqrt{3}\cos 2t\sin\alpha$

$= 6\sin 2t - 2\sqrt{3}\cos 2t$

$\therefore 4\sqrt{3}\cos\alpha = 6 \qquad -2\sqrt{3}\sin\alpha = -4\sqrt{3}$

$\cos\alpha = \frac{6}{4\sqrt{3}} \qquad \sin\alpha = \frac{1}{2}$ (2)

$\alpha = \frac{\pi}{6}$ ✓

ii) ie $x = 4\sqrt{3}\sin(2t - \frac{\pi}{6})$

$\dot{x} = 8\sqrt{3}\cos(2t - \frac{\pi}{6})$ ✓ (2)

$\ddot{x} = -16\sqrt{3}\sin(2t - \frac{\pi}{6})$

$= -16x$ which is in the form $\ddot{x} = -n$ ✓

iii) when $x = 2$

$4\sqrt{3}\sin(2t - \frac{\pi}{6}) = 2$

$\sin(2t - \frac{\pi}{6}) = \frac{1}{2\sqrt{3}}$ ✓

$2t - \frac{\pi}{6} = \sin^{-1}(\frac{1}{2\sqrt{3}})$

$t = \frac{\sin^{-1}(\frac{1}{2\sqrt{3}}) + \frac{\pi}{6}}{2}$ (2)

$= 0.40822...$ ✓

$= 0.41$ seconds. (must be in radians)

d) Let $\angle DAC = x$

then $\angle ADC = x$ Base $\angle$'s isosc $\triangle ADC$

then $\angle CAB = \angle ADC$ ($\angle$ in alt segment

$= x$

$\angle CAB = \angle ACD$ Alt $\angle$'s $AB \parallel EC$

$= x$

Also $\angle DEB = x$ ($=\angle$'s on chord DC

$\therefore ED \parallel AC$ (equal alt $\angle$'s) (2)

There are many variations on this proof

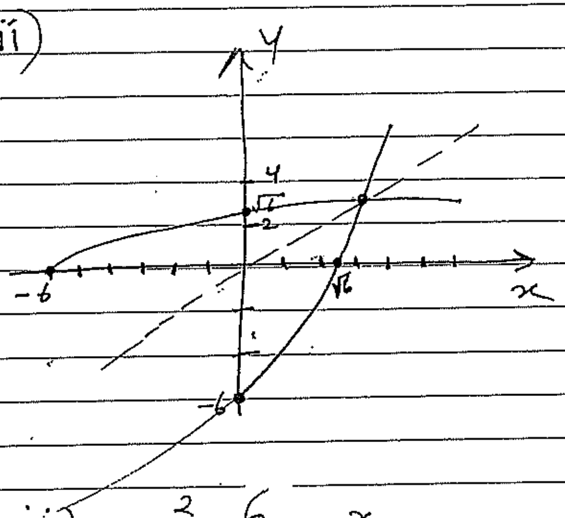
13) i) $y = \sqrt{x+6}$

$x = \sqrt{y+6}$

$y = x^2 - 6$

$D: x \geq 0$

ii)



iii) $x^2 - 6 = x$

$x^2 - x - 6 = 0$

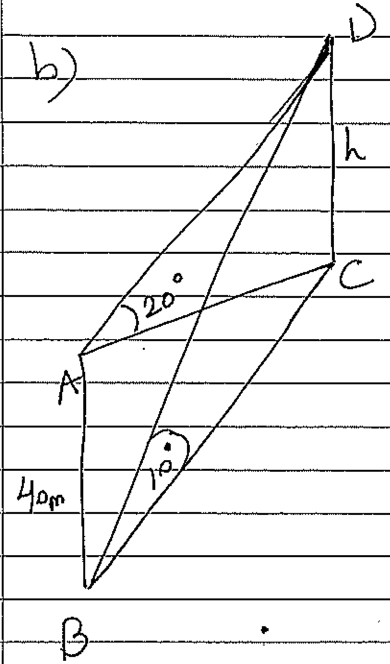
iv) $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad x \neq -2$

$y = 3 \quad P(3,3)$

b)



$\tan 20 = \frac{h}{AC}$

$\tan 10 = \frac{h}{BC}$

$AC^2 + AB^2 = BC^2$

$\left(\frac{h}{\tan 20}\right)^2 + 40^2 = \left(\frac{h}{\tan 10}\right)^2$

$40^2 = \frac{h^2}{(\tan 10)^2} - \frac{h^2}{(\tan 20)^2}$

$= \frac{h^2(\tan 20)^2 - h^2(\tan 10)^2}{\tan(20)^2(\tan 10)^2}$

$h^2\left[(\tan 20)^2 - (\tan 10)^2\right] = 40^2(\tan 10)^2(\tan$ [illegible]

$h = 8\ m$

c) i)

$$\frac{1}{2} 4 \times 4 \sin x - \frac{1}{2} \times 2^2 x = \frac{1}{16} \pi \times 16$$

$$8 \sin x - 2x = \pi$$

$$8 \sin x - 2x - \pi = 0$$

ii) $P(0.5) = 8 \sin(0.5) - 1 - \pi$

$= -0.306$

$P(0.6) = 8(\sin(0.6)) - 1.2 - \pi$

$= 0.1755$

$P(0.5) < 0$, $P(0.6) > 0$

$\therefore x = \alpha$ is a

solution

iii) $x = 0.6 - \frac{f(0.6)}{f'(0.6)}$

$= 0.6 - \frac{0.1755}{4.603}$

$= 0.56$

14) a)

$a = \frac{d}{dx} \frac{1}{2} v^2$

$\frac{1}{2} v^2 = \frac{1}{2}\left(\frac{32}{x} - \frac{x}{2}\right)^2$

$= \frac{1}{2}\left(\frac{1024}{x^2} - 32 + \frac{x^2}{4}\right)$

$= \frac{1024}{2x^2} - 16 + \frac{x^2}{8}$

$= \frac{512}{x^2} - 16 + \frac{x^2}{8}$

$a = -1024x^{-3} + \frac{x}{4}$

$= \frac{-1024}{x^3} + \frac{x}{4}$

ii) $v = \frac{dx}{dt}$

$v = \frac{32}{x} - \frac{x}{2}$

$= \frac{64 - x^2}{2x}$

$\frac{dt}{dx} = \frac{2x}{64 - x^2}$

$t = \int \frac{2x}{64 - x^2}$

$t = -\int \frac{-2x}{64 - x^2}$

$t = -\ln(64 - x^2) + C$

at $t = 0$ $x = 2$

$0 = -\ln(60) + C$

$C = \ln 60$

$t = -\ln(64 - x^2) + \ln 60$

$-t = \ln(64 - x^2) - \ln 60$

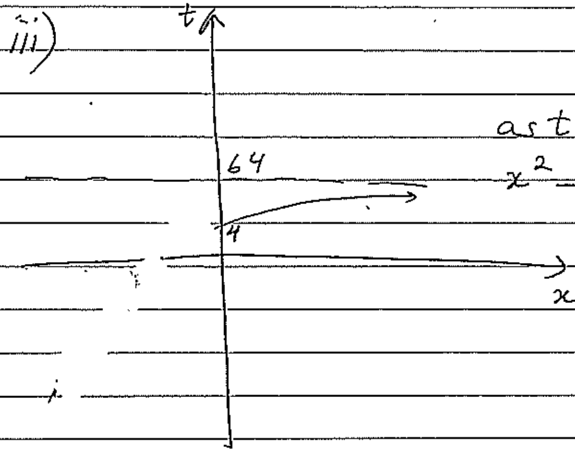
$-t = \ln\left(\frac{64 - x^2}{60}\right)$

$e^{-t} = \frac{64 - x^2}{60}$

$60e^{-t} = 64 - x^2$

$x^2 = 64 - 60e^{-t}$

iii)



as $t \to \infty$

$x^2 \to 64$

b) $M = \frac{-3 \times 2t + 1 \times 0}{-3 + 1}, \frac{-3t^2 + 1}{-3 + 1}$

$(x, y) = \frac{-6t}{-2}, \frac{-3t^2 + 1}{-2}$

$x = 3t \to t = \frac{x}{3}$

$y = \frac{+3t^2 - 1}{2}$

at $t = \frac{x}{3}$

$y = \frac{3\left(\frac{x}{3}\right)^2 - 1}{2} = \frac{\frac{3x^2}{9} - 1}{2}$

$y = \frac{3x^2 - 9}{18}$

$18y = 3x^2 - 9$

$6y = x^2 - 3$

$x^2 = 6y + 3$

c) i)

$$y_B = \frac{-gx^2}{2V^2}\sec^2\beta + x\tan\beta$$

ii) $$\frac{-gx^2}{2V^2}\sec^2\beta + x\tan\beta = \frac{-gx^2}{2V^2}\sec^2\alpha + x\tan\alpha$$

$$\frac{-gx^2}{2V^2}\sec^2\beta + \frac{gx^2}{2V^2}\sec^2\alpha = x\tan\alpha - x\tan\beta$$

$$\frac{gx^2}{2V^2}\left(\sec^2\alpha - \sec^2\beta\right) = x\left(\tan\alpha - \tan\beta\right)$$

$$\frac{gx}{2V^2}\left(1+\tan^2\alpha - 1 - \tan^2\beta\right) = \tan\alpha - \tan\beta$$

$$\frac{gx}{2V^2}\left(\tan\alpha - \tan\beta\right)\left(\tan\alpha + \tan\beta\right) = \tan\alpha - \tan\beta$$

$$\frac{gx}{2V^2} = \frac{1}{\tan\alpha + \tan\beta}$$

$$x = \frac{2V^2}{g}\cdot\frac{1}{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}$$

$$= \frac{2V^2}{g}\cdot\frac{1}{\frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta}}$$

$$= \frac{2V^2\cos\alpha\cos\beta}{g\sin(\alpha+\beta)}$$

iii) $$x_B = V(t-T)\cos\beta$$

iv) $$Vt\cos\alpha = \frac{2V^2\cos\alpha\cos\beta}{g\sin(\alpha+\beta)}$$

$$V(t-T)\cos\beta = \frac{2V^2\cos\alpha\cos\beta}{g\sin(\alpha+\beta)}$$

From ①

$$t = \frac{2V\cos\beta}{g\sin(\alpha+\beta)}$$

From ②

$$t - T = \frac{2V\cos\alpha}{g\sin(\alpha+\beta)}$$

$$T = \frac{2V\cos\beta}{g\sin(\alpha+\beta)} - \frac{2V\cos\alpha}{g\sin(\alpha+\beta)}$$

$$= \frac{2V(\cos\beta - \cos\alpha)}{g\sin(\alpha+\beta)}$$